CHAPTER 5: APPLYING QUADRATIC MODELS

### Specific Expectations Addressed in the Chapter

- Collect data that can be represented as a quadratic relation, from experiments using appropriate equipment and technology (e.g., concrete materials, scientific probes, graphing calculators), or from secondary sources (e.g., the Internet, Statistics Canada); graph the data and draw a curve of best fit, if appropriate, with or without the use of technology. [5.4, Chapter Task]

- Identify, through investigation using technology, the effect on the graph of \( y = x^2 \) of transformations (i.e., translations, reflections in the \( x \)-axis, vertical stretches or compressions) by considering separately each parameter \( a, h, \) and \( k \) [i.e., investigate the effect on the graph of \( y = x^2 \) of \( a, h, \) and \( k \) in \( y = x^2 + k, y = (x - h)^2, \) and \( y = ax^2 \)]. [5.1, 5.2]

- Explain the roles of \( a, h, \) and \( k \) in \( y = a(x - h)^2 + k \), using the appropriate terminology to describe the transformations, and identify the vertex and the equation of the axis of symmetry. [5.1, 5.2, 5.3, 5.5, 5.6, Chapter Task]

- Sketch, by hand, the graph of \( y = a(x - h)^2 + k \) by applying transformations to the graph of \( y = x^2 \). [5.3, Chapter Task]

- Determine the equation, in the form \( y = a(x - h)^2 + k \), of a given graph of a parabola. [5.4, 5.6, Chapter Task]

- Sketch or graph a quadratic relation whose equation is given in the form \( y = ax^2 + bx + c \), using a variety of methods (e.g., sketching \( y = x^2 - 2x - 8 \) using intercepts and symmetry; sketching \( y = 3x^2 - 12x + 1 \) by [completing the square and applying transformations]; graphing \( h = -4.9t^2 + 50t + 1.5 \) using technology). [5.5, 5.6]

- Determine the zeros and the maximum or minimum value of a quadratic relation from its graph (i.e., using graphing calculators or graphing software) or from its defining equation (i.e., by applying algebraic techniques). [5.5, Chapter Task]

- Solve problems arising from a realistic situation represented by a graph or an equation of a quadratic relation, with and without the use of technology (e.g., given the graph or the equation of a quadratic relation representing the height of a ball over elapsed time, answer questions such as the following: What is the maximum height of the ball? After what length of time will the ball hit the ground? Over what time interval is the height of the ball greater than 3 m?). [5.3, 5.4, 5.5, 5.6, Chapter Task]

### Prerequisite Skills Needed for the Chapter

- Create a table of values for a relation, and use it to graph the relation.

- Apply reflections on a coordinate grid.

- Recognize and sketch quadratic relations in standard or factored form.

- Apply translations on a coordinate grid.

- Identify the zero(s), equation of the axis of symmetry, and vertex of a quadratic relation in standard or factored form, based on its graph.

- Understand and apply the order of operations.

- Create a scatter plot, and draw a line or curve of good fit.

- Factor quadratic expressions.
What “big ideas” should students develop in this chapter?

Students who have successfully completed the work of this chapter and who understand the essential concepts and procedures will know the following:

- The graphs of \( y = ax^2 \), \( y = x^2 + k \), and \( y = (x - h)^2 \) are obtained from the graph of \( y = x^2 \) by a vertical stretch/compression and/or reflection in the x-axis, a vertical translation, and a horizontal translation, respectively.
- The vertex form of a quadratic relation is \( y = a(x - h)^2 + k \).
- The equation of a quadratic relation in vertex form can be determined from its graph, given information such as the coordinates of the vertex and one other point on the graph.
- The vertex, equation of the axis of symmetry, maximum or minimum value, and zeros of a quadratic relation can be determined from its graph or from its equation.
- Quadratic relations can be used to model many real-world situations.

### Chapter 5: Planning Chart

<table>
<thead>
<tr>
<th>Lesson Title</th>
<th>Lesson Goal</th>
<th>Pacing 12 days</th>
<th>Materials/Masters Needed</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Getting Started</strong>, pp. 246–249</td>
<td>Use concepts and skills developed prior to this chapter.</td>
<td>2 days</td>
<td>grid paper; ruler; coloured pencils or markers; scissors; Diagnostic Test</td>
</tr>
<tr>
<td><strong>Lesson 5.1</strong>: Stretching/Reflecting Quadratic Relations, pp. 250–258</td>
<td>Examine the effect of the parameter ( a ) in the equation ( y = ax^2 ) on the graph of the equation.</td>
<td>1 day</td>
<td>graphing calculator; dynamic geometry software, or grid paper and ruler; Lesson 5.1 Extra Practice</td>
</tr>
<tr>
<td><strong>Lesson 5.2</strong>: Exploring Translations of Quadratic Relations, pp. 259–262</td>
<td>Investigate the roles of ( h ) and ( k ) in the graphs of ( y = x^2 + k ), ( y = (x - h)^2 ), and ( y = (x - h)^2 + k ).</td>
<td>1 day</td>
<td>grid paper; ruler; graphing calculator</td>
</tr>
<tr>
<td><strong>Lesson 5.3</strong>: Graphing Quadratics in Vertex Form, pp. 263–272</td>
<td>Graph a quadratic relation in the form ( y = a(x - h)^2 + k ) by using transformations.</td>
<td>1 day</td>
<td>grid paper; ruler; Lesson 5.3 Extra Practice</td>
</tr>
<tr>
<td><strong>Lesson 5.4</strong>: Quadratic Models Using Vertex Form, pp. 275–284</td>
<td>Write the equation of the graph of a quadratic relation in vertex form.</td>
<td>1 day</td>
<td>grid paper; ruler; graphing calculator; spreadsheet program (optional); Lesson 5.4 Extra Practice</td>
</tr>
<tr>
<td><strong>Lesson 5.5</strong>: Solving Problems Using Quadratic Relations, pp. 285–295</td>
<td>Model and solve problems using the vertex form of a quadratic relation.</td>
<td>1 day</td>
<td>grid paper; ruler; Lesson 5.5 Extra Practice</td>
</tr>
<tr>
<td><strong>Lesson 5.6</strong>: Connecting Standard and Vertex Forms, pp. 297–302</td>
<td>Sketch or graph a quadratic relation with an equation of the form ( y = ax^2 + bx + c ) using symmetry.</td>
<td>1 day</td>
<td>grid paper; ruler; Lesson 5.6 Extra Practice</td>
</tr>
<tr>
<td><strong>Mid-Chapter Review</strong>: pp. 273–274</td>
<td></td>
<td>4 days</td>
<td>Mid-Chapter Review Extra Practice; Chapter Review Extra Practice; Chapter Test</td>
</tr>
</tbody>
</table>
Introduce the chapter by discussing the photograph on pages 244 and 245 of the Student Book. The parabolic arches have a curve that is tighter near the vertex, whereas circular arches have the same curvature all the way around. A parabola is the strongest shape for the arch of a bridge or a similar structure, such as the cables of a suspension bridge. The reason for this could be a small research challenge for curious students.

Ask students to think about how they could use a quadratic relation to model the parabolas in the photograph. Ask questions such as: What information would you need to know? Would a photograph taken from a different angle be easier to use? Would \( y = x^2 \) be the best relation to start with, or would \( y = -x^2 \) be more useful?
GETTING STARTED

Using the Words You Need to Know

Even though students may match every term with the description that most closely represents it, they may need to use the process of elimination to match some of the terms. If students are unsure about the definition of a term, suggest that they look up the definition in the Glossary, and write the definition in their notes. Ask them to provide their own example for the term as well. Ask questions such as these: How are a reflection and a translation the same? How are they different?

Using the Skills and Concepts You Need

Work through each of the examples in the Student Book (or similar examples, if you would like students to see more examples), and answer any questions that students have. Ask students to relate the solution to the sketch of the graph. Also ask students to look over the Practice questions to see if there are any questions they do not know how to solve. Be sure to refer students to the Study Aid chart in the margin of the Student Book for more help. Allow students to work on the Practice questions in class. Assign any unfinished questions for homework.

Using the Applying What You Know

Have students work in pairs on the activity. Have them read the whole activity before beginning their work. One partner could complete part A while the other partner completes part B. For the rest of the activity, one partner could apply the transformations and the other partner could record, switching roles for each figure. In part D, students should be thinking about whether a translation by itself is enough, or whether other transformations (such as reflections) are needed. Use part H as the basis for a class discussion about the most efficient way to choose a sequence of transformations. Extend the discussion, if desired, to a non-algebraic example of transformations from one position of the U-shaped figure to another, mentioning that this figure is somewhat similar to a parabola.

