

# Teaching Notes: Geometric Design

## Components

- *Application Question: Designing a Quilt*
  - To develop Competency 2: Uses mathematical reasoning
  - To develop Competency 3: Communicates by using mathematical language
  - Can be used after completing *Nelson Mathematics Secondary Year One, Cycle One* Chapters 5 and 7
- *Situational Problem: Old Into New*
  - To develop Competency 1: Solves a situational problem
  - Can be used after completing *Nelson Mathematics Secondary Year One, Cycle One* Chapters 5 and 6

## Broad Area of Learning: Environmental Awareness; Personal and Career Planning

### Educational Aim

- To enable students to use their mathematical abilities to demonstrate their understanding of environmental characteristics, the phenomena in the world around them or the interdependence of the environment and human activity
- To enable students to discover the role of mathematics in society by carrying out interdisciplinary projects involving related strategies and mathematical knowledge, while continuing to develop on a personal level

### Focus of Development

- To encourage students to develop an active relationship with their environment
- To enable students to use their skills to solve situational problems, making them aware of their potential and identity

## Cross-Curricular Competencies

- Uses information
- Solves problems
- Uses creativity
- Adopts effective work methods

## Concepts

### *Arithmetic: Operations Involving Numbers Written in Decimal and Fractional Notation*

- Written computation: the four operations involving numbers that are easy to work with (including large numbers) and sequences of simple operations performed in the proper order (numbers written in decimal notation), using equivalent ways of writing numbers and the properties of operations

### *Geometry: Geometric Figures and Spatial Sense*

#### – Plane figures

- Measurement
  - Perimeter, circumference
  - Area, lateral area, total area
- Congruent and similar figures

## Processes

### *Geometry*

- Geometric transformations
  - Translation, rotation, reflection
- Finding unknown measurements
  - Lengths
    - Perimeter of a plane figure
  - Areas
    - Area of polygons that can be split into triangles and quadrilaterals

## Application Question: Designing a Quilt

<b>Preparation and Planning</b>	
Pacing	10–15 min    Introduction 30–45 min    Individual work
Materials	<ul style="list-style-type: none"> <li>• a ruler</li> <li>• scissors</li> <li>• extra paper for tracing and cutting</li> </ul>
Masters	<ul style="list-style-type: none"> <li>• Designing a Quilt</li> </ul>
Can be done after completing	<i>Nelson Mathematics Secondary Year One, Cycle One</i> Chapters 5 and 7

### Introduction (Whole Class) 10–15 min

Discuss different geometric characteristics such as translations, rotations, and reflections. Ask students to identify such characteristics in the things around them. Tell students how these same characteristics are often used in art to create an appealing look.

