CHAPTER 2: ANALYTIC GEOMETRY: LINE SEGMENTS AND CIRCLES

Specific Expectations Addressed in the Chapter

- Develop the formula for the midpoint of a line segment, and use this formula to solve problems (e.g., determine the coordinates of the midpoints of the sides of a triangle, given the coordinates of the vertices, and verify concretely or by using dynamic geometry software). **[2.1]**
- Develop the formula for the length of a line segment, and use this formula to solve problems (e.g., determine the lengths of the line segments joining the midpoints of the sides of a triangle, given the coordinates of the vertices of the triangle, and verify using dynamic geometry software). **[2.2]**
- Develop the equation for a circle with centre (0, 0) and radius *r*, by applying the formula for the length of a line segment. **[2.3]**
- Determine the radius of a circle with centre (0, 0), given its equation; write the equation of a circle with centre (0, 0), given the radius; and sketch the circle, given the equation in the form $x^2 + y^2 = r^2$. **[2.3]**
- Solve problems involving the slope, length, and midpoint of a line segment (e.g., determine the equation of the right bisector of a line segment, given the coordinates of the endpoints; determine the distance from a given point to a line whose equation is given, and verify using dynamic geometry software). **[2.1, 2.2, 2.4, 2.5, 2.7, Chapter Task]**
- Determine, through investigation (e.g., using dynamic geometry software, by paper folding), some characteristics and properties of geometric figures (e.g., medians in a triangle, similar figures constructed on the sides of a right triangle). **[2.6]**
- Verify, using algebraic techniques and analytic geometry, some characteristics of geometric figures (e.g., verify that two lines are perpendicular, given the coordinates of two points on each line; verify, by determining side length, that a triangle is equilateral, given the coordinates of the vertices). **[2.4, 2.5, 2.7]**
- Plan and implement a multi-step strategy that uses analytic geometry and algebraic techniques to verify a geometric property (e.g., given the coordinates of the vertices of a triangle, verify that the line segment joining the midpoints of two sides of the triangle is parallel to the third side and half its length, and check using dynamic geometry software; given the coordinates of the vertices of a rectangle, verify that the diagonals of the rectangle bisect each other). **[2.4, 2.5, 2.7]**

Prerequisite Skills Needed for the Chapter

- Apply a translation to a point or figure.
- Determine the equation of a linear relation, given two points or one point and the slope. Apply the slope formula.
- Construct the perpendicular bisector of a line segment.
- Draw a line on a coordinate grid, given its equation, or its slope and one point.
- Understand and apply the Pythagorean theorem.
- Construct a line through a point so that the line intersects and is perpendicular to another line.
- Solve a system of equations by substitution or elimination.
- Determine whether two lines are parallel, perpendicular, or neither.
- Determine the area of a triangle.
- Classify triangles and quadrilaterals.

What "big ideas" should students develop in this chapter?

Students who have successfully completed the work of this chapter and who understand the essential concepts and procedures will know the following:

- The coordinates of the midpoint of a line segment are the means of the coordinates of the endpoints of the line segment, as in the formula $(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.
- The distance between two points, or the length of a line segment, can be determined using the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- A circle with radius r and its centre at the origin can be described by the equation $x^2 + y^2 = r^2$.
- Triangles and quadrilaterals can be classified using properties of their sides, if the coordinates of their vertices are known.
- The perpendicular bisector of a line segment, a median or altitude of a triangle, or a midsegment of a triangle or quadrilateral can be constructed if the coordinates of the endpoints or vertices are known.
- Medians, altitudes, and perpendicular bisectors of triangles, and chords of circles, intersect with certain properties.

Chapter 2: Planning Chart

Lesson Title	Lesson Goal	Pacing 13 days	Materials/Masters Needed
Getting Started, pp. 68–71	Use concepts and skills developed prior to this chapter.	2 days	grid paper, ruler, and protractor, or dynamic geometry software; Diagnostic Test
Lesson 2.1: Midpoint of a Line Segment, pp. 72–80	Develop and use the formula for the midpoint of a line segment.	1 day	grid paper, ruler, and compass, or dynamic geometry software; Lesson 2.1 Extra Practice
Lesson 2.2: Length of a Line Segment, pp. 81–87	Determine the length of a line segment.	1 day	grid paper; ruler; Lesson 2.2 Extra Practice
Lesson 2.3: Equation of a Circle, pp. 88–93	Develop and use an equation for a circle.	1 day	graphing calculator; grid paper, ruler, compass, or dynamic geometry software; Lesson 2.3 Extra Practice
Lesson 2.4: Classifying Figures on a Coordinate Grid, pp. 96–103	Use properties of line segments to classify two-dimensional figures.	1 day	grid paper and ruler, or dynamic geometry software; Lesson 2.4 Extra Practice
Lesson 2.5: Verifying Properties of Geometric Figures, pp. 104–110	Use analytic geometry to verify properties of geometric figures.	1 day	grid paper and ruler, or dynamic geometry software; Lesson 2.5 Extra Practice
Lesson 2.6: Exploring Properties of Geometric Figures, pp. 111–114	Investigate intersections of lines or line segments within triangles and circles.	1 day	grid paper and ruler, or dynamic geometry software
Lesson 2.7: Using Coordinates to Solve Problems, pp. 115–121	Use properties of lines and line segments to solve problems.	1 day	grid paper; ruler; Lesson 2.7 Extra Practice
Mid-Chapter Review, pp. 94–95 **Chapter Review**, pp. 122–125 **Chapter Self-Test**, p. 126 **Curious Math**, p. 114 **Chapter Task**, p. 127		4 days	Mid-Chapter Review Extra Practice; Chapter Review Extra Practice; Chapter Test

CHAPTER OPENER

Using the Chapter Opener

Introduce the chapter by discussing the photographs on pages 66 and 67 of the Student Book.

Both photographs show famous architectural structures located in France: the Eiffel Tower in Paris and a Roman aqueduct, called the Pont du Gard, near Nîmes. Both structures have line segments and circular arches. Ask students how they might create a coordinate grid so that the centre of the circle forming one of the arches in the aqueduct is at the origin. Ask: How could you then determine the coordinates of the endpoints and the highest point of the arch?

Direct students to the question on page 67. Then ask students what other questions they could pose about the structures (such as the lengths of line segments in the tower, or the outer radii of the arcs made by the angled bricks in the aqueduct).

Discuss the central idea of this chapter: to determine facts about line segments and circles algebraically and then use these facts to determine properties of geometric figures. For example, the second stage of the Eiffel Tower is approximately the shape of a quadrilateral, but what type of quadrilateral? Encourage students to think about how to use coordinate geometry to answer questions like this as they work through the chapter.

GETTING STARTED

Using the Words You Need to Know

Ask students to share their knowledge about the terms, as well as their strategies for matching the terms to the diagrams. Remind students that measuring is not necessary when equal side lengths are marked with ticks.

Using the Skills and Concepts You Need

Discuss the examples as a class. Refer students to the Study Aid chart in the Student Book. Have students work on the Practice questions in class and then complete any unfinished questions for homework.

Using the Applying What You Know

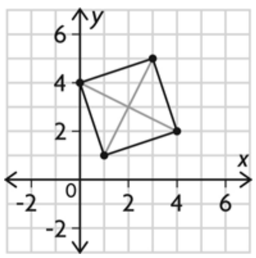
Arrange students in pairs. Ask them to read through the whole activity before they begin. As they work, prompt them, when necessary, to remember what they have learned about lines that are perpendicular. After students have finished, have a class discussion about the lengths of the diagonals, the angle relationships they observed, and their ideas for part H.

Student Book Pages 68–71

Preparation and Planning	
Pacing	
5–10 min	**Words You Need to Know**
40–45 min	**Skills and Concepts You Need**
45–55 min	**Applying What You Know**
Materials	
▪ grid paper, ruler, and protractor, or dynamic geometry software	
Nelson Website	
http://www.nelson.com/math	

Answers to Applying What You Know

A., C. Answers may vary, e.g., similar to the diagram at the right



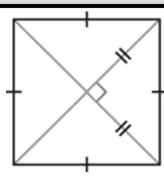
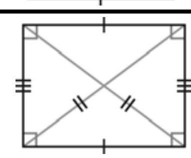
B. Answers may vary, e.g., all sides measure about 3.2 cm; all angles measure 90°.

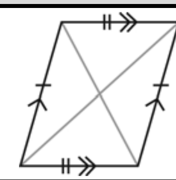
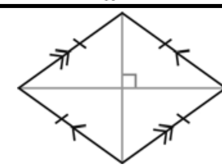
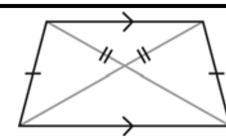
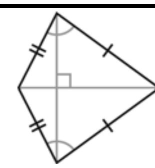
C. i) Answers may vary, e.g., both diagonals measure about 4.5 cm.
ii) 90°, 90°

D. They are congruent and perpendicular.

E. All sides are congruent, and the diagonals are congruent and perpendicular for all the squares.

F.–G.

Type of Quadrilateral	Side Relationships	Interior Angle Relationships	Diagonal Relationships	Relationship of Angles Formed by Intersecting Diagonals	Diagram
square	all sides congruent; adjacent sides perpendicular	all 90°	congruent	all angles 90°	
rectangle	adjacent sides perpendicular; opposite sides parallel and congruent	all 90°	congruent	congruent opposite angles	

Type of Quadrilateral	Side Relationships	Interior Angle Relationships	Diagonal Relationships	Relationship of Angles Formed by Intersecting Diagonals	Diagram
parallelogram	opposite sides parallel and congruent	opposite interior angles congruent	not congruent	congruent opposite angles	
rhombus	all sides congruent; opposite sides parallel	opposite interior angles congruent	not congruent	all angles 90°	
isosceles trapezoid	one pair of opposite sides parallel; other pair of opposite sides congruent	two pairs of adjacent interior angles congruent	congruent	congruent opposite angles	
kite	two pairs of adjacent sides congruent	one pair of opposite interior angles congruent	not congruent	all angles 90°	

H. i) In a rhombus that is not a square, adjacent sides are not perpendicular and the diagonals are not congruent.

ii) In a rectangle that is not a square, adjacent sides are not congruent and the diagonals are not perpendicular.

iii) In a parallelogram that is not a rhombus, adjacent sides are not congruent and the diagonals are not perpendicular.

iv) In a kite that is not a rhombus, two pairs of adjacent sides are congruent and two opposite interior angles are congruent.

