

Direct Variation

You will need

- a calculator

► GOAL

Understand and apply direct variation.

Learn about the Math

Shannon and Kate are making cookies for their school bake sale. The table shows the number of cookies, y , that the girls can produce in x minutes.

Time in minutes, x	Number of cookies, y
11	33
12	36
15	45
33	99

The girls would like to determine how long it will take them to bake 300 cookies. Kate recognizes this as a **direct variation** and knows that if she can find a relationship between the number of cookies they have baked and the time it has taken to bake them, she will be able to determine the amount of time it will take to make 300 cookies.

direct variation

a relationship between two variables in which the ratio between the two variables is a constant

? What relationship can be found between the number of cookies made and the time needed to make them?

Example 1: Determining a relationship between two variables

Determine how long it will take the girls to bake 300 cookies.

Qi's Solution

$$\frac{y}{x} = \frac{33}{11} = \frac{3}{1} = 3; \frac{y}{x} = \frac{36}{12} = \frac{3}{1} = 3;$$

$$\frac{y}{x} = \frac{45}{15} = \frac{3}{1} = 3; \frac{y}{x} = \frac{99}{33} = \frac{3}{1} = 3;$$

I determined the ratio of the number of cookies made to the number of minutes it took to make them. In each case, the ratio was 3.



3 is the **constant of variation**

$\frac{y}{x} = \frac{3}{1}$ or $y = 3x$ (using cross multiplication)

I know that this means that y is directly proportional to x . This relationship can be shown by the equation, $\frac{y}{x} = k$, or $y = kx$, where k is the constant of variation.

$$(\text{the number of cookies}) = 3(\text{the number of minutes})$$

$$y = 3x$$

This equation summarizes the relationship between the two variables.

$$\begin{aligned} 300 &= 3 \times x \\ 300 \div 3 &= x \\ 100 &= x \end{aligned}$$

I used this equation to estimate how long it will take to produce 300 cookies. I replaced the number of cookies (y) with 300 and solved for the time in minutes (x).

It will take 100 min (or 1 h 40 min) to make 300 cookies.

Reflecting

1. Which of these statements does *not* describe a direct variation? Explain how you know.
 - a) y varies directly as x .
 - b) y is directly proportional to x .
 - c) *The product of x and y is constant.*
2. Explain what you think the "direct" in "direct variation" means.
3. Complete this statement. In a direct variation the ratio between two variables is _____.

constant of variation

the constant ratio in a direct variation (usually represented by k)

Work with the Math

Example 2: Determining if a table of values shows a direct variation

Determine whether this table of values shows a direct variation. If so, find the constant of variation.

x	y
2	10
3	15
4	20

Maggie's Solution

$$\frac{10}{2} = 5; \frac{15}{3} = 5; \frac{20}{4} = 5$$

I determined whether $\frac{y}{x}$ is a constant, by substituting each pair of the values in the table for x and y .

This shows that y and x have a direct variation, and the constant of variation is 5.



A Checking

4. Determine if this table of values shows a direct variation. If so, find the constant of variation.

x	y
3	6
4	8
5	15

B Practising

5. In each of the following, determine if the table of values shows a direct variation. If so, find the constant of variation.

a)

x	y
2	8
3	12
8	24

c)

x	y
2	12
4	24
6	36

b)

x	y
6	18
10	30
14	42

d)

x	y
1	4
3	9
4	16

6. In each of the following, x and y have a direct variation.

a) y is 48 when x is 24. Find y when x is 6.

b) y is 12 when x is 3. Find y when x is 8.

c) y is 32 when x is 4. Find y when x is 5.

d) y is 15 when x is 5. Find y when x is 7.

7. The number of books sold and the total cost have a direct variation. If 5 books cost \$36.25, find the total cost of 8 books.

8. The following table of values shows Chris's time travelling, x , and his distance travelled, y . Assume he is travelling at a constant rate. Determine his rate of speed in kilometres per hour (km/h).

Time (h), x	Distance (km), y
1	20
2	40
3	60

9. In each of the following, x and y have a direct variation. Find the constant of variation for each table of values.

a)

x	y
1	3
2	6
3	9

b)

x	y
2	10
3	15
4	20

c)

x	y
3	21
4	28
6	42

10. In the following table of values, x and y have a direct variation. Find the constant of variation and then use it to determine the missing value in the table.

x	y
6	12
8	16
9	?

11. Determine if the table of values shows a direct variation. If so, find the constant of variation, and then use it to complete the table.

x	y
2	6
6	18
7	?
?	27
11	33

© Extending

12. Create a set of variables with a direct variation. Make a table of values for the variables that displays the direct relationship.