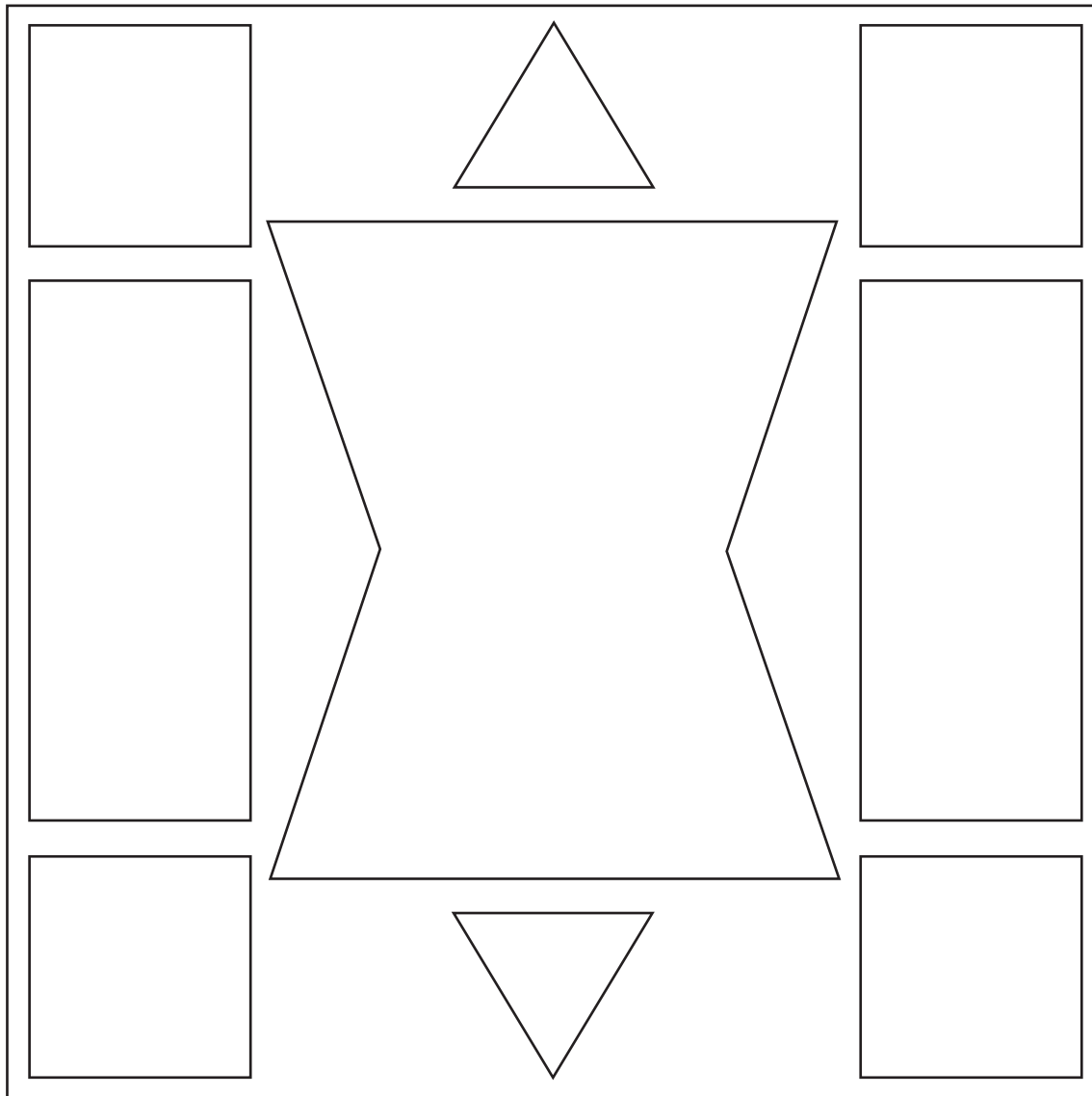
### Using the Application Question (Individual) 30–45 min

As a class, read the introduction and criteria for Designing a Quilt together. Also, remind students to use the Evaluation Criteria Checklist to help them complete the activity.

If students are having difficulty . . .	What you can do to help
Students may have difficulty designing their quilt square to include a transformation.	<ul style="list-style-type: none"> <li>• Suggest that students start with a blank square and focus on one type of transformation making sure that every part of the design that follows does not interfere with that transformation.</li> </ul>

## Sample Solution (Thorough)

A. For example,



B. For example, I started by determining the side length for the quilt square. The square root of 225 is 15, so I drew a 15 cm by 15 cm square. I formed my complex shape by combining two congruent isosceles trapezoids at the short base. I placed this shape centred vertically and horizontally within the large square. The complex shape has a perimeter of about 35 cm; I measured the side lengths and added them to determine the perimeter. The complex shape has an area of about  $56 \text{ cm}^2$ . I also estimated this once I drew the shape, but to make sure that the complex shape would not have an area greater than  $100 \text{ cm}^2$ , I made sure that it was no taller than 10 cm and no wider than 10 cm. My design includes four congruent squares, two congruent triangles, and two congruent rectangles. I placed each of these shapes so that the whole design could be reflected or rotated and still produce the same design. The design can be reflected across a horizontal line through the centre of the square or reflected across a vertical line through the centre of the square. The design can also be rotated through  $180^\circ$  cw or ccw and be identical to the original design.