Initial Assessment	What You Will See Students Doing ...
When students understand ...	**If students misunderstand ...**
Students check that sides are equal lengths and interior angles are 90º, ensuring that adjacent sides are perpendicular.	Students may not recognize that adjacent sides are perpendicular when interior angles are 90º.
Students construct diagonals as line segments connecting non-adjacent vertices.	Students may not know how to construct diagonals.
Students understand that diagonals are perpendicular if they meet at 90º.	Students may not realize that diagonals are perpendicular if they meet at 90º, or students may not draw the diagonals correctly.
Students correctly identify differences between types of quadrilaterals in terms of diagonal relationships.	Students may incorrectly identify differences in terms of diagonal relationships, or they may identify differences that do not involve diagonal relationships.

2.1 MIDPOINT OF A LINE SEGMENT
Lesson at a Glance

GOAL

Develop and use the formula for the midpoint of a line segment.

Prerequisite Skills/Concepts

- Apply a translation to a point or figure.
- Determine the equation of a linear relation, given two points or one point and the slope. Apply the slope formula.
- Construct the perpendicular bisector of a line segment.

Specific Expectations

- Develop the formula for the midpoint of a line segment, and use this formula to solve problems (e.g., determine the coordinates of the midpoints of the sides of a triangle, given the coordinates of the vertices, and verify concretely or by using dynamic geometry software).
- Solve problems involving the slope, [length,] and midpoint of a line segment (e.g., determine the equation of the right bisector of a line segment, given the coordinates of the endpoints; determine the distance from a given point to a line whose equation is given, and verify using dynamic geometry software).

Mathematical Process Focus

- Reasoning and Proving
- Selecting Tools and Computational Strategies
- Connecting

Student Book Pages 72–80

Preparation and Planning

Pacing

5 min	**Introduction**
15–20 min	**Teaching and Learning**
35–40 min	**Consolidation**

Materials
- grid paper, ruler, and compass, or dynamic geometry software

Recommended Practice
Questions 4, 5, 7, 12, 14, 15, 18

Key Assessment Question
Question 12

Extra Practice
Lesson 2.1 Extra Practice

Nelson Website
http://www.nelson.com/math

MATH BACKGROUND | LESSON OVERVIEW

- Students develop the formula for the midpoint of a line segment and investigate the reasoning for this formula:
  - If the endpoints are (x_1, y_1) and (x_2, y_2), the formula for the midpoint is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.
  - Each coordinate is the mean of the corresponding coordinates of the endpoints.
- Students use the coordinates of the midpoint and one endpoint of a line segment to determine the coordinates of the other endpoint.
- Students use their knowledge of geometry to solve problems that involve midpoints.

1 Introducing the Lesson

(5 min)

Show students two objects, such as two tacks, that could be endpoints of a line segment, or draw two points on the board. Have students show where they would draw a point halfway between the two objects or points. Ask: How did you decide? Encourage discussion about students' answers and reasoning. Repeat this a few times, including some pairs of points that are in a vertical line and other pairs of points that are in a horizontal line.

2 Teaching and Learning

(15 to 20 min)

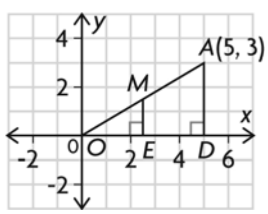
Investigate the Math

Read through the patio problem with the class at the beginning of the lesson. Ask questions such as these: Where will the centre of each semicircle be on the diameter? What information will help you determine the centre of the semicircle? Have students work in pairs and record their responses to the prompts.

Answers to Investigate the Math

Answers may vary, e.g.,

A.–C.



C. Both triangles are right triangles. The measures of the matching angles are equal because both triangles have a right angle and a common angle at *O*. The third angle of $\triangle AOD$ and $\triangle MOE$ must be the same because the sum of the three angles in any triangle is 180°. The sides of the smaller triangle are half as long as the sides of the larger triangle. The coordinates of point *M* are about (2.5, 1.5).

D. The coordinates of point *M* are (2.5, 1.5); *M* is the midpoint of the diameter *OA* of the required semicircle.

Technology-Based Alternative Lesson

This investigation offers an opportunity for using dynamic geometry software. You might do a class demonstration or have students work in pairs.

Answers to Reflecting

E. Since point M is the midpoint of line segment OA, point M is halfway between points O and A. The horizontal position of point M is halfway between the horizontal positions of points O and A, or halfway between the x-coordinates of points O and A, so it is represented by the mean of the x-coordinates of points O and A. The vertical position of point M is halfway between the vertical positions of points O and A, or halfway between the y-coordinates of points O and A, so it is represented by the mean of the y-coordinates of points O and A.

F. $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$; the x-coordinate is the mean of the x-coordinates x_1 and x_2, and the y-coordinate is the mean of the y-coordinates y_1 and y_2.

3 Consolidation

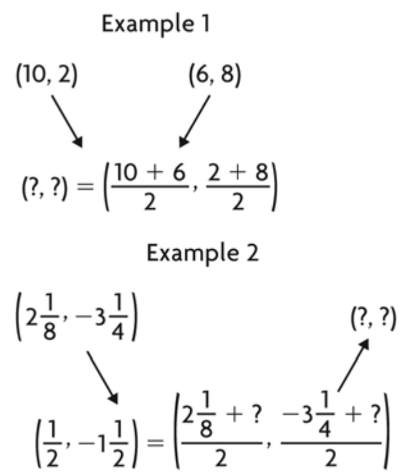
(35 to 40 min)

Apply the Math

Using the Solved Examples

Example 1 presents a general midpoint problem, in which neither endpoint is (0, 0). Discuss the advantages and disadvantages of reasoning about translations versus using a formula. If possible, demonstrate Sarah's diagram with dynamic geometry software.

In *Example 2*, the midpoint formula is used to determine the coordinates of an endpoint of a line segment, given the coordinates of the midpoint and the other endpoint. Demonstrate the reasoning, using a diagram such as the one at the right.



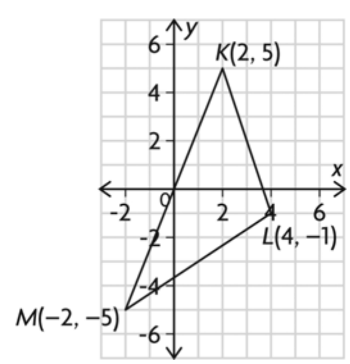
In *Example 3*, the midpoint of one side of a triangle is used to determine the equation of the median from the opposite vertex. Emphasize the connection between the midpoint formula and the concept of a median.

Example 4 presents a problem that involves applying the midpoint formula to determine the equation of a perpendicular bisector. Ask students what information they can use to determine the equation of the perpendicular bisector.

Answer to the Key Assessment Question

For question 12, students may find it helpful to refer to *Example 2*, in which the equation of one median of a triangle is determined.

12. The equation of the median from K is $y = 8x - 11$. The equation of the median from L is $y = -\frac{1}{4}x$. The equation of the median from M is $y = \frac{7}{5}x - \frac{11}{5}$.



Closing

Read question 18 as a class. After students describe the two different strategies to a partner, have them share their ideas with the class. Discuss how descriptions of the same strategy can vary.

Assessment and Differentiating Instruction

What You Will See Students Doing ...

When students understand...	If students misunderstand...
Students communicate their reasoning about the midpoint of a line segment.	Students cannot explain why the coordinates of the midpoint of a line segment are the means of the corresponding coordinates of the endpoints, or they may not understand the reasoning.
Students reason about and apply the midpoint formula correctly.	Students who do not understand the midpoint formula may make errors selecting the coordinates to use or calculating.
Students solve midpoint problems, such as problems involving triangles, circles, or perpendicular bisectors.	Students cannot identify the required midpoint in midpoint problems involving triangles, circles, or perpendicular bisectors, or they may misidentify the required midpoint.
Key Assessment Question 12	
Students understand that they need to determine the midpoints of the sides to calculate the equations of the medians.	Students do not include calculations of the midpoints of the sides of $\triangle KLM$.
Students correctly determine the midpoint of each side of ΔKLM.	Students use the wrong formula or make calculation errors when determining the midpoints.
Students determine the equations of the medians.	Students use incorrect pairs of points to calculate the equations, or they make errors in their calculations. Students may not remember the meaning of a median.

Differentiating Instruction | How You Can Respond

EXTRA SUPPORT

1. Some students may have difficulty conceptually connecting the medians of a triangle with the midpoints of its sides. Remind them that the median from a vertex goes through the middle of the opposite side.
2. Some students may need to be reminded how to calculate the equation of a line: given two points on the line. Using a diagram, guide them with the steps of beginning by determining the slope m, then substituting the coordinates of either point into $y = mx + b$ to determine the value of b. Have students explain as many of the steps as possible.

EXTRA CHALLENGE

1. If students are confident with algebra, challenge them to demonstrate algebraically that, given a line segment from $P(x_1, y_1)$ to $Q(x_2, y_2)$ with midpoint M, the slope of PM is the same as the slope of PQ.

2.2 LENGTH OF A LINE SEGMENT

Lesson at a Glance

GOAL

Determine the length of a line segment.

Prerequisite Skills/Concepts

- Determine the equation of a linear relation, given two points or one point and the slope.
- Draw a line on a coordinate grid, given its equation, or its slope and one point.
- Understand and apply the Pythagorean theorem.
- Construct a line through a point so that the line intersects and is perpendicular to another line.
- Solve a system of equations by substitution or elimination.

Specific Expectations

- Develop the formula for the length of a line segment, and use this formula to solve problems (e.g., determine the lengths of the line segments joining the midpoints of the sides of a triangle, given the coordinates of the vertices of the triangle, and verify using dynamic geometry software).
- Solve problems involving the slope, length, [and midpoint of a line segment] (e.g., determine the equation of the right bisector of a line segment, given the coordinates of the endpoints; determine the distance from a given point to a line whose equation is given, and verify using dynamic geometry software).

Mathematical Process Focus

- Reasoning and Proving
- Selecting Tools and Computational Strategies
- Connecting

Student Book Pages 81–87

Preparation and Planning

Pacing

5 min	**Introduction**
20–25 min	**Teaching and Learning**
30–35 min	**Consolidation**

Materials
- grid paper
- ruler

Recommended Practice
Questions 5, 6, 7, 13, 14, 16

Key Assessment Question
Question 7

Extra Practice
Lesson 2.2 Extra Practice

Nelson Website
http://www.nelson.com/math

MATH BACKGROUND | LESSON OVERVIEW

- Students use the Pythagorean theorem to develop the distance formula:
- The distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, gives the distance between two points, $A(x_1, y_1)$ and $B(x_2, y_2)$, which is the length of line segment AB.
- Students determine the distance from a point, A, to a line by constructing a perpendicular line through A, locating the intersection, B, of the two lines, and then calculating the distance, AB.