Answers to Applying What You Know

A.–D.
E.–G. Answers may vary, e.g.,

<table>
<thead>
<tr>
<th>Move</th>
<th>Figure</th>
<th>Original Coordinates</th>
<th>Transformation</th>
<th>New Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>yellow</td>
<td>A(–6, 4), B(–3, 7)</td>
<td>translation 5 units right, 6 units down</td>
<td>A'(–1, –2), B'(2, 1)</td>
</tr>
<tr>
<td>2</td>
<td>purple</td>
<td>A(–2, 3), B(0, 1)</td>
<td>reflection in y-axis</td>
<td>A'(2, 3), B'(0, 1)</td>
</tr>
<tr>
<td>3</td>
<td>purple</td>
<td>A'(2, 3), B'(0, 1)</td>
<td>translation 1 unit right, 1 unit down</td>
<td>A'(3, 2), B'(1, 0)</td>
</tr>
<tr>
<td>4</td>
<td>green</td>
<td>A(–4, 5), B(–3, 1)</td>
<td>reflection in x-axis</td>
<td>A'(–4, –5), B'(–3, –1)</td>
</tr>
<tr>
<td>5</td>
<td>green</td>
<td>A'(–4, –5), B'(–3, –1)</td>
<td>translation 2 units right, 2 units up</td>
<td>A''(–2, –3), B''(–1, 1)</td>
</tr>
<tr>
<td>6</td>
<td>blue</td>
<td>A(–1, 5), B(1, 8)</td>
<td>translation 1 unit left, 7 units down</td>
<td>A'(–2, –2), B'(0, 1)</td>
</tr>
<tr>
<td>7</td>
<td>blue</td>
<td>A'(–2, –2), B'(0, 1)</td>
<td>reflection in x-axis</td>
<td>A''(–2, 2), B''(0, –1)</td>
</tr>
<tr>
<td>8</td>
<td>orange</td>
<td>A(–8, 1), B(–5, 3)</td>
<td>reflection in x-axis</td>
<td>A'(–8, –1), B'(–5, –3)</td>
</tr>
<tr>
<td>9</td>
<td>orange</td>
<td>A'(–8, –1), B'(–5, –3)</td>
<td>translation 5 units right</td>
<td>A''(–3, –1), B''(0, –3)</td>
</tr>
<tr>
<td>10</td>
<td>orange</td>
<td>A''(–3, –1), B''(0, –3)</td>
<td>reflection in y-axis</td>
<td>A'''(3, --1), B'''(0, –3)</td>
</tr>
</tbody>
</table>

H. 10 moves; 9 moves are possible if the orange figure is translated and then rotated.

<table>
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<tr>
<th>Initial Assessment</th>
<th>What You Will See Students Doing ...</th>
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<tbody>
<tr>
<td><strong>When students understand ...</strong></td>
<td><strong>If students misunderstand ...</strong></td>
</tr>
<tr>
<td>Students locate points using coordinates and identify the coordinates of points on a coordinate grid.</td>
<td>Students have difficulty working with coordinates. They may not realize that the first coordinate describes the horizontal position and the second coordinate describes the vertical position. They may confuse the direction for positive and negative coordinates.</td>
</tr>
<tr>
<td>Students correctly perform translations, reflections, and rotations.</td>
<td>Students cannot perform translations, reflections, or rotations correctly. The positions or orientations may be incorrect.</td>
</tr>
<tr>
<td>Students correctly describe a sequence of transformations.</td>
<td>Students cannot describe a sequence of transformations correctly. They may state the distance or direction of a translation incorrectly, or they may name the wrong axis for a reflection.</td>
</tr>
</tbody>
</table>
5.1 STRETCHING/REFLECTING QUADRATIC RELATIONS

Lesson at a Glance

GOAL
Examine the effect of the parameter $a$ in the equation $y = ax^2$ on the graph of the equation.

Prerequisite Skills/Concepts
- Create a table of values for a relation, and use it to graph the relation.
- Apply reflections on a coordinate grid.

Specific Expectations
- Identify, through investigation using technology, the effect on the graph of $y = x^2$ of transformations (i.e., translations, reflections in the $x$-axis, vertical stretches or compressions) by considering separately each parameter $a$, $h$, and $k$ [i.e., investigate the effect on the graph of $y = x^2$ of $a$, $h$, and $k$ in $y = x^2 + k$, $y = (x – h)^2$, and $y = ax^2$].
- Explain the roles of $a$, $h$, and $k$ in $y = a(x – h)^2 + k$, using the appropriate terminology to describe the transformations [and identify the vertex and the equation of the axis of symmetry].

Mathematical Process Focus
- Reasoning and Proving
- Connecting

Student Book Pages 250–258

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<td>10–15 min Introduction</td>
</tr>
<tr>
<td>30–40 min Teaching and Learning</td>
</tr>
<tr>
<td>10–15 min Consolidation</td>
</tr>
</tbody>
</table>

Materials
- graphing calculator
- dynamic geometry software, or grid paper and ruler

Recommended Practice
Questions 4, 5, 6, 8, 11, 12

Key Assessment Question
Question 5

Extra Practice
Lesson 5.1 Extra Practice

New Vocabulary/Symbols
parameter
vertical stretch
vertical compression

Nelson Website
http://www.nelson.com/math

MATH BACKGROUND | LESSON OVERVIEW

- Students should be able to graph points on a coordinate grid.
- Students investigate the relationship between the value of $a$ in $y = ax^2$ and the shape of the graph of $y = ax^2$.
- Students determine that the graph of $y = x^2$ is vertically stretched or compressed by a factor of $a$ to produce the graph of $y = ax^2$.
- Students determine that the graph of a parabola is stretched vertically when $a > 1$ or $a < -1$. They determine that the graph is also reflected in the $x$-axis when $a$ is negative.
- Students determine that the graph of a parabola is compressed vertically when $-1 < a < 1$. They determine that the graph is also reflected in the $x$-axis when $a$ is negative.
- Students learn the technique of modelling a natural or artificial parabolic form in a photograph by superimposing a grid with the origin at the vertex and adjusting the value of $a$ in the relation $y = ax^2$. 
Introducing the Lesson

(10 to 15 min)

Have students bring in some appropriate real-data graphs (such as graphs of weather data or exchange-rate fluctuations). Students can sketch parabolas, as done in the stock chart at the beginning of the lesson (page 250). Discuss the advantages and disadvantages of fitting parabolas to these kinds of data, compared with drawing straight-line graphs.

Teaching and Learning

(30 to 40 min)

Investigate the Math

Have students work in pairs, and record their responses to the prompts.

- Students should think about $a$ as greater than 1 in parts C and D, between 0 and 1 in parts E and F, and negative in parts G and H. Make sure that students understand the role of $a$ in the transformation from $y = x^2$ to $y = ax^2$. Part I focuses on this, as does part L in Reflecting.
- Ask students about the special cases $a = 1$ and $a = -1$: How are they like the other values of $a$? How are they unlike the other values of $a$?

Answers to Investigate the Math

A.–B. 

As $a$ increases, the graphs appear to be getting narrower.

D. I would expect the graph of $y = 3x^2$ to appear between the graphs of $y = 2x^2$ and $y = 5x^2$.

My conjecture was correct.

E. I would expect the graphs to appear between the graph of $y = x^2$ and the $x$-axis.

When $0 < a < 1$, the parabola gets wider.
F. I would expect the graph of \( y = \frac{3}{4} x^2 \) to appear between the graphs of \( y = \frac{1}{2} x^2 \) and \( y = x^2 \).

![Graph showing the relationship between the graphs of \( y = \frac{3}{4} x^2 \), \( y = \frac{1}{2} x^2 \), and \( y = x^2 \).]

My conjecture was correct.

G. When \( a < 0 \), the parabola is reflected in the \( x \)-axis.

![Graph showing the reflection of a parabola when \( a < 0 \).]

H. Answers may vary, e.g., I would expect the graph of \( y = -2x^2 \) to open down and lie between the graphs of \( y = -4x^2 \) and \( y = -\frac{1}{4} x^2 \).

![Graph showing the relationship between the graphs of \( y = -2x^2 \), \( y = -4x^2 \), and \( y = -\frac{1}{4} x^2 \).]

My conjecture was correct.

I. When compared with the graph of \( y = x^2 \), the graph of \( y = ax^2 \) is a parabola that has been stretched or compressed vertically by a factor of \( a \). If \( a > 1 \), the graph has been stretched vertically. If \( 0 < a < 1 \), the graph has been compressed vertically. When \( a < 0 \), the graph has still been stretched/compressed vertically but it has also been reflected across the \( x \)-axis.

**Technology-Based Alternative Lesson**

Use dynamic geometry software, such as *The Geometer’s Sketchpad*, to show students the parabola defined by \( y = ax^2 \) for many values of \( a \) as described in Appendix B-16.

If TI-nspire calculators are available, have students enter the equation \( y = x^2 \) in the entry line of a Graphs & Geometry application for the investigation. Students can see the equation change as they change the shape of the graph. To change the shape of the graph, have students hold the cursor over the graph so it changes to a double-headed arrow. Then, they can hold down the Click button until the cursor changes to a closed hand, and use the arrow keys to move the parabola. Students will have the opportunity to see what happens when \( a = 0 \). To compare the \( y \)-values of several relations, have students enter a variety of equations, such as \( y = x^2 \), \( y = 2x^2 \), \( y = -x^2 \), \( y = 3x^2 \), and \( y = -2x^2 \), in the entry line of a Graphs & Geometry application. Students can refer to Appendix B-42.
Answers to Reflecting

J. The orange parabola has the greatest value of $a$, since it is the narrowest graph. The green parabola has the least value of $a$, since it is the widest graph. Both the green and red parabolas have negative $a$ values, because they open downward.