## Assessment of Learning: Designing a Quilt

Level	Competency	Overall judgment at end of cycle
5	Advanced	The student's competency <b>exceeds</b> the requirements.
4	Thorough	The student's competency <b>clearly meets</b> the requirements.
3	Acceptable	The student's competency <b>barely meets</b> the requirements.
2	Partial	The student's competency <b>fails to meet</b> the requirements.
1	Minimal	The student's competency <b>clearly fails to meet</b> the requirements.

Competency 2: Uses mathematical reasoning	
Evaluation criteria for the competency: Uses mathematical reasoning	Observable elements The student . . .
CR 3- Proper application of mathematical reasoning suited to the situation	<ul style="list-style-type: none"> <li>determines measurements for the complex shape directly or provides estimates that show that actual measurements are not necessary</li> <li>provides justification for their choices and statements</li> </ul>
CR 2- Correct use of concepts and processes appropriate to the situation	<ul style="list-style-type: none"> <li>uses estimation or actual measurements to justify satisfaction of perimeter and area criteria</li> <li>makes use of rotation, reflection, or translation in designing the quilt square</li> </ul>
CR 4- Proper organization of the steps in an appropriate procedure shows his/her work in a clear and organized manner	<ul style="list-style-type: none"> <li>correctly draws a quilt square matching all of the given criteria</li> <li>shows his/her work or estimates for the measurements of the complex shape in a clear and organized manner</li> </ul>
CR 5- Correct justification of the steps in an appropriate procedure	<ul style="list-style-type: none"> <li>makes statements that justify his/her meeting all the given criteria</li> <li>uses solid mathematical arguments in explanation of determining that the perimeter and the area of the complex shape were within the required range</li> </ul>
CR 1- Formulation of a conjecture	

<b>Competency 2: Uses mathematical reasoning</b>					
<b>Evaluation Criteria</b>	<b>Advanced</b>	<b>Thorough</b>	<b>Acceptable</b>	<b>Partial</b>	<b>Minimal</b>
<b>Formulation of a conjecture appropriate to the situation</b>					
<b>Correct use of the concepts and processes appropriate to the situation</b>	<ul style="list-style-type: none"> <li>• chose appropriate geometrical concepts and applied the chosen geometrical concepts appropriately</li> </ul>	<ul style="list-style-type: none"> <li>• chose appropriate geometrical concepts and applied the geometrical concepts appropriately, but made minor errors</li> </ul>	<ul style="list-style-type: none"> <li>• chose some appropriate geometrical concepts and applied the chosen concepts, but made some conceptual errors</li> </ul>	<ul style="list-style-type: none"> <li>• chose few appropriate geometrical concepts and applied them, but made several conceptual errors</li> </ul>	<ul style="list-style-type: none"> <li>• chose inappropriate geometrical concepts and/or applied geometrical concepts inappropriately, making many conceptual errors</li> </ul>
<b>Proper application of mathematical reasoning suited to the situation</b>	<ul style="list-style-type: none"> <li>• used efficient strategies in applying his/her mathematical reasoning to the design of the quilt square</li> </ul>	<ul style="list-style-type: none"> <li>• used effective strategies in applying his/her mathematical reasoning to the design of the quilt square</li> </ul>	<ul style="list-style-type: none"> <li>• used a few effective strategies for certain steps in applying his/her mathematical reasoning to the design of the quilt square</li> </ul>	<ul style="list-style-type: none"> <li>• used few appropriate strategies in applying his/her mathematical reasoning to the design of the quilt square</li> </ul>	<ul style="list-style-type: none"> <li>• used inappropriate strategies in applying his/her mathematical reasoning to the design of the quilt square</li> </ul>
<b>Proper organization of the steps in an appropriate procedure</b>	<ul style="list-style-type: none"> <li>• presented a complete and organized procedure that clearly outlines how the quilt square was planned and created</li> </ul>	<ul style="list-style-type: none"> <li>• presented a clear and organized procedure for how the quilt square was planned and created although some of the steps were not fully explained</li> </ul>	<ul style="list-style-type: none"> <li>• presented a procedure for how the quilt square was planned and created that was unclear or for which some steps were not fully explained</li> </ul>	<ul style="list-style-type: none"> <li>• presented little procedure for how the quilt square was planned and created</li> </ul>	<ul style="list-style-type: none"> <li>• did not present any procedure for how the quilt square was planned and created</li> </ul>
<b>Correct justification of the steps in an appropriate procedure</b>	<ul style="list-style-type: none"> <li>• rigorously observed the rules and conventions of mathematical language in the explanation about meeting all criteria</li> </ul>	<ul style="list-style-type: none"> <li>• used appropriate mathematical language in the explanation about meeting all criteria</li> </ul>	<ul style="list-style-type: none"> <li>• used some appropriate or rudimentary mathematical language in the explanation about meeting all criteria</li> </ul>	<ul style="list-style-type: none"> <li>• used only slightly appropriate mathematical language in the explanation about meeting all criteria</li> </ul>	<ul style="list-style-type: none"> <li>• used erroneous or inappropriate mathematical language in the explanation about meeting all criteria</li> </ul>