1 Introducing the Lesson

(5 min)

Invite students to share any knowledge or experience they have with technology that translates handwriting into typed letters. If this technology is not familiar to students, explain that it is possible. Then ask: How would this technology be useful?

2 Teaching and Learning

(20 to 25 min)

Investigate the Math

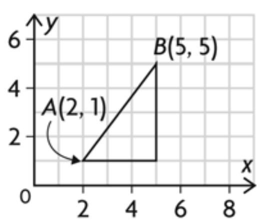
Relate the photograph and descriptive text at the beginning of the lesson to students' understanding of technology. Discuss how handwriting translation to a computer is related to coordinate geometry.

Have students work in pairs to answer the prompts in the investigation. As students share their work, discuss how the positions of the chosen points affect their answers. The distance formula in part E will be the same, however, up to the order of the coordinates in the squared terms. It may be helpful to have some students choose points in different quadrants or points that give *PQ* a negative slope. A class discussion can highlight how the choice of points does not affect the outcome of the formula.

Answers to Investigate the Math

Answers may vary, e.g.,

A.

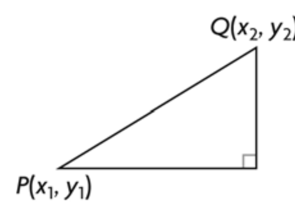


B. (5, 1); the *x*-coordinate is same as the *x*-coordinate of *B*, and the *y*-coordinate is same as the *y*-coordinate of *A*.

C. 3 units, 4 units; the lengths are the differences between the *x*-coordinates and between the *y*-coordinates when the values are subtracted, so the final result is positive.

D. 5 units

E. (x_2, y_1); the *x*-coordinate is same as the *x*-coordinate of *Q*, and the *y*-coordinate is same as the *y*-coordinate of *P*
$x_2 - x_1$ units, $y_2 - y_1$ units; the lengths are the positive differences between the *x*-coordinates and between the *y*-coordinates: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



Technology-Based Alternative Lesson

The investigation can be done using dynamic geometry software. You could do a class demonstration or have students work in pairs. In part D, however, make sure that students use the Pythagorean theorem first and use the software to measure only as a check.

Answers to Reflecting

F. No. When numbers are squared, the result is positive. Therefore, $(x_1 - x_2)^2$ and $(y_1 - y_2)^2$ are the same as $(x_2 - x_1)^2$ and $(y_2 - y_1)^2$.

G. Determine the difference between the *x*-coordinates and the difference between the *y*-coordinates. Add the squares of these differences, and then calculate the square root of the sum.

H. The length of line segment *PQ* is the distance between points *P* and *Q*.

3 Consolidation

(30 to 35 min)

Apply the Math (Whole Class/Think, Pair, Share)

Using the Solved Examples

In *Example 1*, students determine the lengths of horizontal and vertical line segments in parts a) and b), and use the distance formula in part c). Have students work in pairs, with one partner explaining parts a) and b) and the other partner explaining part c). Then, in a class discussion, have students connect the distance formula to the Pythagorean theorem.

In *Example 2*, driving distances are represented on a coordinate grid, giving a real-life context for using the distance formula. As a class, compare the sketch in the solution with a map. Discuss why decimal digits are kept during calculations—to avoid errors due to rounding.

Example 3 extends the concept of determining distance on a coordinate grid to the case of a point and a line. Discuss with the class how, in this context, distance means the shortest, or perpendicular, distance. Have students work in pairs, with one partner summarizing the strategy and the other partner checking the calculations. Then work through the example as a class.

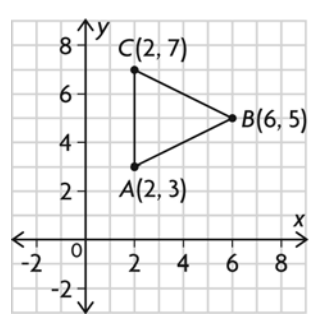
Answer to the Key Assessment Question

If students calculate the square roots before adding for question 7, they should round the distances for *AB* and *BC* to two decimal places to avoid errors due to rounding to one decimal place for the answer. Some may add the distances; others may multiply by 2 since the distances are equal.

7. Distance from *A* to *B* $= \sqrt{(6-2)^2 + (5-3)^2} = \sqrt{20}$

Distance from *B* to *C* $= \sqrt{(2-6)^2 + (7-5)^2} = \sqrt{20}$

Total distance $= 2\sqrt{20}$, or about 8.9 units



Closing

Ask students to read question 16. Have students work in pairs, with each partner explaining his or her strategy to the other. Then discuss the suggested strategies as a class. Ask whether any other strategies are possible.

Assessment and Differentiating Instruction	
What You Will See Students Doing ...	
When students understand...	**If students misunderstand...**
Students reason about the length of a line segment, connecting it to the distance between two points and to the Pythagorean theorem.	Students cannot construct the right triangle associated with a line segment in order to connect it to the Pythagorean theorem.
Students select strategies to calculate lengths of line segments.	Students cannot select strategies to calculate lengths of line segments, or they might apply the strategies incorrectly.
Students apply the distance formula effectively in a range of mathematical and real-world problems.	Students may make errors when using the distance formula, choosing points, doing calculations, interpreting given information or the results of calculations, or recording units.
Key Assessment Question 7	
Students recognize that the answer is the length of *AB* plus the length of *BC*.	Students may calculate the length of *AC* instead, or they may not calculate both lengths and determine the sum.
Students apply the distance formula correctly to both *AB* and *BC*, or they apply the distance formula to *AB* and then reason that *BC* = *AB* because $\triangle ABC$ is isosceles.	Students may make errors when applying the distance formula. For example, they may incorrectly calculate $\sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$.

Differentiating Instruction | How You Can Respond

EXTRA SUPPORT

1. Have students work with a 3-4-5 right triangle, applying the Pythagorean theorem and taking the square root of both sides. Then assign suitable coordinates to each vertex, to help students make the connection to the coordinate grid. Rework the triangle in terms of coordinate differences and the distance formula.
2. Introduce *Example 3* by considering the easier case of a point and a vertical or horizontal line.

EXTRA CHALLENGE

1. Have students solve, or create and solve, distance problems like *Example 2* using maps. The problems can be related to places they are studying, places they would like to visit, stories they are reading, or locations in their community.

2.3 EQUATION OF A CIRCLE
Lesson at a Glance

GOAL

Develop and use an equation for a circle.

Prerequisite Skills/Concepts

- Understand and apply the Pythagorean theorem.
- Understand and apply the distance formula.

Specific Expectations

- Develop the equation for a circle with centre (0, 0) and radius r, by applying the formula for the length of a line segment.
- Determine the radius of a circle with centre (0, 0), given its equation; write the equation of a circle with centre (0, 0), given the radius; and sketch the circle, given the equation in the form $x^2 + y^2 = r^2$.

Mathematical Process Focus

- Reasoning and Proving
- Selecting Tools and Computational Strategies
- Connecting

Student Book Pages 88–93

Preparation and Planning

Pacing

5–10 min	**Introduction**
25 min	**Teaching and Learning**
25–30 min	**Consolidation**

Materials
- graphing calculator
- grid paper, ruler, and compass, or dynamic geometry software

Recommended Practice
Questions 4, 7, 8, 12, 13, 17

Key Assessment Question
Question 12

Extra Practice
Lesson 2.3 Extra Practice

Nelson Website
http://www.nelson.com/math

MATH BACKGROUND | LESSON OVERVIEW

- Students use the distance formula to develop the equation of a circle with radius r and its centre at the origin: $x^2 + y^2 = r^2$.
- If point (x, y) lies on a circle that is centred at the origin, then the points $(–x, y)$, $(–x, –y)$, and $(x, –y)$ also lie on the circle. Each pair of points (x, y) and $(–x, –y)$ define a diameter of the circle that passes through the origin.

1 Introducing the Lesson

(5 to 10 min)

Draw a circle on the board. Ask for a volunteer to mark the centre of the circle and a point on its circumference. Then discuss what changes and what stays the same as the point on the circumference moves around the circumference.

If dynamic geometry software is available, display the coordinates of the point on the circumference, as well as its distance to the centre. Drag the point around the circle so that students can see the coordinates changing and the distance staying constant.

2 Teaching and Learning

(25 min)

Investigate the Math

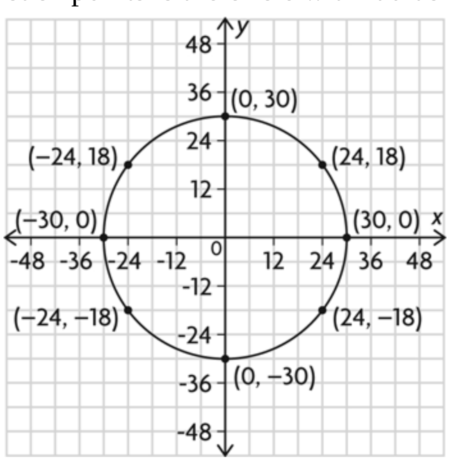
Initiate a discussion about seismographs, including the technology involved and the need for such technology.

Have students work in pairs to record their responses to the prompts in the investigation. Make sure that students understand the connection between the equation of a circle with its centre at the origin, the distance formula, and the Pythagorean theorem.

Answers to Investigate the Math

A. The epicentre is a point that is 30 km away from the seismograph. The set of all such points is the circle with radius 30 and its centre at (0, 0).

B.–C. i)



ii) The distance from (0, 0) to (±24, ±18) is $\sqrt{(\pm 24 - 0)^2 + (\pm 18 - 0)^2} = 30$.

D. 30; the seismograph and the epicentre are 30 km apart.

E. $\sqrt{x^2 + y^2}$

F. $x^2 + y^2 = 900$; this equation describes all the points that are 30 km away from the seismograph at (0, 0).

Technology-Based Alternative Lesson

Parts A, B, and C are appropriate for dynamic geometry software. You could do a class demonstration, or you could have students work in pairs or on their own. Make sure snapping points to the grid position is enabled.

Answer to Reflecting

G. If $x^2 + y^2 = 900$, then $x^2 + (-y)^2 = x^2 + y^2 = 900$, $(-x)^2 + y^2 = x^2 + y^2 = 900$, and $(-x)^2 + (-y)^2 = x^2 + y^2 = 900$.