K. The $x$-coordinates remain the same. The $y$-coordinates are multiplied by $a$. The shape of the graph near the vertex remains almost the same.

L. i) $a < -1$ or $a > 1$  
   ii) $-1 < a < 0$ or $0 < a < 1$  
   iii) $a < 0$

3 Consolidation

(10 to 15 min)

Apply the Math

Using the Solved Examples

*Example 1* introduces the idea of plotting parabolas of form $y = ax^2$ by transformations, specifically stretches, compressions, and/or reflections. Read the problem and solution with the class. Discuss the five-point technique with the whole class.

*Example 2* introduces the technique of fitting a quadratic curve to a photograph. If dynamic geometry software is available, have students work through the second solution in pairs or participate in a class demonstration of the solution. Otherwise, have one partner in each pair describe the technique used in the first solution to the other partner. Encourage students to make predictions about the effects of different values of $a$.

Answer to the Key Assessment Question

After students complete question 5, provide an opportunity for them to share their descriptions.

5. a) The point (1, 1) on $y = x^2$ corresponds to the point (1, 4) on the black graph, so it is a vertical stretch by a factor of 4. The equation of the black graph is $y = 4x^2$.

   b) The point (2, 4) on $y = x^2$ corresponds to the point (2, –2) on the black graph, so it is a vertical compression by a factor of $\frac{1}{2}$, followed by a reflection in the $x$-axis. The equation of the black graph is $y = -\frac{1}{2}x^2$.

   c) The point (2, 4) on $y = x^2$ corresponds to the point (2, –10) on the black graph, so it is a vertical stretch by a factor of 2.5, followed by a reflection in the $x$-axis. The equation of the black graph is $y = -2.5x^2$.

   d) The point (2, 4) on $y = x^2$ corresponds to the point (2, 1) on the black graph, so it is a vertical compression by a factor of $\frac{1}{4}$. The equation of the black graph is $y = \frac{1}{4}x^2$. 

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**Closing**

Have students read question 12. Ask students to work in pairs, with each partner explaining part a) or part b) to the other. Bring the class together to share and discuss some of their explanations. Discuss part c) as a class, covering graphical as well as numerical explanations.

### Assessment and Differentiating Instruction

**What You Will See Students Doing ...**

<table>
<thead>
<tr>
<th>When students understand...</th>
<th>If students misunderstand...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students correctly apply vertical stretches/compressions and reflections to the graph of ( y = x^2 ).</td>
<td>Students may apply stretches when the value of ( a ) should result in a compression, or vice versa. Students may not apply the same factor for each point on the graph, resulting in a graph that is not a parabola.</td>
</tr>
<tr>
<td>Students correctly predict the shapes (width, direction of opening) of the graph of ( y = ax^2 ) based on the value of ( a ).</td>
<td>Students cannot predict, or they predict incorrectly, the shape (width, direction of opening) of the graph of ( y = ax^2 ) based on the value of ( a ). They may predict that the graph will be narrower when the value of ( a ) will actually result in a wider graph.</td>
</tr>
</tbody>
</table>

**Key Assessment Question 5**

| Students correctly describe the transformation(s) that produce each graph. | Students do not include a negative value of \( a \) when the graph is a reflection, or they do not use the correct factor for the stretch or compression. |
| Students write the correct equation for each graph. | Students may confuse compressions with stretches when writing the equation for each graph. |

### Differentiating Instruction | How You Can Respond

**EXTRA SUPPORT**

1. If students are having difficulty applying vertical stretches/compressions and reflections, remind them to multiply the \( y \)-values in the table of values by the scale factor \( a \).
2. Remind students that the scale factor \( a \) can be read from the graph by determining points on the two parabolas that are vertically in line and have integer coordinates (e.g., the points \((2, 4)\) and \((2, 8)\) tell you that \( a = 2 \) because the \( y \)-coordinates are doubled).
3. If students are having difficulty predicting the shape of the graph of \( y = ax^2 \), use a technology demonstration so they can see the effects of changing the value of \( a \). (This is essentially a repeat of the investigation, but struggling students may not have useful results from the investigation or they may benefit from repetition. Repeating the activity one-on-one can provide useful reinforcement.)

**EXTRA CHALLENGE**

1. Ask students who show a good grasp of the material what would happen if they applied a horizontal stretch or compression: Which coordinates would change? How would the equation change? How would the graph change? Then ask what would happen if they applied a reflection about the \( y \)-axis.
5.2 EXPLORING TRANSLATIONS OF QUADRATIC RELATIONS

Lesson at a Glance

Prerequisite Skills/Concepts
- Recognize and sketch quadratic relations in standard or factored form.
- Apply translations on a coordinate grid.
- Identify the equation of the axis of symmetry and vertex of a quadratic relation in standard or factored form, based on its graph.

Specific Expectations
- Identify, through investigation using technology, the effect on the graph of $y = x^2$ of transformations (i.e., translations, reflections in the $x$-axis, vertical stretches or compressions) by considering separately each parameter $[a,] h$, and $k$ [i.e., investigate the effect on the graph of $y = x^2$ of $[a,] h$, and $k$ in $y = x^2 + k$, $y = (x - h)^2$, and $y = ax^2$].
- Explain the roles of $[a,] h$, and $k$ in $y = a(x - h)^2 + k$, using the appropriate terminology to describe the transformations, and identify the vertex and the equation of the axis of symmetry.

Mathematical Process Focus
- Reasoning and Proving
- Connecting

MATH BACKGROUND | LESSON OVERVIEW

- Students explore the connections between quadratic relations of the form $y = x^2 + k$, $y = (x - h)^2$, and $y = (x - h)^2 + k$, transformations of the graphs of these relations, and the location of each vertex and axis of symmetry.
- Students determine that the graph of $y = (x - h)^2 + k$ is obtained from the graph of $y = x^2$ by a horizontal translation $h$ units right (or $-h$ units left if $h < 0$) and $k$ units up (or $-k$ units down if $k < 0$).
- Students determine that the vertex of the parabola of $y = (x - h)^2 + k$ is at $(h, k)$ and that its line of symmetry is $x = h$.
- It may seem counter-intuitive to students that the value of $h$ is subtracted from $x$ to move the graph in the direction of increasing $x$-values. One way to conceptualize this is to think of the axis of symmetry passing through the point where $x = 0$ for $y = x^2$ and passing through the point where $x - h = 0$ for $y = (x - h)^2$. 

GOAL
Investigate the roles of $h$ and $k$ in the graphs of $y = x^2 + k$, $y = (x - h)^2$, and $y = (x - h)^2 + k$. 

Student Book Pages 259–262

Preparation and Planning

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<td>Teaching and Learning</td>
<td></td>
</tr>
<tr>
<td>10–15 min</td>
<td>Consolidation</td>
<td></td>
</tr>
</tbody>
</table>

Materials
- grid paper
- ruler
- graphing calculator

Recommended Practice
Questions 1, 3, 4, 5

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Introducing the Lesson

(5 to 10 min)

Have students look at the parabola design on page 259. Questions you might ask include the following: What kinds of symmetry does the design have? Have any of the parabolas been translated from \( y = x^2 \)? Have any been stretched or compressed from \( y = x^2 \)? Have any been reflected from \( y = x^2 \)? How do you know?

Teaching and Learning

(35 to 45 min)

Explore the Math

Have students work in pairs and record their responses to the prompts in the investigation.

- As students complete the investigation, ask appropriate questions to make sure that they understand the connections between the equation
  \[ y = (x - h)^2 + k \]
  and translations of the graph of \( y = x^2 \), as well as the location of the vertex and axis of symmetry.
- You may want to point out that these connections also apply to linear equations. For example, the graph of \( y = mx \) is a straight line through the origin with slope \( m \). You can think of the origin as the vertex and translate it to \((h, k)\). The graph of \( y = m(x - h) + k \) is the straight line through \((h, k)\) with slope \( m \).
- For part H, students may benefit from trying their equations with a graphing calculator to see the effect.

Answers to Explore the Math

A. The graph of \( y = x^2 \) is translated 3 units down to obtain the graph of \( y = x^2 - 3 \).

B. Answers may vary, e.g.,
C. Answers may vary, e.g.,

<table>
<thead>
<tr>
<th>Value of k</th>
<th>Equation</th>
<th>Distance and Direction from ( y = x^2 )</th>
<th>Vertex</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( y = x^2 )</td>
<td>not applicable</td>
<td>(0, 0)</td>
</tr>
<tr>
<td>-3</td>
<td>( y = x^2 - 3 )</td>
<td>down 3</td>
<td>(0, -3)</td>
</tr>
<tr>
<td>1</td>
<td>( y = x^2 + 1 )</td>
<td>up 1</td>
<td>(0, 1)</td>
</tr>
<tr>
<td>4</td>
<td>( y = x^2 + 4 )</td>
<td>up 4</td>
<td>(0, 4)</td>
</tr>
</tbody>
</table>

D. Answers may vary, e.g.,

The graph of \( y = x^2 \) is translated 3 units right to obtain the graph of \( y = (x - 3)^2 \).