<b>Competency 3: Communicates using mathematical language</b>	
<b>Evaluation criteria for the competency: Communicates by using mathematical language</b>	<b>Observable elements The student . . .</b>
CR 1- Correct interpretation of a message involving at least one type of mathematical representation suited to the situation	<ul style="list-style-type: none"> <li>• recognizes the purpose of the message</li> <li>• states valid explanation of how to produce a quilt square that can be described using translations, rotations, or reflections</li> </ul>
CR 2- Production of a message suited to the context, using appropriate mathematical terminology and following mathematical rules and conventions	<ul style="list-style-type: none"> <li>• accurately draws a quilt square that meets the given criteria</li> <li>• provides accurate measurements, estimates, and calculations to justify meeting area and perimeter criteria</li> </ul>

<b>Competency 3: Communicates by using mathematical language</b>					
<b>Evaluation Criteria</b>	<b>Advanced</b>	<b>Thorough</b>	<b>Acceptable</b>	<b>Partial</b>	<b>Minimal</b>
<b>Correct interpretation of a message involving at least one type of mathematical representation suited to the situation</b>	<ul style="list-style-type: none"> <li>• used appropriate elements of mathematical language and everyday language to clearly describe the geometric characteristics of his/her quilt square and how they were utilized</li> </ul>	<ul style="list-style-type: none"> <li>• used elements of mathematical language and everyday language to describe the geometric characteristics of his/her quilt square and how they were utilized</li> </ul>	<ul style="list-style-type: none"> <li>• used some appropriate elements of mathematical language and everyday language to describe the geometric characteristics of his/her quilt square and how they were utilized</li> </ul>	<ul style="list-style-type: none"> <li>• used a few appropriate elements of mathematical language and everyday language to describe the geometric characteristics of his/her quilt square and how they were utilized</li> </ul>	<ul style="list-style-type: none"> <li>• used inappropriate elements of mathematical language and everyday language to describe the geometric characteristics of his/her quilt square and how they were utilized</li> </ul>
<b>Production of a message suited to the context, using appropriate mathematical terminology and following mathematical rules and conventions</b>	<ul style="list-style-type: none"> <li>• produced an articulate, coherent, message that includes all relevant information in his/her explanations</li> </ul>	<ul style="list-style-type: none"> <li>• produced a clear, well-organized message that includes the relevant information in his/her explanation</li> </ul>	<ul style="list-style-type: none"> <li>• included elementary, ambiguous, or repetitive information in his/her explanations</li> </ul>	<ul style="list-style-type: none"> <li>• included confusing and unconnected information in his/her explanations</li> </ul>	<ul style="list-style-type: none"> <li>• included erroneous or unrelated information in his/her explanations</li> </ul>