H. The x and y terms have the exponent 2 for the equation of a circle but the exponent 0 or 1 for a linear relation.

I. $x^2 + y^2 = r^2$

3 Consolidation

(25 to 30 min)

Apply the Math

Using the Solved Examples

Example 1 presents a simple application of the general equation of a circle, $x^2 + y^2 = r^2$. Ensure that students understand the use of the units.

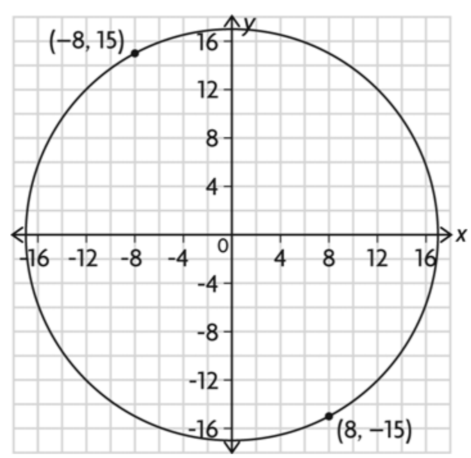
Example 2 introduces a strategy for graphing a circle and integrates the formula for a midpoint.

In *Example 3*, an equation of a circle that is centred at the origin is determined using the coordinates of a point known to be on the circle. The opposite endpoint of the diameter is also determined. Ask a few students to explain the diagram.

Answer to the Key Assessment Question

Students can reason that the coordinates of the points in question 12 are opposites, so the centre of the circle is (0, 0). Since the points lie on the circle, they can substitute the coordinate of either point into $x^2 + y^2 = r^2$ resulting in $8^2 + (-15)^2 = 289$, then substitute 289 into $x^2 + y^2 = r^2$. A diagram would be helpful.

12. The equation of the circle is $x^2 + y^2 = 289$.



Closing

Have students read question 17 and record some reasons individually. Then they could brainstorm in small groups to get as many reasons as possible. You may wish to assign one member of each group to act as a person who is unwilling to be convinced and raises objections to all the reasons. Bring the groups together for a class discussion of possible reasons.

Assessment and Differentiating Instruction

What You Will See Students Doing ...

When students understand...	If students misunderstand...
Students recognize and effectively apply the connection between the distance formula and the equation of a circle.	Students may not connect the distance formula to the radius, possibly because they do not understand that a circle is defined as the points at a fixed distance from the centre.
Students correctly write the equation of a circle that is centred at the origin, given either its graph or one or more points on its circumference.	Students may not use the distance formula, or they may not use it correctly, to calculate the radius.
Students correctly sketch or graph a circle that is centred at the origin, given its equation.	Students may interpret the radius in the equation incorrectly. For example, they may think that the radius is 16 when given the equation $x^2 + y^2 = 16$.
Students apply the general form of the equation of a circle that is centred at the origin to real-world problems.	Students may misinterpret the given information. For example, when locating the epicentre of an earthquake, they may treat the location of the seismograph as being on the circle, rather than at its centre.
Key Assessment Question 12	
Students recognize that the centre of the circle is (0, 0) because the diameter has midpoint (0, 0).	Students may not recognize that the centre of the circle is (0, 0). They may still be able to complete the problem, however, so it is necessary to look for a good explanation of how to use the given information.
Students correctly substitute $x = \pm 8$ and $y = \pm 15$ into the equation $x^2 + y^2 = r^2$ to determine the equation $r^2 = 289$.	Students may not substitute the coordinates into $x^2 + y^2 = r^2$, or they may make errors when doing so. They may not use the correct formula.

Differentiating Instruction | How You Can Respond

EXTRA SUPPORT

1. If students are having difficulty graphing equations given in the form $x^2 + y^2 = C$, suggest that they start by determining the radius r as the square root of C and then plot the points $(r, 0)$, $(–r, 0)$, $(0, r)$, and $(0, –r)$.
2. If students are having conceptual difficulty with the equation form $x^2 + y^2 = C$, emphasize the connection to the Pythagorean theorem by guiding them through a discussion about a right triangle with non-right-angle vertices at the origin/centre and at a point on the circle. Include them in constructing the diagram as much as appropriate.

EXTRA CHALLENGE

1. Have students investigate the distance from a point to a circle that is centred at the origin, just as they investigated the distance from a point to a line.
2. Students could research data about earthquakes and relate the data to the use of a seismograph. Students could use the data for calculations and write statements explaining the calculations.

MID-CHAPTER REVIEW

Big Ideas Covered So Far

- The coordinates of the midpoint of a line segment are the means of the coordinates of the endpoints of the line segment, as in the formula $(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.
- The distance between two points, or the length of a line segment, can be determined using the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- A circle with radius r and its centre at the origin has the equation $x^2 + y^2 = r^2$.

Using the Frequently Asked Questions

Have students keep their Student Books closed. Display the Frequently Asked Questions on a board. Have students discuss the questions and use the discussion to draw out what the class thinks are good answers. Then have students compare the class answers with the answers on Student Book page 94. Students can refer to the answers to the Frequently Asked Questions as they work through the Practice Questions.

Using the Mid-Chapter Review

Ask students if they have any questions about any of the topics covered so far in the chapter. Review any topics that students would benefit from considering again. Assign Practice Questions for class work and for homework.

To gain greater insight into students' understanding of the material covered so far in the chapter, you may want to ask them questions such as the following:

- How are the midpoint formula and the distance formula similar? How are they different?
- How would you determine the coordinates of the midpoint of a line segment if you forgot the midpoint formula?
- What are some practical uses of the distance formula?
- Given that $P(a, b)$ and $Q(–a, –b)$ are points on a circle, what do you know about the centre of the circle? (It is at the origin.) What do you know about the line segment PQ? (It is a diameter.)
- What steps would you follow to sketch a circle if you knew the equation of the circle?

2.4 CLASSIFYING FIGURES ON A COORDINATE GRID

Lesson at a Glance

GOAL

Use properties of line segments to classify two-dimensional figures.

Prerequisite Skills/Concepts

- Determine the midpoint of a line segment.
- Apply the distance formula.
- Determine whether two lines are parallel, perpendicular, or neither.
- Determine the area of a triangle.

Specific Expectations

- Solve problems involving the slope, length, and midpoint of a line segment (e.g., determine the equation of the right bisector of a line segment, given the coordinates of the endpoints; determine the distance from a given point to a line whose equation is given, and verify using dynamic geometry software).
- Verify, using algebraic techniques and analytic geometry, some characteristics of geometric figures (e.g., verify that two lines are perpendicular, given the coordinates of two points on each line; verify, by determining side length, that a triangle is equilateral, given the coordinates of the vertices).
- Plan and implement a multi-step strategy that uses analytic geometry and algebraic techniques to verify a geometric property (e.g., given the coordinates of the vertices of a triangle, verify that the line segment joining the midpoints of two sides of the triangle is parallel to the third side and half its length, and check using dynamic geometry software; given the coordinates of the vertices of a rectangle, verify that the diagonals of the rectangle bisect each other).

Mathematical Process Focus

- Problem Solving
- Reasoning and Proving
- Selecting Tools and Computational Strategies
- Representing

Student Book Pages 96–103

Preparation and Planning

Pacing
5 min **Introduction**
15–20 min **Teaching and Learning**
35–40 min **Consolidation**

Materials
- grid paper and ruler, or dynamic geometry software

Recommended Practice
Questions 5, 6, 8, 10, 14, 18, 19

Key Assessment Question
Question 14

New Vocabulary
analytic geometry

Extra Practice
Lesson 2.4 Extra Practice

Nelson Website
http://www.nelson.com/math

MATH BACKGROUND | LESSON OVERVIEW

- Students use analytic geometry, specifically the algebra of linear relations and the slope, distance, and midpoint formulas, to classify quadrilaterals and triangles.
- Students classify triangles as equilateral, isosceles, or scalene according to the number of equal side lengths. They classify triangles as right triangles if two sides are perpendicular.
- Students classify quadrilaterals according to which pairs of sides, if any, are equal length and/or are parallel or perpendicular.

1 Introducing the Lesson

(5 min)

Describe several properties, such as the following, and ask students to identify the type(s) of quadrilateral or triangle. Then challenge students to describe quadrilaterals and triangles for classmates to identify.

- quadrilateral with exactly one pair of opposite sides parallel (trapezoid)
- quadrilateral with corners that are right angles (square, rectangle)
- triangle with two sides of equal length (isosceles triangle)
- quadrilateral with opposite sides parallel (parallelogram, rhombus, square, rectangle)
- triangle with two sides perpendicular (right triangle)
- quadrilateral with opposite sides parallel and all sides of equal length (rhombus)
- triangle with two angles of equal measure (isosceles triangle)

2 Teaching and Learning

(15 to 20 min)

Learn About the Math

Introduce the context, inviting students to explain what they know about the work of a surveyor and the education required to become a surveyor. Relate the discussion to the problem presented in the introduction.

Example 1 introduces the idea that coordinate-based calculations, such as the calculations to determine the side lengths of the lot, can give information about a geometric figure. Ask students why it is necessary to calculate the slopes of the sides as well as their lengths. Elicit from students that they need to determine whether the sides are parallel. Ensure that students can explain each step in the calculations.

Technology-Based Alternative Lesson

Dynamic geometry software can be used to check the examples and complete the questions. You may want to use dynamic geometry software and a projector to work through examples with the class.

Answers to Reflecting

A. Anita had to be sure that the side lengths were exactly equal and the sides were parallel. A diagram only shows whether side lengths are almost equal and whether sides are almost parallel.

B. Anita had to calculate the slopes and lengths of all four sides to determine that the shape of the lot was a rhombus. If only two sides were parallel, the shape would be a trapezoid. If only opposite sides were the same length, the shape would be a parallelogram or, if the sides were perpendicular, a rectangle.

3 Consolidation

(35 to 40 min)

Apply the Math

Using the Solved Examples

Work through *Example 2* with the class. Begin by asking students what type of triangle △*ABC* appears to be, and why (isosceles triangle since *AB* and *BC* appear to be equal in length; right triangle since *AB* and *BC* appear to be perpendicular). Then ask which information they need to calculate (lengths and slopes of *AB* and *BC*). You might ask one volunteer to explain the calculations for the side lengths and another volunteer to explain the calculations for the slopes. Discuss that either the side lengths or the slopes could be calculated first.