<table>
<thead>
<tr>
<th>Value of ( h )</th>
<th>Equation</th>
<th>Distance and Direction from ( y = x^2 )</th>
<th>Vertex</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( y = x^2 )</td>
<td>not applicable</td>
<td>(0, 0)</td>
</tr>
<tr>
<td>3</td>
<td>( y = (x - 3)^2 )</td>
<td>right 3</td>
<td>(0, 3)</td>
</tr>
<tr>
<td>-1</td>
<td>( y = (x + 1)^2 )</td>
<td>left 1</td>
<td>(0, -1)</td>
</tr>
<tr>
<td>2</td>
<td>( y = (x - 2)^2 )</td>
<td>right 2</td>
<td>(0, 2)</td>
</tr>
</tbody>
</table>

E. The type of transformation that has been applied to \( y = x^2 \) to obtain each of the graphs in these tables is a translation.

F. If the graph of \( y = x^2 \) is translated \( k \) units up, add \( k \) to the equation; if it is translated \( k \) units down, subtract \( k \) from the equation. If the graph of \( y = x^2 \) is translated \( h \) units right, replace \( x \) in the equation with \( (x - h) \); if the graph of \( y = x^2 \) is translated \( h \) units left, replace \( x \) with \( (x + h) \).

G. The type of transformation that has been applied to \( y = x^2 \) to obtain each of the graphs in these tables is a translation.

<table>
<thead>
<tr>
<th>Value of ( h )</th>
<th>Value of ( k )</th>
<th>Equation</th>
<th>Relationship to ( y = x^2 )</th>
<th>Vertex</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>( y = x^2 )</td>
<td>not applicable</td>
<td>(0, 0)</td>
</tr>
<tr>
<td>-3</td>
<td>-5</td>
<td>( y = (x + 3)^2 - 5 )</td>
<td>left 3, down 5</td>
<td>(-3, -5)</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>( y = (x - 4)^2 + 1 )</td>
<td>right 4, up 1</td>
<td>(4, 1)</td>
</tr>
<tr>
<td>-2</td>
<td>6</td>
<td>( y = (x + 2)^2 + 6 )</td>
<td>left 2, up 6</td>
<td>(-2, 6)</td>
</tr>
<tr>
<td>-5</td>
<td>-3</td>
<td>( y = (x + 5)^2 - 3 )</td>
<td>left 5, down 3</td>
<td>(-5, -3)</td>
</tr>
</tbody>
</table>

H. The equations are \( y = x^2 \), \( y = -x^2 \), \( y = x^2 - 4 \), \( y = -x^2 + 4 \), \( y = (x - 4)^2 \), \( y = -(x - 4)^2 \), \( y = (x + 4)^2 \), and \( y = -(x + 4)^2 \).

I. The vertex is located at \((h, k)\).
Answers to Reflecting

J.  i) The graph is translated up as \( k \) increases from 0 and down as \( k \) decreases from 0.
   ii) The value of \( k \) is added to the \( y \)-coordinate, so \( (x, y) \) becomes \( (x, y + k) \).
   iii) The vertex at \( (0, k) \) moves up as \( k \) increases and down as \( k \) decreases.
       The equation of the axis of symmetry, \( x = 0 \), is not affected.

K.  i) The graph is translated right as \( h \) increases from 0 and left as \( h \) decreases from 0.
    ii) The value of \( h \) is added to the \( x \)-coordinate, so \( (x, y) \) becomes \( (x + h, y) \).
    iii) The vertex at \( (h, 0) \) and the equation of the axis of symmetry, \( x = h \),
        move right as \( h \) increases and left as \( h \) decreases.

L.  i) Their shapes are the same as the parabola defined by \( y = x^2 \).
    ii) The equation of the axis of symmetry is \( x = h \).
    iii) The coordinates of the vertex are \( (h, k) \).

Technology-Based Alternative Lesson

If TI-nspire calculators are available, students can enter the relation \( y = x^2 \) on
the entry line of a Graphs & Geometry application. Have them move the
cursor to the vertex of the parabola and check that the cursor looks like the
coordinate axes, as shown in the screen at the right, and that the parabola is
flashing. Ask students to hold the Click button until a closed hand appears,
and then use the arrow keys to move the parabola. The equation will change
as the parabola is moved.

To compare the \( y \)-values of several relations, have students enter a variety of
equations, such as \( y = x^2, y = x^2 + 3, y = x^2 - 2, y = x^2 - 3, \) and \( y = x^2 + 2, \) in
the entry line of a Graphs & Geometry application. They can then add a Lists
& Spreadsheet application and change to a Function Table. This will allow
them to see the \( y \)-values of each relation and look for patterns. Students can
refer to Appendix B-37 and B-42.

Repeat the same process for the following groups of relations:
\[
\begin{align*}
y &= x^2, \quad y = (x + 1)^2, \quad y = (x - 1)^2, \quad y = (x + 4)^2, \quad \text{and} \quad y = (x - 4)^2 \\
y &= x^2, \quad y = (x + 1)^2 + 3, \quad y = (x - 1)^2 - 3, \quad y = (x + 4)^2 - 2, \quad \text{and} \\
y &= (x - 4)^2 + 2
\end{align*}
\]

3 Consolidation

(10 to 15 min)

Students should be able to match equations of the form \( y = (x - h)^2 + k \) with
the corresponding graphs and sketch the graph of a quadratic relation of this
form. They should also be able to determine the values of \( h \) and \( k \) and the
equation of a quadratic relation from given transformations, and vice versa.
5.3 GRAPHING QUADRATICS IN VERTEX FORM

Lesson at a Glance

Prerequisite Skills/Concepts
• Apply reflections of a parabola on a coordinate grid.
• Apply translations of a parabola on a coordinate grid.
• Recognize and sketch quadratic relations in standard or factored form.
• Identify the equation of the axis of symmetry and the coordinates of the vertex of a quadratic relation in standard or factored form, based on its graph.
• Understand and apply the order of operations.

Specific Expectations
• Explain the roles of \( a, h, \) and \( k \) in \( y = a(x - h)^2 + k \), using the appropriate terminology to describe the transformations, and identify the vertex and the equation of the axis of symmetry.
• Sketch, by hand, the graph of \( y = a(x - h)^2 + k \) by applying transformations to the graph of \( y = x^2 \).
• Solve problems arising from a realistic situation represented by a graph or an equation of a quadratic relation, with and without the use of technology (e.g., given the graph or the equation of a quadratic relation representing the height of a ball over elapsed time, answer questions such as the following: What is the maximum height of the ball? After what length of time will the ball hit the ground? Over what time interval is the height of the ball greater than 3 m?).

Mathematical Process Focus
• Problem Solving
• Connecting
• Representing

GOAL
Graph a quadratic relation in the form \( y = a(x - h)^2 + k \) by using transformations.

MATH BACKGROUND | LESSON OVERVIEW
• Students are introduced to the vertex form of a quadratic relation: \( y = a(x - h)^2 + k \).
• Students use what they have learned in Lessons 5.1 and 5.2 to graph a quadratic relation, given its equation in vertex form, and to relate equations to graphs.
• Students apply the concept of vertex form to determine and graph relations in problems with real-world contexts.
Introducing the Lesson
(5 to 10 min)

To get students thinking about transformations, tell them that they will be combining the ideas in the previous two lessons. As a visual introduction, you might show the graph of \( y = x^2 \) and another parabola (without the equation stated) on the same grid and ask students to suggest transformations that could move the first parabola to the second parabola.

Teaching and Learning
(15 to 20 min)

Learn About the Math

Two main points are developed in this lesson:

- The graph of a quadratic relation in vertex form can be sketched by applying a sequence of transformations to the graph of \( y = x^2 \).
- The order of the transformations can vary slightly, but any vertical stretch or compression and any reflection about the \( x \)-axis must be done before any vertical translation. This follows from using the correct order of operations, multiplying by \( a \) before adding \( k \).

*Example 1* demonstrates two possible orders of transformations for a particular relation. It also demonstrates the five-point technique that was used in Lesson 5.1. You might have students pair up, with each partner working through one solution and then explaining it to the other. As a class, discuss the similarities and differences between the two solutions.

Answers to Reflecting

A. After Kevin stretched the graph by a factor of 2, he was able to shift the graph 8 units down and 3 units right in a single translation.

B. Srinithi’s solution follows the order of operations exactly. The first step is the operations inside the brackets, which results in a translation 3 units right. The next step is multiplication, which results in a vertical stretch by a factor of 2. The next step is subtraction, which results in a translation 8 units down. So, Srinithi’s solution requires three steps. Kevin’s solution requires only two steps, but it does not follow the order of operations exactly. (It is still a correct order, however.)