## Situational Problem: Old Into New

<b>Preparation and Planning</b>	
Pacing	15–20 min    Introduction 45–60 min    Individual work
Materials	<ul style="list-style-type: none"><li>• grid paper</li><li>• a ruler</li></ul>
Masters	<ul style="list-style-type: none"><li>• Old Into New</li></ul>
Can be done after completing	<i>Nelson Mathematics Secondary Year One, Cycle One</i> Chapters 5 and 6

### Introduction (Whole Class) 15–20 min

Discuss with your class the importance of reusing or recycling materials. Ask students why large amounts of landfill wastes are harmful to the environment.

### Using the Situational Problem (Individual) 45–60 min

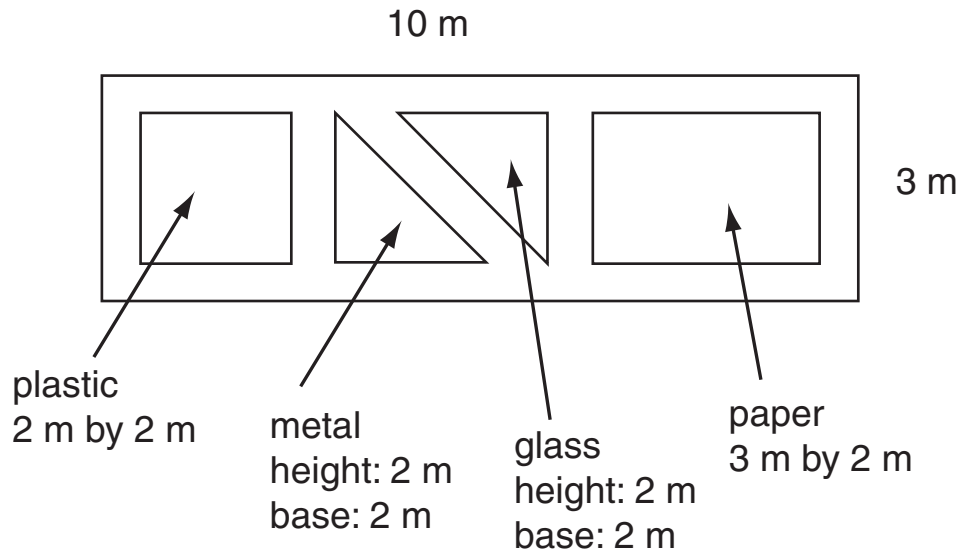
Together, read the introduction and the criteria for Old Into New. Remind students to use the Evaluation Criteria Checklist to help them complete the activity.

If students are having difficulty . . .	What you can do to help
Students may have difficulty determining acceptable dimensions for the concrete pad.	<ul style="list-style-type: none"><li>• Suggest that students focus first on the area of the pad and then check which dimensions that fit the area criterion also satisfy the perimeter criterion.</li></ul>

### Sample Solution (Thorough)

- A. For example, I will first determine the size of the rectangular area where the concrete will be poured. I will determine dimensions that satisfy the area and perimeter conditions. Then I will begin determining appropriate shapes and sizes for the four collection bins. After I have determined the size of the concrete pad and the size and shape of the collection bins, I will check that the amount of concrete not covered by the collection bins is  $20 \text{ m}^2$  or less. I will probably use polygons for the shape of the collection bins, rather than a complex shape; this will make it easier to check that the amount of concrete that is not covered is under the limit.