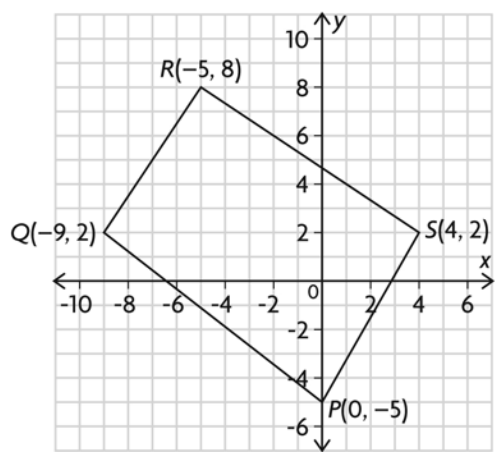
Example 3 introduces the calculation of area, which requires students to realize that they need to construct an altitude of the triangle. Have students work in pairs, with one partner explaining the solution to part a) and the other partner explaining the solution to part b). Encourage students to summarize their discussions for the class. If possible, demonstrate the check with dynamic geometry software.

Answer to the Key Assessment Question

Arrange for students to share answers for question 14, and discuss different ways to explain that *PQRS* is not a rectangle.

14. Answers may vary, e.g., $m_{PQ} = -\frac{7}{9}$ and $m_{QR} = \frac{3}{2}$, so $m_{PQ} \neq -\frac{1}{m_{QR}}$.

Since the slopes are not negative reciprocals, the sides are not perpendicular. Since the sides are not perpendicular, *PQRS* is not a rectangle.



Closing

Students can work on their own to create a flow chart for question 19, or they can first brainstorm, in small groups, to determine the information they could calculate and the properties they could identify for special types of quadrilaterals.

Assessment and Differentiating Instruction

What You Will See Students Doing ...

When students understand...	If students misunderstand...
Students recognize that the midpoint formula and the distance formula give information that can be used to classify quadrilaterals and triangles, given the coordinates of their vertices.	Students have difficulty connecting the ideas from previous lessons to the properties of quadrilaterals and triangles, or they do not know the properties.
Students correctly formulate strategies to determine perimeters, areas, and types of quadrilaterals and triangles.	Students make errors, such as using the perpendicular bisector of the base of a triangle when trying to determine its altitude and area.
Students correctly determine needed information and draw appropriate conclusions about quadrilaterals and triangles.	Students use inappropriate information to draw conclusions, draw inaccurate conclusions based on the information available, or communicate their reasoning ineffectively.
Key Assessment Question 14	
Students formulate an appropriate strategy, such as comparing the slopes of adjacent and opposite sides.	Students calculate irrelevant information, such as the coordinates of the midpoints of the sides.
Students correctly determine the information they need, draw a correct conclusion, and communicate how the information supports the conclusion.	Students make conclusions that are not supported by the properties they discovered. For example, they may discover that *PQRS* is not a rhombus and then incorrectly deduce that *PQRS* is not a rectangle.

Differentiating Instruction | How You Can Respond

EXTRA SUPPORT

1. To help students recall the properties of different types of quadrilaterals, suggest that they use their tables from Applying What You Know in Getting Started, the Glossary, or In Summary for reference.
2. Help students summarize the information they can determine from a given geometric figure (e.g., the length of a median of a triangle).

EXTRA CHALLENGE

1. Ask students to create a strategy for determining whether a given quadrilateral is an isosceles trapezoid without using its diagonals and then using only its diagonals. Have students share their strategies and justify their thinking.

2.5 VERIFYING PROPERTIES OF GEOMETRIC FIGURES

Lesson at a Glance

GOAL

Use analytic geometry to verify properties of geometric figures.

Prerequisite Skills/Concepts

- Determine the midpoint of a line segment.
- Understand and apply the distance formula.
- Determine whether two lines are parallel, perpendicular, or neither.
- Classify triangles and quadrilaterals.
- Apply the general equation of a circle that is centred at the origin.

Specific Expectations

- Solve problems involving the slope, length, and midpoint of a line segment (e.g., determine the equation of the right bisector of a line segment, given the coordinates of the endpoints; determine the distance from a given point to a line whose equation is given, and verify using dynamic geometry software).
- Verify, using algebraic techniques and analytic geometry, some characteristics of geometric figures (e.g., verify that two lines are perpendicular, given the coordinates of two points on each line; verify, by determining side length, that a triangle is equilateral, given the coordinates of the vertices).
- Plan and implement a multi-step strategy that uses analytic geometry and algebraic techniques to verify a geometric property (e.g., given the coordinates of the vertices of a triangle, verify that the line segment joining the midpoints of two sides of the triangle is parallel to the third side and half its length, and check using dynamic geometry software; given the coordinates of the vertices of a rectangle, verify that the diagonals of the rectangle bisect each other).

Mathematical Process Focus

- Problem Solving
- Reasoning and Proving
- Selecting Tools and Computational Strategies
- Representing

Student Book Pages 104–110

Preparation and Planning

Pacing
10 min **Introduction**
15–20 min **Teaching and Learning**
30–35 min **Consolidation**

Materials
- grid paper and ruler, or dynamic geometry software

Recommended Practice
Questions 4, 6, 8, 10, 13, 15, 16

Key Assessment Question
Question 10

New Vocabulary
midsegment of a quadrilateral

Extra Practice
Lesson 2.5 Extra Practice

Nelson Website
http://www.nelson.com/math

MATH BACKGROUND | LESSON OVERVIEW

- Students use slopes and/or lengths of sides, diagonals, and midsegments to verify properties of quadrilaterals, circles, and triangles.
- The concept of a midsegment is extended from triangles to quadrilaterals.
- The midsegments of a quadrilateral always form a parallelogram (which may also be a rhombus, a rectangle, and/or a square). Students verify this for a particular quadrilateral in *Example 1*.

1 Introducing the Lesson
(10 min)

Before beginning this lesson, set up a quadrilateral with its midsegments constructed. Use dynamic geometry software, if available. If dynamic geometry software is not available, draw on a grid or use pins and string on a grid to show the quadrilateral.

Have students move the vertices of the quadrilateral to various locations, and discuss what happens to the opposite midsegments. Ask questions such as these: Do the opposite midsegments appear to remain parallel? How could the software be used to check? Then invite students to pose questions for the class.

2 Teaching and Learning
(15 to 20 min)

Learn About the Math

Relate the diagram to a landscape design, ensuring that students realize which part of the diagram is the lawn area. *Example 1* introduces a practical application of ideas from Lesson 2.4. It also hints, as does Introducing the Lesson, that joining the adjacent midpoints of any quadrilateral creates a parallelogram.

Students must be able to follow both solutions. Have students work in pairs, with each partner explaining one of the solutions to the other.

Technology-Based Alternative Lesson

Dynamic geometry software can be used to demonstrate or check the examples, following the steps in the examples or just the verification. If students use the software to complete the questions, remind them about Appendix B.

Answers to Reflecting

A. Ed focused on properties of the sides of a parallelogram. He showed that opposite sides have the same slope, so they are parallel. Grace used properties of the diagonals of a parallelogram. She showed that the diagonals bisect each other.

B. Verify that the lengths of opposite sides are equal by using the distance formula to calculate the length of each side of *JKLM*.

3 **Consolidation**

(30 to 35 min)

Apply the Math

Using the Solved Examples

In *Example 2*, the required information is given in the problem statement. Begin by discussing with the class exactly what information needs to be calculated. You could then either continue working through the solution with the class or have students work in pairs. If students work in pairs, ask for volunteers to summarize their results for the class.

Example 3 deals with properties of a circle. For the second part, remind students that the equation of a line through the origin has the form $y = mx$.

Answer to the Key Assessment Question

Encourage discussion for solutions of question 10 about ways details of the strategies are the same and ways they are different.

10. Determine the midpoints.

$$M_{RS} = \left(\frac{-5+(-1)}{2}, \frac{2+3}{2}\right) = (-3, 2.5) = A$$

Let the midpoint of *RS* be *A*.

$$M_{ST} = \left(\frac{-1+(-2)}{2}, \frac{3+(-1)}{2}\right) = (-1.5, 1) = B$$

Let the midpoint of *ST* be *B*.

$$M_{TU} = \left(\frac{-2+(-6)}{2}, \frac{-1+(-2)}{2}\right) = (-4, -1.5) = C$$

Let the midpoint of *TU* be *C*.

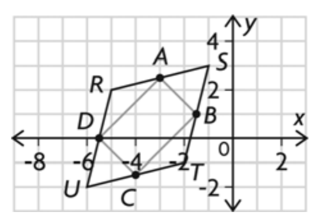
$$M_{UR} = \left(\frac{-6+(-5)}{2}, \frac{-2+2}{2}\right) = (-5.5, 0) = D$$

Let the midpoint of *UR* be *D*.

$$d_{AC} = \sqrt{[-3-(-4)]^2 + [2.5-(-1.5)]^2} = \sqrt{17} \doteq 4.12, \text{ or about 4.12 units}$$

$$d_{BD} = \sqrt{[-1.5-(-5.5)]^2 + (1-0)^2} = \sqrt{17} \doteq 4.12, \text{ or about 4.12 units}$$

The diagonals of *ABCD* are equal length, so the midsegments form a rectangle.



Closing

Discuss question 16. Arrange for students to post their flow charts or discuss their flow charts in groups.

Assessment and Differentiating Instruction

What You Will See Students Doing ...

When students understand...	If students misunderstand...
Students formulate correct strategies to verify geometric properties.	Students may confuse analytic geometry strategies, or they may be unsure of what steps are required. They may not know some of the required properties of figures.
Students correctly calculate the information needed to construct a figure or verify geometric properties.	Students do not recognize, or cannot accurately calculate, the information needed.
Key Assessment Question 10	
Students correctly determine the midpoints of all four sides of the given rhombus.	Students make errors when determining the midpoints of the sides of the given rhombus. For example, they may make transposition errors, such as $\left(\frac{x_1 + y_1}{2}, \frac{x_2 + y_2}{2}\right)$, use incorrect vertices, or make mistakes when calculating.
Students use an appropriate strategy to verify that the quadrilateral is a rectangle. For example, they determine that opposite sides are parallel and adjacent sides are perpendicular, or that diagonals are equal in length and perpendicular.	Students may verify only some of the necessary properties, or they may make mistakes when calculating. They may not know the properties of a rectangle.

Differentiating Instruction | How You Can Respond

EXTRA SUPPORT

1. Encourage students to sketch diagrams for problems. Even if they have a good conceptual understanding, sketching will help them avoid calculation errors.
2. If students are struggling with dynamic geometry software, remind them how to use the key operations.