C. Based on the order of operations, the transformations are done in the following order: brackets, multiplication, and then addition/subtraction. A similar order for transformations is horizontal translation, vertical stretch/rotation, and then vertical translation. This order allows you to draw the new graph correctly.
Consolidation

(30 to 40 min)

Apply the Math

Using the Solved Examples

*Example 2* develops two strategies for sketching the graph of a quadratic relation in vertex form. One strategy involves using transformations. The other strategy applies the properties of the parabola as defined by its equation. Have students work in pairs with each partner explaining one of the solutions to the other. Discuss the advantages of each method with the class. The different solutions provide alternative strategies for students.

*Example 3* applies the ideas and techniques for graphing a quadratic relation in vertex form to solving a realistic problem. It also introduces the idea of manipulating the equation in vertex form, based on the data in the problem. Ask students to predict how the changing data in each part will affect both the equation and the graph. Then guide them as they read the example. As a class, discuss which predictions were correct.

Answer to the Key Assessment Question

After students complete question 13, invite a few students to read their answers for each part. Discuss differences in reasoning and differences in communicating reasons.

13. The equation in part c), \( y = -\frac{2}{3} (x - 3)^2 + 5 \), represents the graph.
   - The vertex is at (3, 5), so the equation is of the form \( y = a(x - 3)^2 + 5 \). This rules out \( y = -\frac{2}{3} x^2 + 5 \), the equation in part a).
   - The parabola opens downward, so \( a \) must be negative. This rules out \( y = \frac{2}{3} (x - 3)^2 + 5 \), the equation in part d).
   - The parabola has the vertex at (3, 5). If the value of \( a \) in the equation was 1 or \(-1\), the parabola would pass through (1, 1) and (5, 1). The graph is wider than a parabola passing through these points, so the graph is a result of a vertical compression of \( y = x^2 \). This rules out \( y = -(x - 3)^2 + 5 \), the equation in part b).

Closing

Have students read question 18. Students should be able to connect the vertex form of an equation with transformations of the graph of \( y = x^2 \) and with the vertex of the transformed graph. You might discuss with the class how the given equation compares to the general vertex form of an equation and how the location of the vertex is affected by the transformations.
### Assessment and Differentiating Instruction

#### What You Will See Students Doing ...

<table>
<thead>
<tr>
<th>When students understand...</th>
<th>If students misunderstand...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students correctly identify the transformations of ( y = x^2 ) that are needed to draw the graph of a quadratic relation, given its equation in vertex form.</td>
<td>Students may not relate the value of ( h ) to a horizontal translation and/or the value of ( k ) to a horizontal transformation. They may not correctly determine whether the value of ( h ) is positive or negative as they interpret ((x - h)^2).</td>
</tr>
<tr>
<td>Students apply transformations in a correct order.</td>
<td>Students apply transformations in an order that is not correct, without considering the order of operations.</td>
</tr>
<tr>
<td>Students effectively communicate their reasoning about applying transformations.</td>
<td>Students may use incorrect terms to describe transformations.</td>
</tr>
</tbody>
</table>

#### Key Assessment Question 13

| Students correctly identify the equation that corresponds with the vertex of the parabola to eliminate part a). | Students may not identify the coordinates of the vertex of the parabola correctly, or they may not relate the coordinates to the correct equation. |
| Students understand the direction of opening of the parabola to eliminate part d). | Students may not remember that the equation of a parabola that opens downward has a negative value of \( a \). |
| Students correctly determine that the graph represents a vertical compression of \( y = x^2 \) to eliminate part b). | Students may not determine that the graph represents a vertical compression of \( y = x^2 \). |

#### Differentiating Instruction | How You Can Respond

**EXTRA SUPPORT**
1. If students are having difficulty connecting the vertex form of a quadratic relation to transformations of the graph of \( y = x^2 \), help them break down the process so they can see where the information comes from and how it is used. For example, start by writing the general vertex form next to the specific equation so that \( a, h, \) and \( k \) can be clearly identified.
2. If students are confused about the correct order of transformations, encourage them to connect the parts of a quadratic relation in vertex form to the BEDMAS order of operations:
   - \( B: (x - h) \rightarrow \) translation right/left,
   - \( M: a \rightarrow \) vertical stretch/compression,
   - \( AS + k \rightarrow \) translation up/down

**EXTRA CHALLENGE**
1. Have students create a design, perhaps similar to the design in Lesson 5.2 (page 259), using different combinations of transformations. Students could then trade designs to determine the transformations used.
### MID-CHAPTER REVIEW

#### Big Ideas Covered So Far

- The graphs of \( y = ax^2 \), \( y = x^2 + k \), and \( y = (x - h)^2 \) are obtained from the graph of \( y = x^2 \) by a vertical stretch/compression and/or reflection in the \( x \)-axis, a vertical translation, and a horizontal translation, respectively.
- The vertex form of a quadratic relation is \( y = a(x - h)^2 + k \).

#### Using the Frequently Asked Questions

Have students keep their Student Books closed. Display the Frequently Asked Questions on a board. Have students discuss the questions and use the discussion to draw out what the class thinks are good answers. Then have students compare the class answers with the answers on Student Book page 273. Students can refer to the answers to the Frequently Asked Questions as they work through the Practice Questions.

#### Using the Mid-Chapter Review

Ask students if they have any questions about any of the topics covered so far in the chapter. Review any topics that students would benefit from considering again. Assign Practice Questions for class work and for homework.

To gain greater insight into students’ understanding of the material covered so far in the chapter, you may want to ask them questions such as the following:

- What do you know about the value of \( a \) if the graph of \( y = ax^2 \) is narrower than the graph of \( y = x^2 \) and opens upward? What do you know about the value of \( a \) if the graph of \( y = ax^2 \) is wider than the graph of \( y = x^2 \) and opens downward?
- Describe how you would draw a parabola if you were given the following information: its vertex is \((3, -1)\), it is stretched from the graph of \( y = x^2 \) by a factor of 2, and it opens upward.
- How would you decide which order to apply translations to the graph of \( y = x^2 \) to draw the graph of \( y = -3(x - 4)^2 - 2 \)?
5.4 QUADRATIC MODELS USING VERTEX FORM

Lesson at a Glance

**Goal**
Write the equation of the graph of a quadratic relation in vertex form.

**Prerequisite Skills/Concepts**
- Recognize and sketch quadratic relations in standard or factored form.
- Identify the zero(s), equation of the axis of symmetry, and vertex of a quadratic relation in standard or factored form, based on its graph.
- Create a scatter plot and draw a line or curve of good fit.

**Specific Expectations**
- Collect data that can be represented as a quadratic relation, from experiments using appropriate equipment and technology (e.g., concrete materials, scientific probes, graphing calculators), or from secondary sources (e.g., the Internet, Statistics Canada); graph the data and draw a curve of best fit, if appropriate, with or without the use of technology.
- Determine the equation, in the form $y = a(x - h)^2 + k$, of a given graph of a parabola.
- Solve problems arising from a realistic situation represented by a graph or an equation of a quadratic relation, with and without the use of technology (e.g., given the graph or the equation of a quadratic relation representing the height of a ball over elapsed time, answer questions such as the following: What is the maximum height of the ball? After what length of time will the ball hit the ground? Over what time interval is the height of the ball greater than 3 m?).

**Mathematical Process Focus**
- Problem Solving
- Reasoning and Proving
- Selecting Tools and Computational Strategies
- Connecting
- Representing

**Student Book Pages 275–284**

<table>
<thead>
<tr>
<th>Preparation and Planning</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pacing</strong></td>
</tr>
<tr>
<td>5–10 min  Introduction</td>
</tr>
<tr>
<td>20–25 min  Teaching and Learning</td>
</tr>
<tr>
<td>25–35 min  Consolidation</td>
</tr>
<tr>
<td><strong>Materials</strong></td>
</tr>
<tr>
<td>grid paper</td>
</tr>
<tr>
<td>ruler</td>
</tr>
<tr>
<td>graphing calculator</td>
</tr>
<tr>
<td>spreadsheet program (optional)</td>
</tr>
</tbody>
</table>

**Recommended Practice**
Questions 3, 6, 11, 13, 14, 15, 16

**Key Assessment Question**
Question 15

**Extra Practice**
Lesson 5.4 Extra Practice

**Nelson Website**
http://www.nelson.com/math

**Math Background | Lesson Overview**
- Students determine the equation of a quadratic relation in vertex form from a graph of the relation by locating the vertex and substituting one other point to determine the value of $a$ in the vertex form, by using other partial information, or by reasoning about transformations.
- Students determine quadratic models for data using trial values and/or quadratic regression with grid paper, a spreadsheet, or a graphing calculator.
Ask students to think about what happens to sales as the price of a product such as bread increases. Ask: As the price falls, why might revenue increase? Why might revenue decrease? What does the graph on Student Book page 275 suggest about the answers to these questions?