B. For example,



- C. For example, I found that it was good to determine the size of the concrete base first. Because there were no criteria about the size of the collection bins, except that the bin for paper needed to be the largest, it was easy to focus on the size of the concrete and then determine appropriate sizes for the collection bins. I did follow my plan. I tried several different possibilities for the size of the concrete before deciding on one and then tried several different shapes and arrangements for the collection bins before deciding on the one that I drew.
- D. The concrete pad that I decided on is 10 m by 3 m. So the perimeter of the concrete pad is 26 m, which is less than 40 m, as required. The area of the concrete pad is  $30 \text{ m}^2$ , which is less than  $50 \text{ m}^2$ , as required. To determine the amount of concrete that is not covered by the recycling collection bins, I must first determine the area of each of the bins. The bins for paper and plastic are rectangular, so the area for each is found by multiplying the length and width. The area for the plastic bin is  $4 \text{ m}^2$ ; the area for the paper bin is  $6 \text{ m}^2$ . The bins for metal and glass are congruent triangles, each with a base and height of 2 m. The area of each is  $\frac{1}{2} \times 2 \times 2$ , or  $2 \text{ m}^2$ . The total area of the bins is  $4 + 6 + 2 + 2$ , or  $14 \text{ m}^2$ . This means that the amount of concrete not covered by the bins is  $30 - 14$ , or  $16 \text{ m}^2$ , which is less than  $20 \text{ m}^2$ , as required. I could have used less concrete and fencing without changing the size of the bins by reducing the 1.5 m space between the bins and the edge of the concrete; I included this space to make it practical for people to get in to deposit their recyclable material. I could have also used a concrete pad with different dimensions. For example, a concrete pad 5 m by 5 m would have an area of  $25 \text{ m}^2$ , less than in my solution, and a perimeter of 20 m, also less than in my solution. However, this would have required me to use different bin shapes and/or sizes.

## Assessment of Learning: Old Into New

<b>Competency 1: Solves a situational problem</b>					
<b>Evaluation Criteria</b>	<b>Advanced</b>	<b>Thorough</b>	<b>Acceptable</b>	<b>Partial</b>	<b>Minimal</b>
<b>Oral or written explanation showing that the student understands the situational problem</b>	<ul style="list-style-type: none"> <li>took all of the criteria into account in designing the bins and their arrangement</li> </ul>	<ul style="list-style-type: none"> <li>took most of the criteria into account in designing the bins and their arrangement</li> </ul>	<ul style="list-style-type: none"> <li>took some of the criteria into account in designing the bins and their arrangement</li> </ul>	<ul style="list-style-type: none"> <li>identified some of the criteria but was unable to use it in designing the bins and their arrangement</li> </ul>	<ul style="list-style-type: none"> <li>took no more than one of the criteria into account in designing the bins and their arrangement</li> </ul>
<b>Mobilization of mathematical knowledge appropriate to the situational problem</b>	<ul style="list-style-type: none"> <li>presented a complete design for the bins and their arrangement in accordance with the given criteria</li> </ul>	<ul style="list-style-type: none"> <li>presented a design for the bins and their arrangement with few errors related to the given criteria</li> </ul>	<ul style="list-style-type: none"> <li>presented a design for the bins and their arrangement with errors related to the given criteria</li> </ul>	<ul style="list-style-type: none"> <li>presented a partial design for the bins and their arrangement with consideration of only the easiest given criteria</li> </ul>	<ul style="list-style-type: none"> <li>presented a partial design for the bins and their arrangement with no consideration of the given criteria</li> </ul>
<b>Development of a solution (i.e. a procedure and a final answer) appropriate to the situational problem</b>	<ul style="list-style-type: none"> <li>checked his/her solution and corrected it, if necessary</li> </ul>	<ul style="list-style-type: none"> <li>checked the main steps involved in his/her procedure for completing the given task</li> </ul>	<ul style="list-style-type: none"> <li>checked some of the steps involved in his/her procedure for completing the given task</li> </ul>	<ul style="list-style-type: none"> <li>made little attempt to reflect on his/her procedure for completing the given task</li> </ul>	<ul style="list-style-type: none"> <li>did not reflect on his/her procedure for completing the given task</li> </ul>