EXTRA CHALLENGE

1. Have students verify that the midpoint quadrilateral of a rectangle is always a rhombus, and vice versa.

2.6 EXPLORING PROPERTIES OF GEOMETRIC FIGURES

Lesson at a Glance

GOAL

Investigate intersections of lines or line segments within triangles and circles.

Prerequisite Skills/Concepts

- Construct the perpendicular bisector of a line segment.
- Construct a line through a point so that the line intersects and is perpendicular to another line.
- Determine the midpoint of a line segment.
- Understand and apply the distance formula.
- Apply the general equation of a circle that is centred at the origin.

Specific Expectation

- Determine, through investigation (e.g., using dynamic geometry software, by paper folding), some characteristics and properties of geometric figures (e.g., medians in a triangle, similar figures constructed on the sides of a right triangle).

Mathematical Process Focus

- Reasoning and Proving
- Representing

Student Book Pages 111–114

Preparation and Planning

Pacing

5 min	**Introduction**
35–40 min	**Teaching and Learning**
15–20 min	**Consolidation**

Materials
- grid paper and ruler, or dynamic geometry software

New Vocabulary
circumcentre
altitude
orthocentre

Recommended Practice
Questions 1, 2, 3, 4

Nelson Website
http://www.nelson.com/math

MATH BACKGROUND | LESSON OVERVIEW

- Students construct and identify all three centres of a triangle: the centroid, the circumcentre, and the orthocentre.
- Students verify that the vertices of a triangle lie on a circle that is centred at the circumcentre.
- Students verify that all three medians of a triangle meet at a common point, the centroid.
- Students verify that all three perpendicular bisectors of the sides of a triangle meet at a common point, the circumcentre, which is the centre of the circle that passes through all three vertices of the triangle.
- Students verify that all three altitudes of a triangle meet at the orthocentre.
- Students verify that the centroid, circumcentre, and orthocentre coincide when the triangle is equilateral.

1 Introducing the Lesson

(5 min)

Ask students what they recall about the centroid of a triangle. Also discuss other significant lines through vertices, besides medians. Ask: Which type of line could help you determine the area of a triangle?

2 Teaching and Learning

(35 to 40 min)

Explore the Math

Direct attention to the diagram of a triangle that is being balanced by a pencil at the centroid. Students might try this at the end of the lesson.

Have students work in pairs to answer the prompts in the exploration. For part D, suggest experimenting with a right triangle. Ask students to explain why two altitudes of a right triangle are along sides intersecting at the vertex of the right angles.

For the Reflecting questions, ensure that students understand the meaning of the word coincide in part J.

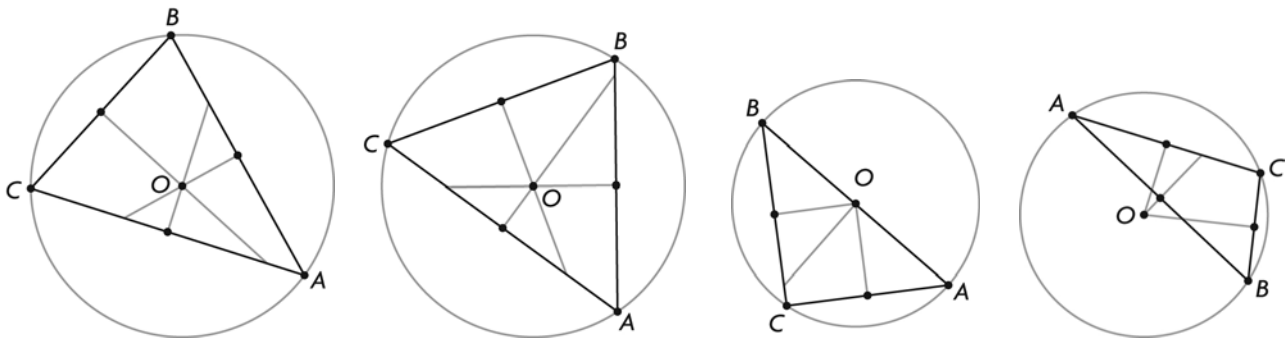
Technology-Based Alternative Lesson

This exploration can be done using dynamic geometry software, with students working in pairs if sufficient technology is available. It is best not to show the grid or, at least, to uncheck Snap Points. Students should begin a new sketch for parts D and G. For parts C, E, and H, rather than constructing new triangles or circles, students may move around the vertices of their original triangles or the centres and edge points of their original circles. The constructions, if done correctly, will adjust automatically.

Answers to Explore the Math

Answers may vary, e.g.,

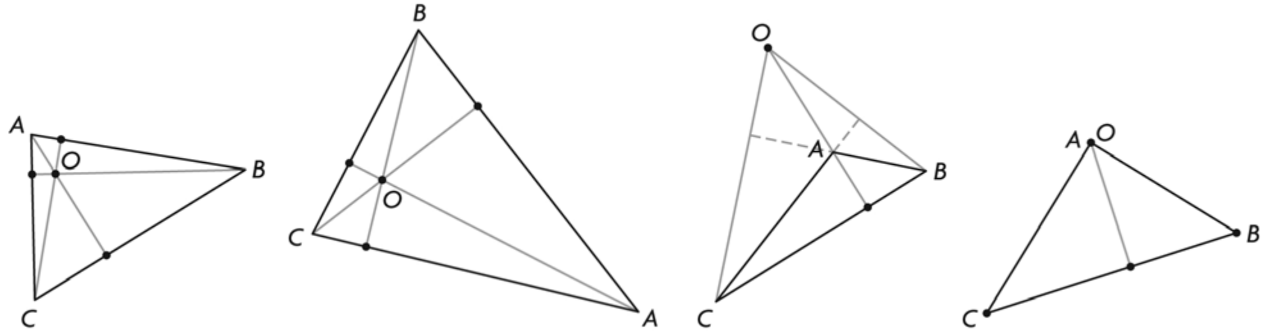
A.–C.



B. The circle passes through all three vertices.

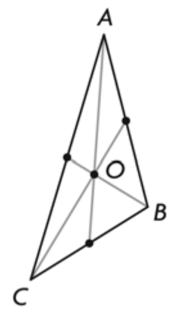
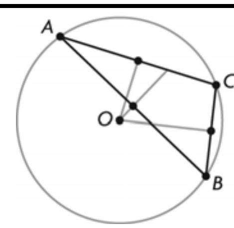
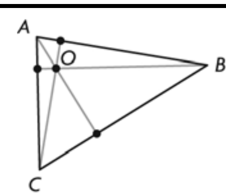
C. Yes, the result is always the same. The circle passes through all three vertices, no matter what triangle I draw.

D.–E.

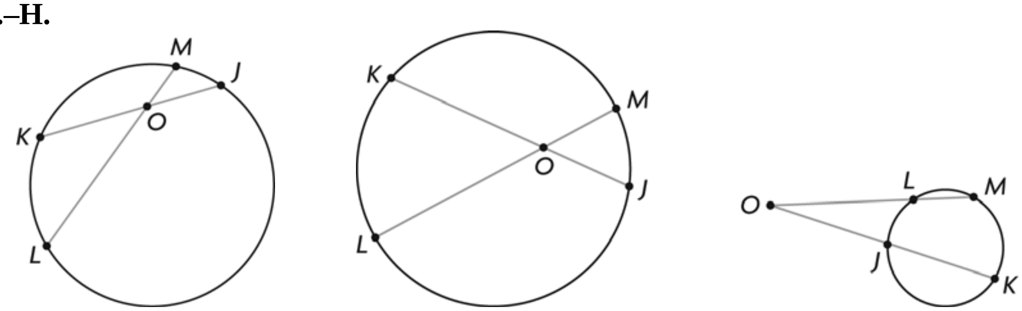


E. No. Answers may vary, e.g., if the triangle is obtuse, the altitudes from the acute vertices lie outside the triangle, so their intersection must also be outside the triangle.

F.

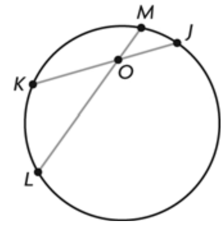
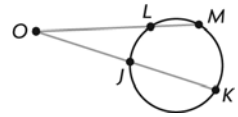
Type of Triangle Centre	Type of Intersecting Lines	Special Property	Diagram
centroid	medians	centre of mass	
circumcentre	perpendicular bisectors of sides	centre of circle that passes through all three vertices	
orthocentre	altitudes	on the line segment representing the height of the triangle drawn from each vertex	

G.–H.



The products $JO \times OK$ and $LO \times OM$ are the same for each circle and pair of chords.

I.

Location of Intersection	Property	Diagram
inside the circle	*JO* × *OK* = *LO* × *OM*	
outside the circle	*JO* × *OK* = *LO* × *OM*	

Answers to Reflecting

J. The medians, perpendicular bisectors, and altitudes coincide in an equilateral triangle.

K. No. The relation *JO* × *OK* = *LO* × *OM* does not depend on the radius of the circle or vary with the location of *O*.

3 Consolidation

(15 to 20 min)

Students should be able to construct the centroid, circumcentre, and orthocentre of a triangle given the coordinates of the vertices.

Students should understand the following properties of the chords of a circle:

- If two chords of a circle intersect inside the circle, the products of the lengths of the two parts of each chord are equal.
- If the intersection of the chords is outside the circle, this equality remains true.

Students should be able to answer the Further Your Understanding questions independently.

Curious Math

This Curious Math feature provides students with an opportunity to explore a remarkable geometric fact: the midpoints of the sides of a triangle, the feet of its altitudes, and the midpoints of the line segments that join the orthocentre to the vertices of the triangle all lie on the same circle. Students could verify their constructions using dynamic geometry software.

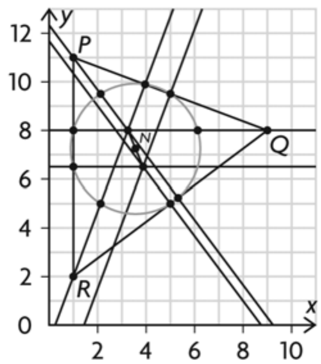
Answers to Curious Math

1. (5, 9.5), (5, 5), (1, 6.5)

2. $y = -1\frac{1}{3}x + 12\frac{1}{3}$, $y = 8$, $y = 2\frac{2}{3}x - \frac{2}{3}$; (5.32, 5.24), (1, 8), $\left(3\frac{70}{73}, 9\frac{65}{73}\right)$

3. orthocentre: (3.25, 8); midpoints: (2.125, 9.5), (6.125, 8), (2.125, 5)

4. circumcentre: (3.875, 6.5); midpoint: (3.5625, 7.25)



5. Midpoints of sides: Use the midpoint formula.
Altitudes: Construct the altitudes (the lines through the vertices, perpendicular to the opposite sides). Determine the intersection of each altitude with the opposite side.
Midpoints of line segments joining the orthocentre to the vertices: Determine the common intersection of the altitudes, and then use the midpoint formula.