Learn About the Math

This lesson develops the idea that the equation of a quadratic relation in vertex form can be deduced from the graph of the relation, given information such as the coordinates of the vertex and one other point on the graph.

*Example 1* presents a realistic situation for the relationship between price and profit. As a class, discuss the important information that is given. Ask students how Sabrina begins her solution (by writing $y = a(x - h)^2 + k$).

Have students work through the example in pairs. Each pair can discuss possible answers to the Reflecting questions. Then initiate a class discussion about the questions. Lead students to discuss how Sabrina substitutes values into $y = a(x - h)^2 + k$. Ask: What other values could have been used?

**Answers to Reflecting**

A. You need the coordinates of the vertex and one other point on the graph.

B. I think that the vertex form is most useful, because you can substitute the coordinates of the vertex for $h$ and $k$ in the equation of the relation.

Apply the Math

**Using the Solved Examples**

You might introduce *Example 2* as a detective story, since it challenges students to use a partial graph and additional clues to write an equation. Have students work in pairs, with one partner spotting the clues and the other partner explaining how the clues are used in the solution.

*Example 3* introduces the idea of fitting a parabola to given data. Have students work in pairs or in groups of three or four to identify and record the steps in Eric’s solution.
Work through the graphing calculator solution (Gillian’s solution) step by step with the class. Discuss how Gillian’s solution compares with the first solution. It is important that each student knows how to use a graphing calculator to solve these questions. As a class, develop a strategy for this type of problem.

**Technology-Based Alternative for Lesson**

If TI-nspire calculators are available, have students check Eric’s solution by using one of the options for quadratic regression discussed in Example 3. Students can refer to Appendix B-37, B-38, B-40, and B-46.

To check Gillian’s solution, have students create their own curve of good fit by following these steps:

- Enter the data from Example 3 in a Lists & Spreadsheet application.
- Add a Graphs & Geometry application to plot the data.
- Change the Graph Type to Scatter Plot.
- Select the appropriate data for x and y.
- Change the Graph Type to Function.
- Enter an estimate for an equation of the curve of good fit.

Students can use the scatter plot to estimate the vertex and the value of a in their initial estimate of the equation. Based on the graph that appears, they can make alterations to get a better fit. Instead of changing the equation on the entry line, they can click twice on the equation on the screen and then move to the number they want to change.

Alternatively, have students try to solve the example.

**Answer to the Key Assessment Question**

For question 15, students could reason that the maximum profit of $1600 occurs when the price is $75, so the point (75, 1600) is the vertex and the parabola opens downward. The equation is of the form \( y = a(x - 75)^2 + 1600 \), where \( a \) is negative. The profit is $1225 when the price is $50, so (50, 1225) is a point on the parabola and the coordinates can be substituted for \( x \) and \( y \) to obtain 1225 = \( a(50 - 75)^2 + 1600 \).

15. The equation is \( p = -0.6(d - 75)^2 + 1600 \).

**Closing**

Have students read question 16 and discuss their ideas in pairs. Then have them complete the concept circle individually. As a class, discuss what information each form of the equation gives, directly and indirectly, and how this information helps to connect the graph to the equation. You might invite students to record their ideas on a displayed concept circle.
### Assessment and Differentiating Instruction

#### What You Will See Students Doing ...

<table>
<thead>
<tr>
<th>When students understand...</th>
<th>If students misunderstand...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students write equations represented by parabolas.</td>
<td>Students do not know what part of a parabola relates to part of an equation, or they do not relate the parts correctly. They may have difficulty determining whether the value of ( h ) is positive or negative.</td>
</tr>
<tr>
<td>Students correctly interpret and apply information about a quadratic relation in a real-world situation.</td>
<td>Students cannot relate features of a model to the appropriate aspect of a situation, or they interpret features of a model incorrectly.</td>
</tr>
<tr>
<td>Students understand the difference between a sketched curve of good fit and the curve of best fit obtained by quadratic regression.</td>
<td>Students do not understand why different curves are possible for a curve of good fit or why the curve obtained by quadratic regression is more exact.</td>
</tr>
</tbody>
</table>

#### Key Assessment Question 15

| Students identify the vertex of the quadratic relation and use it to create the equation \( y = a(x - 75)^2 + 1600 \). | Students cannot identify the vertex of the quadratic relation and/or use it to create an equation. |
| Students substitute the remaining information into the equation to determine the correct value of \( a \). | Students cannot interpret the remaining information to substitute into the equation, or they calculate incorrectly. |

#### Differentiating Instruction | How You Can Respond

**EXTRA SUPPORT**
1. Students who are having difficulty with problems may need guidance to determine which parts of the equation \( y = a(x - h)^2 + k \) are given in a problem. Substituting for \( a, h, \) and/or \( k \) is generally a good place to start.
2. To help students who are struggling with problems that involve scatter plot data, tell them to begin creating a curve of good fit by deciding where to place the vertex for a curve of good fit. Then they can determine the value of \( a \) by substituting one of the data points, preferably one that is close to where they think the curve should go.

**EXTRA CHALLENGE**
1. Challenge confident students to think about the general problem of determining a quadratic relation, given two points on its graph. Ask: When would you determine only one quadratic relation? (when one point is identified as the vertex) When would you determine the equation of the axis of symmetry? (when the two points have the same \( y \)-coordinate) What happens in the remaining cases? Encourage students to explore this problem using appropriate technology.
2. Challenge students to describe situations that can be represented by parabolas in the lesson, such as those in question 3. Ask them to relate the values shown on the graphs to values in the situations they describe.
5.5 SOLVING PROBLEMS USING QUADRATIC RELATIONS

Lesson at a Glance

GOAL
Model and solve problems using the vertex form of a quadratic relation.

Prerequisite Skills/Concepts
- Recognize and sketch quadratic relations in standard or factored form.
- Identify the zero(s), equation of the axis of symmetry, and vertex of a quadratic relation in standard or factored form, based on its graph.
- Factor quadratic expressions.

Specific Expectations
- Explain the roles of $a$, $h$, and $k$ in $y = a(x - h)^2 + k$, using the appropriate terminology to describe the transformations, and identify the vertex and the equation of the axis of symmetry.
- Sketch or graph a quadratic relation whose equation is given in the form $y = ax^2 + bx + c$, using a variety of methods (e.g., sketching $y = x^2 - 2x - 8$ using intercepts and symmetry; sketching $y = 3x^2 - 12x + 1$ by [completing the square and applying transformations]; graphing $h = -4.9t^2 + 50t + 1.5$ using technology).
- Determine the zeros and the maximum or minimum value of a quadratic relation from its graph (i.e., using graphing calculators or graphing software) or from its defining equation (i.e., by applying algebraic techniques).
- Solve problems arising from a realistic situation represented by a graph or an equation of a quadratic relation, with and without the use of technology (e.g., given the graph or the equation of a quadratic relation representing the height of a ball over elapsed time, answer questions such as the following: What is the maximum height of the ball? After what length of time will the ball hit the ground? Over what time interval is the height of the ball greater than 3 m?).

Mathematical Process Focus
- Problem Solving
- Selecting Tools and Computational Strategies

MATH BACKGROUND | LESSON OVERVIEW
- Students connect information in a problem to a quadratic model.
- Students create quadratic models for problems, including maximum and minimum problems, and use their models to solve these problems.
- Students determine the vertex form of the equation of a quadratic relation, given an equation of the relation in a different form.
1 Introducing the Lesson

(5 to 10 min)

Direct students’ attention to the situation, relating the situation to the photograph that illustrates it. Ask students to explain the information in the second paragraph and the central question on Student Book page 285. You might ask students which values they need for the vertex form of a quadratic model and how the given information helps them determine these values. If necessary, prompt students to think about the coordinates of the vertex.

2 Teaching and Learning

(20 to 25 min)

Learn About the Math

Students use information given in the problem to create a quadratic model, which they can then use to answer further questions.

• Translating information into a model usually involves choices, such as where to locate the vertex. (In Example 1, for instance, (0, 554) makes sense.)
• Example 1 is a classic vertical-motion problem, similar to problems encountered in earlier lessons, such as Lesson 5.3 (Example 3). Work through the example as a class, prompting students with questions such as: Why did Connor use the vertex form? Why does it make sense that the vertex is at (0, 554)?

Answers to Reflecting

A. The vertex is the point on the graph that represents the maximum height. This occurs at the starting height, which is the height at time \( t = 0 \).
B. The value of \( h \) would change from 0 to 2, because the vertex is at the jump point.
C. The information in the question tells you that \( a = -4.9 \), \( h = 0 \), and \( k = 554 \). You can substitute these values into the vertex form of a quadratic relation.

3 Consolidation

(25 to 35 min)

Apply the Math

Using the Solved Examples

Have students work in pairs. Assign Example 2, 3, or 4 to each pair before discussing the examples as a class. Pairs who worked on each example prior to the class discussion could take a lead role in the discussion.
In Example 2, the use of a diagram is an integral part of the solution. One student in each pair could draw the diagram, and the other student could add the coordinates for the upper corners of the truck.