<b>Cross-Curricular Competencies</b>					
<b>Evaluation Criteria</b>	<b>Advanced</b>	<b>Thorough</b>	<b>Acceptable</b>	<b>Partial</b>	<b>Minimal</b>
<b>Cross-Curricular Competency 1: Uses information</b>					
<b>Critical analysis of information</b>	<ul style="list-style-type: none"> <li>a clear explanation was provided as to how the design for the bins and their arrangement was created</li> </ul>	<ul style="list-style-type: none"> <li>an explanation was provided as to how the design for the bins and their arrangement was created</li> </ul>	<ul style="list-style-type: none"> <li>some explanation was provided as to how the design for the bins and their arrangement was created</li> </ul>	<ul style="list-style-type: none"> <li>little explanation was provided as to how the design for the bins and their arrangement was created</li> </ul>	<ul style="list-style-type: none"> <li>no explanation was provided as to how the design for the bins and their arrangement was created</li> </ul>
<b>Logical organization of information</b>	<ul style="list-style-type: none"> <li>explanations are clearly structured, well organized, and include all steps</li> </ul>	<ul style="list-style-type: none"> <li>explanations are structured and organized</li> </ul>	<ul style="list-style-type: none"> <li>explanations are somewhat clear and organized but some errors were made</li> </ul>	<ul style="list-style-type: none"> <li>explanations are not very clear or organized</li> </ul>	<ul style="list-style-type: none"> <li>explanations are not clearly organized and were difficult to understand</li> </ul>

<b>Evaluation Criteria</b>	<b>Advanced</b>	<b>Thorough</b>	<b>Acceptable</b>	<b>Partial</b>	<b>Minimal</b>
<b>Cross-Curricular Competency 2: Solves problems</b>					
<b>Accurate definition of the problem</b>	<ul style="list-style-type: none"> <li>• student shows a clear understanding of the problem presented and clearly demonstrates this in the explanation</li> </ul>	<ul style="list-style-type: none"> <li>• student shows a clear understanding of the problem presented</li> </ul>	<ul style="list-style-type: none"> <li>• student shows a somewhat clear understanding of the problem presented</li> </ul>	<ul style="list-style-type: none"> <li>• student shows a lack of complete understanding of the problem presented</li> </ul>	<ul style="list-style-type: none"> <li>• student does not show any understanding of the problem presented</li> </ul>
<b>Variety and relevance of solutions proposed</b>	<ul style="list-style-type: none"> <li>• an accurate solution is proposed for the problem presented with a clear explanation provided</li> </ul>	<ul style="list-style-type: none"> <li>• an accurate solution is proposed for the problem presented</li> </ul>	<ul style="list-style-type: none"> <li>• a solution is proposed for the problem presented containing some errors</li> </ul>	<ul style="list-style-type: none"> <li>• an inaccurate solution is proposed for the problem presented</li> </ul>	<ul style="list-style-type: none"> <li>• no solution is proposed for the problem presented</li> </ul>
<b>Evaluation of possible strategies</b>	<ul style="list-style-type: none"> <li>• an accurate evaluation of the plan is provided and alternative strategies and steps were discussed</li> </ul>	<ul style="list-style-type: none"> <li>• an accurate evaluation of the plan is provided</li> </ul>	<ul style="list-style-type: none"> <li>• an evaluation of the plan is provided but it lacked several details</li> </ul>	<ul style="list-style-type: none"> <li>• an unclear evaluation of a plan is provided</li> </ul>	<ul style="list-style-type: none"> <li>• a poor evaluation of a plan is provided</li> </ul>
<b>Scope of the analysis</b>	<ul style="list-style-type: none"> <li>• an accurate analysis of the plan was performed and the explanation details specific uses of this information</li> </ul>	<ul style="list-style-type: none"> <li>• an accurate analysis of the plan was performed</li> </ul>	<ul style="list-style-type: none"> <li>• an analysis of the plan was performed but contained some errors</li> </ul>	<ul style="list-style-type: none"> <li>• an inaccurate analysis of the plan was performed</li> </ul>	<ul style="list-style-type: none"> <li>• no analysis of the plan was performed</li> </ul>
<b>Application of strategies developed in other situations</b>	<ul style="list-style-type: none"> <li>• there is clear evidence of the use of strategies developed previously with a detailed explanation of their possible use</li> </ul>	<ul style="list-style-type: none"> <li>• there is clear evidence of the use of strategies developed previously</li> </ul>	<ul style="list-style-type: none"> <li>• there is evidence of the use of strategies developed previously</li> </ul>	<ul style="list-style-type: none"> <li>• there is a small amount of evidence of the use of strategies developed previously</li> </ul>	<ul style="list-style-type: none"> <li>• there is no evidence of the use of strategies developed previously</li> </ul>