2.7 USING COORDINATES TO SOLVE PROBLEMS

Lesson at a Glance

GOAL

Use properties of lines and line segments to solve problems.

Prerequisite Skills/Concepts

- Determine the equation of a linear relation, given two points or one point and the slope.
- Construct the perpendicular bisector of a line segment.
- Construct a line through a point so that the line intersects and is perpendicular to another line.
- Solve a system of equations by substitution or elimination.
- Apply the midpoint formula and the distance formula.

Specific Expectations

- Solve problems involving the slope, length, and midpoint of a line segment (e.g., determine the equation of the right bisector of a line segment, given the coordinates of the endpoints; determine the distance from a given point to a line whose equation is given, and verify using dynamic geometry software).
- Verify, using algebraic techniques and analytic geometry, some characteristics of geometric figures (e.g., verify that two lines are perpendicular, given the coordinates of two points on each line; verify, by determining side length, that a triangle is equilateral, given the coordinates of the vertices).
- Plan and implement a multi-step strategy that uses analytic geometry and algebraic techniques to verify a geometric property (e.g., given the coordinates of the vertices of a triangle, verify that the line segment joining the midpoints of two sides of the triangle is parallel to the third side and half its length, and check using dynamic geometry software; given the coordinates of the vertices of a rectangle, verify that the diagonals of the rectangle bisect each other).

Mathematical Process Focus

- Problem Solving
- Reasoning and Proving
- Selecting Tools and Computational Strategies
- Representing

Student Book Pages 115–121

Preparation and Planning

Pacing
10 min **Introduction**
30–35 min **Teaching and Learning**
15–20 min **Consolidation**

Materials
- grid paper
- ruler

Recommended Practice
Questions 7, 9, 10, 12, 13, 16, 19

Key Assessment Question
Question 16

Extra Practice
Lesson 2.7 Extra Practice

Nelson Website
http://www.nelson.com/math

MATH BACKGROUND | LESSON OVERVIEW

- Students apply the analytic geometry concepts they have investigated in previous lessons to mathematical and real-world problems.

1 Introducing the Lesson

(10 min)

Have students work in small groups to list what they think are the important ideas they have learned in this chapter. Then invite a student from each group to read one of the ideas. Continue for several rounds, with different students reading ideas.

2 Teaching and Learning

(30 to 35 min)

Learn About the Math

Direct students' attention to the parking lot problem at the beginning of the lesson. Discuss the property that the mast light must have in relation to the three entrances. Ask students to consider whether the orthocentre, the centroid, or the circumcentre will help them solve this problem.

After students understand that they need to determine the circumcentre in *Example 1*, discuss what information is required (midpoint and slope). After taking the discussion this far, you may want to have students work in pairs to complete the example. Each partner could determine the equation of one of the perpendicular bisectors, and then the partners could work together to calculate the intersection of the two equations. Bring the class back together to discuss students' results.

Technology-Based Alternative Lesson

You may want to use, or have students use, dynamic geometry software to check the calculations in *Examples 1* and *2* or to complete some of the questions.

Answers to Reflecting

A. Any point on the perpendicular bisector of *AB* must be the same distance from *A* and *B*. Any point on the perpendicular bisector of *BC* must be the same distance from *B* and *C*. So the point at their intersection is the same distance from *A*, *B*, and *C*, and must be the centre of the circle that passes through *A*, *B*, and *C*.

B. All three perpendicular bisectors intersect at the same point. So Jack only needed two of the bisectors to determine the coordinates of the point.

3 **Consolidation**

(15 to 20 min)

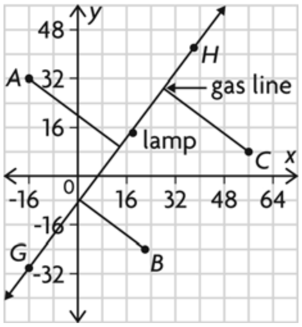
Apply the Math

Using the Solved Examples

Example 2 uses the same context as *Example 1* to apply the concept of a perpendicular from a point to a line. Ensure that students understand the reason for calculating the perpendicular distance from the lamp to the power line. Remind students about the need for units.

Answer to the Key Assessment Question

For question 16, students can refer to *Example 2*. They need to realize that the shortest distance from a home to the gas line is the perpendicular distance and that the distances are in metres.



16. a) To connect home *A*, about 37.2 m of pipe is needed. To connect home *B*, about 26.8 m is needed. To connect home *C*, about 34.8 m is needed. Assuming that the connection charge is proportional to the distance, home *A* will have the highest connection charge.

b) The best location is at approximately (18.1, 14.2).

Closing

Have students read question 19. If necessary, remind students about the difference between the median and the altitude. Students could work on their own or in pairs to describe a strategy they could use to determine the equations and the information they would need to carry out the strategy. Then bring the class back together to discuss students' work.

Assessment and Differentiating Instruction

What You Will See Students Doing ...

When students understand...	If students misunderstand...
Students formulate appropriate analytic geometry strategies to solve problems.	Students may confuse analytic geometry strategies, or they may be unsure of the steps that are required.
Students correctly identify and determine the equations, intersections, and/or distances needed to solve the problems.	Students may misidentify the information required or make errors, such as forgetting to take the negative reciprocal when determining the equation of an altitude or perpendicular bisector. They may not include a concluding sentence or units.
Key Assessment Question 16	
Students formulate an appropriate multi-step strategy to solve each part.	Students may be unsure of the steps needed to solve the problem, or they may simply calculate information without a clear idea of how to use it.
Students accurately determine the distance from each point to *GH* for part a).	Students may make errors when determining the equation of the line that passes through each point and is perpendicular to *GH*, when determining the point of intersection, or when calculating the distances between the points.
Students correctly determine the coordinates of the circumcentre for part b).	Students may calculate the coordinates of the orthocentre, or they may make calculation errors.

Differentiating Instruction | How You Can Respond

EXTRA SUPPORT

1. To help students remember the definitions of median, altitude, and perpendicular bisector, have them work in small groups to talk about strategies they could use to do this. They could draw diagrams or create rhymes, or they could recall that the altitude goes through the vertex, the perpendicular bisector goes through the opposite midpoint, and the median goes through both.
2. The problem-solving steps (Goal, Plan, Needed Information, and Carry Out the Plan) may help students formulate effective strategies. Here is one example:
   Goal: Determine the circumcentre of a triangle (the point that is equidistant from the vertices of the triangle).
   Given: equations of the sides of the triangle
   Plan: Determine the perpendicular bisectors of two sides, and then determine their intersection.
   Needed Information: vertices of the triangle (intersections of equations), midpoints (midpoint formula), and slopes (slope formula) of two sides of the triangle
   Carry Out the Plan: Use the point-slope or slope-intersect form of a linear equation, with the midpoint and the negative reciprocal of the slope of each side, to determine the perpendicular bisectors. Then solve the system of these two equations.

EXTRA CHALLENGE

1. As a tie-in with Chapter 1, ask students to think about the system of three equations for the three perpendicular bisectors of a triangle. Challenge students to use the concept of equivalent systems to explain why this system has a single common solution.

CHAPTER REVIEW

Big Ideas Covered So Far

- The coordinates of the midpoint of a line segment are the means of the coordinates of the endpoints of the line segment, as in the formula $(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.
- The distance between two points, or the length of a line segment, can be determined using the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- A circle with radius *r* and its centre at the origin can be described by the equation $x^2 + y^2 = r^2$.
- Triangles and quadrilaterals can be classified using properties of their sides, if the coordinates of their vertices are known.
- The perpendicular bisector of a line segment, a median or altitude of a triangle, or a midsegment of a triangle or quadrilateral can be constructed if the coordinates of the endpoints or vertices are known.
- Medians, altitudes, and perpendicular bisectors of triangles, and chords of circles, intersect with certain properties.

Using the Frequently Asked Questions

Have students keep their Student Books closed. Display the Frequently Asked Questions on a board. Have students discuss the questions and use the discussion to draw out what the class thinks are good answers. Then have students compare the class answers with the answers on Student Book pages 122 and 123. Students can refer to the answers to the Frequently Asked Questions as they work through the Practice Questions.

Using the Chapter Review

Ask students if they have any questions about any of the topics covered so far in the chapter. Review any topics that students would benefit from considering again. Assign Practice Questions for class work and for homework.

To gain greater insight into students' understanding of the material covered so far in the chapter, you may want to ask them questions such as the following:

- Which types of quadrilateral can be identified using only the slopes of the sides? Using only the lengths of the sides? Using a combination of the slopes and lengths of the sides?
- Which types of quadrilateral have diagonals that do not intersect at their midpoints? (a kite that is not a rhombus)
- *AB* and *CD* are two chords of a circle, and they intersect at *E*. Suppose that you know the lengths of *CE* and *DE*, but all you know about *AE* and *BE* is that they are equal length. How can you determine the common length of *AE* and *BE*? Does it matter whether *E* lies inside or outside the circle?
- What is true about $\triangle ABC$ if the altitude from *A* is also the perpendicular bisector of *BC*?

CHAPTER 2 TEST

For further assessment items, please use Nelson's Computerized Assessment Bank.

1. Determine the midpoint of each line segment, with the given endpoints.
a) endpoints (0, 0) and (–3, 6)
b) endpoints (0, 7) and (–6, 0)
c) endpoints (4, –5) and (–2, 1)
d) endpoints (–3, –4) and (5, 3)

2. Snowy Scarp is located at one end of Long Lake, at map coordinates (–325, 172). Midpoint Island, at map coordinates (–278, 204), is exactly in the middle of Long Lake. If Long Lake is completely straight, what are the coordinates of the other end?

3. A map, with coordinates in kilometres, shows that the Welland Canal meets Lake Erie in Port Colborne, at map coordinates (8, 6). The Welland Canal then goes approximately straight to map coordinates (11, 20), turns slightly, and heads straight to Allanburg, at map coordinates (11, 27). How much longer, to the nearest tenth of a kilometre, is this stretch of the canal than the straight-line distance between Port Colborne and Allanburg?