For Example 3, challenge each pair of students to identify the key idea in changing to vertex form (factoring and then using the zeros to determine the equation of the axis of symmetry). Also, have each pair think about why the value of \( a \) is the same in both standard and vertex form.

For Example 4, have one student in each pair explain Dave’s solution to the other student, and then have them reverse roles for Toni’s solution.

As a class, discuss useful ideas for solving problems that involve quadratic models. Encourage students to share ideas they may have discovered when working on Examples 2, 3, and 4.

Answer to the Key Assessment Question

For question 15, students can use the form \( P = a(x - h)^2 + k \), and rewrite \( P = 20(15 - x)(x + 11) = -20(x - 15)[x - (-11)] \) to obtain \( a = -20 \). They can determine that the equation of the axis of symmetry is \( x = \frac{15 + (-11)}{2} = 2 \); so \( h = 2 \). Then, they can substitute \( x = 2 \) into the given equation for \( P \): \( P = 20(13)(13) = 3380 \); so \( k = 3380 \).

15. a) The vertex form of the profit equation is \( P = -20(x - 2)^2 + 3380 \).
   
   b) 260 tickets will be sold at this price.

Closing

Have students read question 18. Students should think about what they need to know so they can determine the maximum or minimum value. This will help them determine which form is better for this situation. They should also think about which form, standard or vertex, is easier to factor.

### Curious Math

This Curious Math feature provides students with an opportunity to connect a geometric relationship, proportional reasoning, and a quadratic relation to discover the mathematics behind a key aesthetic principle. They employ appropriate technology, as well as an understanding of quadratic relations and their graphs, to determine the value of the golden ratio.

#### Answers to Curious Math

1. The proportion created by comparing the ratios of the side lengths is
   
   \[
   \frac{\text{longer side of golden rectangle}}{\text{longer side of smaller rectangle}} = \frac{\text{shorter side of golden rectangle}}{\text{shorter side of smaller rectangle}}
   \]

2. Let \( x \) represent the longer side of the golden rectangle.
   
   Let \( x - 1 \) represent the shorter side of the smaller rectangle.

   \[
   \frac{x - 1}{x - 1} = 1
   \]

   \[
   x^2 - x - 1 = 0
   \]

3. \[
\text{Graph of a parabola}
\]

4. the positive \( x \)-intercept of the graph

5. The value of the golden ratio is 1.618.
### Assessment and Differentiating Instruction

<table>
<thead>
<tr>
<th>What You Will See Students Doing ...</th>
<th>If students misunderstand...</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>When students understand...</strong></td>
<td></td>
</tr>
<tr>
<td>Students create a quadratic model for a real-world situation and use their model to solve a problem.</td>
<td>Students cannot correctly interpret information about a real-world situation to substitute into the general form of an equation to create a quadratic model. They may not be able to interpret an equation to answer questions about a situation.</td>
</tr>
<tr>
<td>Students determine the maximum or minimum value in a problem that can be modelled by a quadratic relation.</td>
<td>Students may not be able to determine the coordinates of the vertex, or they may not interpret the coordinates correctly to solve a problem.</td>
</tr>
<tr>
<td>Students work with the factored form of a quadratic relation to determine the model in vertex form.</td>
<td>Students may not be able to interpret values in the factored form of a quadratic relation in order to develop the vertex form.</td>
</tr>
</tbody>
</table>

### Key Assessment Question 15

| Students use the factored form of the relation to determine the vertex form. |
| Students interpret the values of the vertex form to determine the maximum profit and the price at which the maximum profit occurs. |
| Students use reasoning to determine the number of tickets sold, based on the other information in the problem and their solution. |

| Students may not calculate correctly to determine the values of $h$, $k$, and $a$ to write the vertex form. |
| Students may not be able to interpret the values of $h$ and $k$ to determine the maximum profit and/or the price at which it occurs. |
| Students do not reason effectively to determine the number of tickets sold, based on the other information in the problem and their solution. |

### Differentiating Instruction | How You Can Respond

**EXTRA SUPPORT**

1. When working with application problems, students often have difficulty interpreting the given information. Some students may find it helpful to draw a picture or a graph of the information before writing the equation.

2. If students are having difficulty creating a quadratic model, suggest a few possible approaches. Usually the key step is to identify (as in Example 1), choose (as in Example 2), or determine (as in Example 3) the location of the vertex.

**EXTRA CHALLENGE**

1. Arrange students who have been successful in pairs. Have each partner create a problem that gives only the minimum information to determine a quadratic model, and then have partners trade problems. Each student can explain the strategy needed to solve his or her partner’s problem.

2. Challenge students who are confident with Example 4 to re-examine Dave’s solution for part b) and give an alternative strategy. (Determine $a$ by expanding the factored form from part a) and equating it to the vertex form; alternatively, substitute either $(-20, 0)$ or $(52, 0)$ into the factored form of an equation.)
5.6 CONNECTING STANDARD AND VERTEX FORMS

Lesson at a Glance

Prerequisite Skills/Concepts
- Create a table of values for a relation, and use it to graph the relation.
- Recognize and sketch quadratic relations in standard or factored form.
- Identify the zero(s), equation of the axis of symmetry, and vertex of a quadratic relation in standard or factored form, based on its graph.
- Factor quadratic expressions when possible.

Specific Expectations
- Explain the roles of \(a\), \(h\), and \(k\) in \(y = a(x - h)^2 + k\), using the appropriate terminology to describe the transformations, and identify the vertex and the equation of the axis of symmetry.
- Determine the equation, in the form \(y = a(x - h)^2 + k\), of a given graph of a parabola.
- Sketch or graph a quadratic relation whose equation is given in the form \(y = ax^2 + bx + c\), using a variety of methods (e.g., sketching \(y = x^2 - 2x - 8\) using intercepts and symmetry; sketching \(y = 3x^2 - 12x + 1\) by completing the square and applying transformations; graphing \(h = -4.9t^2 + 50t + 1.5\) using technology).
- Solve problems arising from a realistic situation represented by a graph or an equation of a quadratic relation, with and without the use of technology (e.g., given the graph or the equation of a quadratic relation representing the height of a ball over elapsed time, answer questions such as the following: What is the maximum height of the ball? After what length of time will the ball hit the ground? Over what time interval is the height of the ball greater than 3 m?).

Mathematical Process Focus
- Connecting
- Representing

GOAL
Sketch or graph a quadratic relation with an equation of the form \(y = ax^2 + bx + c\) using symmetry.

Student Book Pages 297–302

Prerequisite Skills/Concepts
- Rewrite \(y = ax^2 + bx + c\) as \(y = a(x - p)^2 + c\).
- The points \((0, c)\) and \((p, c)\) lie on the graph, and the equation of the axis of symmetry is \(x = \frac{p}{2}\).
- The vertex can be determined by substituting \(x = \frac{p}{2}\) into the original equation.
Introducing the Lesson
(10 min)

Invite students to explain how the equation in the introduction relates to the situation. You might ask them to think about
- what information they can determine from the equation in standard form
- what information they would need if they wanted to write the equation in vertex form

Teaching and Learning
(30 to 40 min)

Investigate the Math
Have students work in pairs, and record their responses to the prompts in the investigation.
- In part D, you may need to prompt students to think about pairs of points in the data table. Which pairs of points are the same distance to the axis of symmetry? What is true about their y-coordinates?
- As students complete the investigation, make sure that they understand why it is useful to convert from standard form to vertex form, and how determining the equation of the axis of symmetry will help them do this.

Answers to Investigate the Math
A. I need to determine the time when the rocket reaches its maximum height.
B. | Time (s) | Height (m) |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>37</td>
</tr>
<tr>
<td>2</td>
<td>62</td>
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<tr>
<td>3</td>
<td>77</td>
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<tr>
<td>4</td>
<td>82</td>
</tr>
<tr>
<td>5</td>
<td>77</td>
</tr>
<tr>
<td>6</td>
<td>62</td>
</tr>
<tr>
<td>7</td>
<td>37</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>-43</td>
</tr>
</tbody>
</table>

C. The rocket hits the water.
D. Examine the points in the data table. Determine two points that have the same height, and determine the mean of their t-coordinates.
E. 8 s after it is launched; (0, 2), (8, 2)
F. because they have the same y-coordinate; \( t = 4 \)
G. because the axis of symmetry passes through the vertex; (4, 82)
H. 3 min 17 s after the start of the program
Answers to Reflecting

I. no, because the standard form of the equation gives no direct information about the vertex

J. By determining the equation of the axis of symmetry, I determined the $t$-coordinate of the vertex. Then I determined the $y$-coordinate by substitution.

K. $h = -5(t - 4)^2 + 82$; expand and simplify, and then compare coefficients

Consolidation

(10 to 20 min)

Apply the Math

Using the Solved Examples

Example 1 introduces the partial factoring technique. Have students work in pairs. One partner could explain how the technique works with an equation in standard form, and the other partner could explain the translation in the diagram. As a class, discuss why partial factoring is a good approach for determining the maximum value.