Evaluation Criteria	Advanced	Thorough	Acceptable	Partial	Minimal
<b>Cross-Curricular Competency 4: Uses creativity</b>					
<b>Exploration of new ideas</b>	<ul style="list-style-type: none"> <li>there is evidence of the exploration of new ideas in the design for the bins and their arrangement and these ideas are clearly expressed in the explanation</li> </ul>	<ul style="list-style-type: none"> <li>there is evidence of the exploration of new ideas in the design for the bins and their arrangement</li> </ul>	<ul style="list-style-type: none"> <li>there is some evidence of the exploration of new ideas in the design for the bins and their arrangement</li> </ul>	<ul style="list-style-type: none"> <li>there is little evidence of the exploration of new ideas in the design for the bins and their arrangement</li> </ul>	<ul style="list-style-type: none"> <li>there is no evidence of the exploration of new ideas in the design for the bins and their arrangement</li> </ul>
<b>Exploration of different ways of doing things</b>	<ul style="list-style-type: none"> <li>there is evidence of the exploration of different ways of doing things in the design for the bins and their arrangement and these ways are clearly expressed in the explanation</li> </ul>	<ul style="list-style-type: none"> <li>there is evidence of the exploration of different ways of doing things in the design for the bins and their arrangement</li> </ul>	<ul style="list-style-type: none"> <li>there is some evidence of the exploration of different ways of doing things in the design for the bins and their arrangement</li> </ul>	<ul style="list-style-type: none"> <li>there is little evidence of the exploration of different ways of doing things in the design for the bins and their arrangement</li> </ul>	<ul style="list-style-type: none"> <li>there is no evidence of the exploration of different ways of doing things in the design for the bins and their arrangement</li> </ul>
<b>Originality of connections among the elements of a situation</b>	<ul style="list-style-type: none"> <li>there is originality in the design for the bins and their arrangement and this is clearly expressed in the explanation</li> </ul>	<ul style="list-style-type: none"> <li>there is originality in the design for the bins and their arrangement</li> </ul>	<ul style="list-style-type: none"> <li>there is some originality in the design for the bins and their arrangement</li> </ul>	<ul style="list-style-type: none"> <li>there is little originality in the design for the bins and their arrangement</li> </ul>	<ul style="list-style-type: none"> <li>there is no originality in the design for the bins and their arrangement</li> </ul>
<b>Cross-Curricular Competency 5: Adopts effective work methods</b>					
<b>Appropriate choice of methods</b>	<ul style="list-style-type: none"> <li>computed area of concrete not covered indirectly by subtracting total area of bins from total area of concrete and recognized that the two triangles combine to form a square</li> </ul>	<ul style="list-style-type: none"> <li>computed area of concrete not covered indirectly by subtracting total area of bins from total area of concrete</li> </ul>	<ul style="list-style-type: none"> <li>computed area of concrete not covered directly by dividing uncovered area into smaller shapes</li> </ul>	<ul style="list-style-type: none"> <li>attempted to compute area directly or indirectly but made some errors in computation</li> </ul>	<ul style="list-style-type: none"> <li>attempted to compute area directly or indirectly but made multiple errors in computation or made conceptual errors about relationship between covered and uncovered areas</li> </ul>