4. Determine the distance between each point and line.
a) (3, 4) and $y = 4x - 2$
b) (–5, 2) and the line through (2, 7) and (8, –1)

5. Determine the equation of the circle that is centred at the origin and passes through each given point.
a) (–4, 7)
b) (2, –1)
c) (12, –5)
d) (–8, –2)

6. The Chicxulub crater in Mexico is believed to be the site of an asteroid impact that ended the dinosaur era. Although this crater is invisible to the eyes, gravity mapping shows that it has an inner circular ring with a radius of about 40 km and an outer ring, also circular, with a radius of about 90 km. Determine the equations of these two rings on a grid, placing the centre of both rings at (0, 0) and using the scale 1 unit represents 10 km. Then graph both rings.

7. A manufacturing template for a tent is drawn on a grid with coordinates in metres. One panel of the tent has coordinates $P(6, -1)$, $Q(2, -4)$, $R(6, 4)$, and $S(-8, 6)$. What type of quadrilateral is this panel?

8. A triangle has vertices at $A(-2, 0)$, $B(0, -2\sqrt{3})$, and $C(2, 0)$. Classify the triangle according to its side lengths.

9. A quadrilateral has vertices at $A(4, -3)$, $B(1, 6)$, $C(-5, 4)$, and $D(-2, -5)$. Verify that the midsegments of the quadrilateral form a rhombus.

10. A triangle has vertices at $X(-1, 3)$, $Y(5, 8)$, and $Z(3, 2)$. Verify that the perimeter of the triangle formed by the midsegments of $\triangle XYZ$ is half the perimeter of $\triangle XYZ$.

11. The points $A(8, -15)$, $B(0, 17)$, and $C(-8, 15)$ lie on the same circle, which is centred at the origin.
a) Explain why AC is a diameter of the circle.
b) Verify that AB and BC are perpendicular.

12. Suppose that the city councils of Renberg at $R(2, 4)$, Scritz at $S(0, -10)$, and Tzipdorf at $T(-16, -2)$ are planning to build an airport that is an equal distance from all three cities. Determine the coordinates of the site, A, of the new airport.

CHAPTER 2 TEST ANSWERS

1. **a)** (–1.5, 3) **b)** (–3, 3.5) **c)** (1, –2) **d)** (1, –0.5)

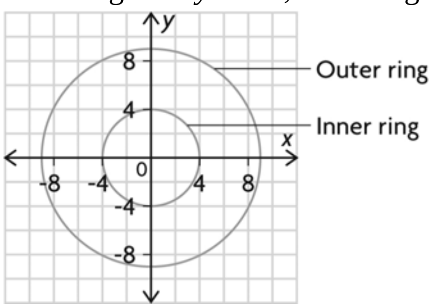
2. (–231, 236)

3. 0.1 km

4. **a)** about 1.5 units **b)** 8.6 units

5. **a)** $x^2 + y^2 = 65$ **b)** $x^2 + y^2 = 5$ **c)** $x^2 + y^2 = 169$ **d)** $x^2 + y^2 = 68$

6. inner ring: $x^2 + y^2 = 16$; outer ring: $x^2 + y^2 = 81$



7. kite

8. equilateral

9. The midpoints of the sides are $P(-2, 5)$, $Q(2.5, 1.5)$, $R(1, -4)$, and $S(-3.5, -0.5)$.
$PQ = RS = \sqrt{4.5^2 + 3.5^2} = \sqrt{32.5} \doteq 5.70,$ or about 5.70 units
$QR = PS = \sqrt{1.5^2 + 5.5^2} = \sqrt{32.5} \doteq 5.70,$ or about 5.70 units
Since all the side lengths are equal, *PQRS* is a rhombus.

10. The midpoints are (2, 5.5), (4, 5), and (1, 2.5). The perimeter of $\triangle XYZ$ is 18.26; the perimeter of the midsegment triangle is 9.13 = 0.5(18.26).

11. **a)** Answers may vary, e.g., the midpoint of *AC* is (0, 0), which is the centre of the circle; or, by the symmetry of a circle about the origin, points (x, y) and $(-x, -y)$ on the circle are always opposite each other.

b) $m_{AB} = \frac{17-(-15)}{0-8} = -4$ and $m_{BC} = \frac{15-17}{-8-0} = \frac{1}{4}$ are negative reciprocals, so *AB* and *BC* are perpendicular.

12. $A(-6, -2)$

CHAPTER TASK

X Marks the Spot

Specific Expectation

- Solve problems involving the slope, length, and midpoint of a line segment (e.g., determine the equation of the right bisector of a line segment, given the coordinates of the endpoints; determine the distance from a given point to a line whose equation is given, and verify using dynamic geometry software).

Introducing the Chapter Task (Whole Class)

Discuss the introduction and the photograph. Then have students complete part A. Ask them to think about the locations of the walk-in clinics. Is there an obvious location for the centre that is equidistant from all four clinics? If so, how could they check this location? If not, invite students to suggest strategies they could use to place the centre as near as possible to being equidistant from all four clinics.

Using the Chapter Task

Have students work individually. Students should produce a grid with points *A*, *B*, *C*, and *D* correctly plotted, and they should estimate the location of the centre, close to (10, 9) or (10, 10). Students should then use analytic geometry to determine the circumcentres of $\triangle ABC$ (the simplest to calculate), $\triangle ABD$, $\triangle ACD$, and $\triangle BCD$, and to determine the distance from each circumcentre to the clinic not used to calculate it. While students' final location for the centre should be somewhere in the quadrilateral formed by these circumcentres (suggest that students plot them on the same grid they used in part A), a wide variety of locations are possible.

Assessing Students' Work

Use the Assessment of Learning chart as a guide for assessing students' work.

Adapting the Task

You can adapt the task in the Student Book to suit the needs of your students. For example:

- Have students work in pairs so they can check and review each other's calculations.
- Have students check the coordinates of the circumcentres using dynamic geometry software, if available. (You could do this with the whole class, if you prefer.)

Student Book Page 127

Preparation and Planning

Pacing

10–15 min	**Introducing the Chapter Task**
45–50 min	**Using the Chapter Task**

Materials
- grid paper
- ruler
- dynamic geometry software (optional)

Nelson Website
http://www.nelson.com/math

Assessment of Learning—What to Look for in Student Work...				
Assessment Strategy: Interview/Observation and Product Marking				
Level of Performance	1	2	3	4
Knowledge and Understanding Knowledge of content	demonstrates **limited** knowledge of content	demonstrates **some** knowledge of content	demonstrates **considerable** knowledge of content	demonstrates **thorough** knowledge of content
Understanding of mathematical concepts	demonstrates **limited** understanding of concepts (e.g., does not understand that circumcentres are required and/or is unable to determine them)	demonstrates **some** understanding of concepts (e.g., understands that circumcentres are required but is unsure how to determine them)	demonstrates **considerable** understanding of concepts (e.g., understands that circumcentres are required and knows how to determine them, but may have difficulty identifying combinations of clinic locations)	demonstrates **thorough** understanding of concepts (e.g., understands exactly which circumcentres are required and knows how to determine them efficiently)
Thinking Use of planning skills • understanding the problem • making a plan for solving the problem	uses planning skills with **limited** effectiveness	uses planning skills with **some** effectiveness	uses planning skills with **considerable** effectiveness	uses planning skills with a **high degree** of effectiveness
Use of processing skills • carrying out a plan • looking back at the solution	uses processing skills with **limited** effectiveness	uses processing skills with **some** effectiveness	uses processing skills with **considerable** effectiveness	uses processing skills with a **high degree** of effectiveness
Use of critical/creative thinking processes	uses critical/creative skills with **limited** effectiveness (e.g., does not present a logical justification for the choice of location)	uses critical/creative skills with **some** effectiveness (e.g., presents a justification for the choice of location that makes a little sense)	uses critical/creative skills with **considerable** effectiveness (e.g., presents a reasonable justification for the choice of location)	uses critical/creative skills with a **high degree** of effectiveness (e.g., presents an exceptionally logical justification for the choice of location)

Assessment of Learning—What to Look for in Student Work...				
Assessment Strategy: Interview/Observation and Product Marking				
Level of Performance	1	2	3	4
Communication Expression and organization of ideas and mathematical thinking, using oral, visual, and written forms	expresses and organizes mathematical thinking with **limited** effectiveness	expresses and organizes mathematical thinking with **some** effectiveness	expresses and organizes mathematical thinking with **considerable** effectiveness	expresses and organizes mathematical thinking with a **high degree** of effectiveness
Communication for different audiences and purposes in oral, visual, and written forms	communicates for different audiences and purposes with **limited** effectiveness	communicates for different audiences and purposes with **some** effectiveness	communicates for different audiences and purposes with **considerable** effectiveness	communicates for different audiences and purposes with a **high degree** of effectiveness
Use of conventions, vocabulary, and terminology of the discipline in oral, visual, and written forms	uses conventions, vocabulary, and terminology of the discipline with **limited** effectiveness	uses conventions, vocabulary, and terminology of the discipline with **some** effectiveness	uses conventions, vocabulary, and terminology of the discipline with **considerable** effectiveness	uses conventions, vocabulary, and terminology of the discipline with a **high degree** of effectiveness
Application Application of knowledge and skills in familiar contexts	applies knowledge and skills in familiar contexts with **limited** effectiveness (e.g., completely fails to determine equations and/or intersections of perpendicular bisectors)	applies knowledge and skills in familiar contexts with **some** effectiveness (e.g., makes many errors when determining equations and/or intersections of perpendicular bisectors)	applies knowledge and skills in familiar contexts with **considerable** effectiveness (e.g., makes few errors when determining equations and/or intersections of perpendicular bisectors)	applies knowledge and skills in familiar contexts with a **high degree** of effectiveness (e.g., devises an efficient strategy using only four perpendicular bisectors; makes few or no errors when determining equations and/or intersections of perpendicular bisectors)
Transfer of knowledge and skills to new contexts	transfers knowledge and skills to new contexts with **limited** effectiveness	transfers knowledge and skills to new contexts with **some** effectiveness	transfers knowledge and skills to new contexts with **considerable** effectiveness	transfers knowledge and skills to new contexts with a **high degree** of effectiveness
Making connections within and between various contexts	makes connections within and between various contexts with **limited** effectiveness	makes connections within and between various contexts with **some** effectiveness	makes connections within and between various contexts with **considerable** effectiveness	makes connections within and between various contexts with a **high degree** of effectiveness