Next, ask students to close their Student Books. Go through Example 2 with the whole class, prompting students for ideas at each step, based on what they learned from Example 1. At various steps, ask students whether it is possible to determine the maximum or minimum value yet (and why or why not). As a conclusion, have the class compare and contrast Example 2 with Example 1.

Answer to the Key Assessment Question

For question 11, students can factor to get $h = -5t(t - 30)$, which shows that $t = \frac{0 + 30}{2} = 15$ is the equation of the axis of symmetry and 15 is the $t$-coordinate of the vertex. Then they can substitute $t = 15$ into the equation to get $h = -5(15)(-15) = 1125$, which is the $h$-coordinate of the vertex, and the maximum height.

11. The maximum height reached by the rocket is 1125 m.

Closing

As students read question 15, discuss what happens if the standard form cannot be factored. Invite some students to present their concept webs to the class, or have students share their concept webs in groups.
### Assessment and Differentiating Instruction

**What You Will See Students Doing ...**

<table>
<thead>
<tr>
<th>When students understand...</th>
<th>If students misunderstand...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students sketch a quadratic relation given in standard form by applying techniques such as partial factoring.</td>
<td>Students are unable to sketch a quadratic relation given in standard form and/or apply partial factoring correctly.</td>
</tr>
<tr>
<td>Students explain the process of partial factoring, its purpose (to determine the equation of the axis of symmetry), when it would be used (when the standard form does not factor), and why it works.</td>
<td>Students are unable to explain the process of partial factoring, its purpose, when it would be used, and/or why it works.</td>
</tr>
</tbody>
</table>

**Key Assessment Question 11**

| Students form an effective strategy to determine the maximum height. | Students may not know how to begin the solution, or they may not be able to interpret their results. |
| Students correctly apply partial factoring to the given equation. | Students may not understand how to factor or they may factor incorrectly. |
| Students correctly determine the equation of the axis of symmetry and, thus, the x-coordinate of the vertex. | Students cannot correctly determine the equation of the axis of symmetry and/or the x-coordinate of the vertex. |

### Differentiating Instruction | How You Can Respond

**EXTRA SUPPORT**

1. If students are having difficulty with partial factoring, have them think about graphing a simpler quadratic relation, such as \( y = 2x(x - 3) \) or a similar relation with parameter values that are appropriate for the problem. If students can graph this simpler relation, ask them what is different when they graph the more difficult relation.

2. If students have trouble using partial factoring to determine the vertex, remind them about the symmetry between the two points, \((0, c)\) and \((p, c)\), that can be determined from the partially factored form \( y = ax(x - p) + c \). A sketch of these points on the graph should suggest where the axis of symmetry is located.

**EXTRA CHALLENGE**

1. Ask students to pose and solve different problems that could be modelled by equations in the problems in this lesson. Have students share their problems for others to solve.

2. Challenge students to develop a general strategy for determining the zeros of a quadratic relation, beginning with partial factoring of the standard form. (For example, determine the line of symmetry, determine the vertex by substituting from the line of symmetry, convert to vertex form, set \( y = 0 \), rearrange the vertex form to \( a(x - h)^2 = -k \), divide both sides by \( a \), take the square root, and add \( h \) to both sides.)
## Chapter Review

### Big Ideas Covered So Far

- The graphs of $y = ax^2$, $y = x^2 + k$, and $y = (x - h)^2$ are obtained from the graph of $y = x^2$ by a vertical stretch/compression and/or reflection in the $x$-axis, a vertical translation, and a horizontal translation, respectively.

- The vertex form of a quadratic relation is $y = a(x - h)^2 + k$.

- The equation of a quadratic relation in vertex form can be determined from its graph, given information such as the vertex and one other point on the graph.

- The vertex, equation of the axis of symmetry, maximum or minimum value, and zeros of a quadratic relation can be determined from its graph or from its equation.

- Quadratic relations can be used to model many real-world situations.

### Using the Frequently Asked Questions

Have students keep their Student Books closed. Display the Frequently Asked Questions on a board. Have students discuss the questions and use the discussion to draw out what the class thinks are good answers. Then have students compare the class answers with the answers on Student Book page 303. Students can refer to the answers to the Frequently Asked Questions as they work through the Practice Questions.

### Using the Chapter Review

Ask students if they have any questions about any of the topics covered so far in the chapter. Review any topics that students would benefit from considering again. Assign Practice Questions for class work and for homework.

To gain greater insight into students’ understanding of the material covered so far in the chapter, you may want to ask questions such as the following:

- If you are given the coordinates of the vertex of a parabola, what else do you need if you want to determine the equation in vertex form?

- Describe the process for determining a curve of good fit as a parabola in vertex form.

- If you are given the equation of a quadratic relation in standard form, but not the coordinates of the vertex, what options do you have to determine the maximum or minimum value of the relation?
CHAPTER 5 TEST

For further assessment items, please use Nelson’s Computerized Assessment Bank.

1. Identify the transformations you must apply to the graph of \( y = x^2 \) to create each new graph. Then state the image of the point \((-3, 9)\).
   a) \( y = 5x^2 \)  
   b) \( y = \frac{3}{4}x^2 \)  
   c) \( y = -3.5x^2 \)  
   d) \( y = -0.25x^2 \)

2. For the parabola defined by \( y = -2.5x^2 \),
   a) state the direction of opening
   b) determine whether a stretch or a compression must be applied to the graph of \( y = x^2 \) to obtain it
   c) determine whether it is wider or narrower than the graph of \( y = x^2 \)
   d) sketch the parabola

3. Determine the vertex and the equation of the axis of symmetry for each parabola.
   a) \( y = x^2 + 4 \)  
   b) \( y = (x + 3)^2 \)  
   c) \( y = (x - 0.5)^2 - 7.5 \)

4. Describe the transformations you would apply to the graph of \( y = x^2 \), in the order you would apply them, to obtain the graph of each quadratic relation.
   a) \( y = -(x - 3)^2 - 5 \)  
   b) \( y = \frac{1}{2} (x + 6)^2 + 3 \)

5. A parabola is obtained from the graph of \( y = x^2 \) by a reflection in the x-axis, a vertical stretch by a factor of 3, and a translation 3 units left and 4 units up. Write the equation of the relation in vertex form.

6. A flare is fired into the air. Its height, \( h \), in metres at \( t \) seconds after it is launched is given by \( h = -5(t - 5)^2 + 130 \).
   a) Sketch a graph that represents the height of the flare.
   b) Determine the maximum height of the flare.
   c) Approximately when did the flare hit the ground?

7. Determine the equation of each quadratic relation in vertex form.
   a) The vertex is at \((4, -3)\), and the point \((6, -1)\) is on the parabola.
   b) The vertex is at \((1, 12)\), and the point \((-2, 9)\) is on the parabola.

8. The predicted revenue trend for a popcorn stand at a fair, as the price of a container of popcorn changes, is shown in the table.

<table>
<thead>
<tr>
<th>Price ($)</th>
<th>1.50</th>
<th>1.75</th>
<th>2.00</th>
<th>2.25</th>
<th>2.50</th>
<th>2.75</th>
<th>3.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue ($)</td>
<td>252</td>
<td>265</td>
<td>272</td>
<td>280</td>
<td>285</td>
<td>278</td>
<td>270</td>
</tr>
</tbody>
</table>

   a) Create a scatter plot, and draw a quadratic curve of good fit.
   b) Estimate the coordinates of the vertex.
   c) Determine, in vertex form, an algebraic relation that models the data.
   d) Check the accuracy of your model using quadratic regression.
9. The manager of a community theatre wants to model the theatre’s average profit per show. Looking at the profit for the last several years, she has noticed that the maximum average profit of $7500 occurred when the ticket price was $15. Currently, with a ticket price of $20, the average profit is $6300. Create an equation of a quadratic relation to represent the theatre’s average profit in terms of its ticket price.

10. Determine the equation, in vertex form, of a quadratic relation with zeros at 1 and 5 and a y-intercept of 10.

11. Determine the vertex form of the relation \( y = -2x^2 + 6x - 7 \) by partial factoring.

12. Determine the values of \( a \) and \( b \) in the relation \( y = ax^2 + bx - 29 \) if the vertex is located at \((-4, 3)\).

13. A football is punted. Its height, \( H \), in metres is given by the relation \( H = -5t^2 + 21t + 1 \), where \( t \) is the time in seconds after the punt.
   a) What is the maximum height of the football?
   b) Assuming that the football is caught at a height of 1 m, what is the length of time that the football is in the air?
1. a) vertical stretch by a factor of 5; (–3, 45)
   b) vertical compression by a factor of \( \frac{3}{4}; \left( -3, \frac{27}{4} \right) \)
   c) vertical stretch by a factor of 3.5, reflection in the x-axis; (–3, –31.5)
   d) vertical compression by a factor of 0.25 or \( \frac{1}{4} \), reflection in the x-axis; (–3, –2.25)

2. a) downward  b) stretch  c) narrower  d) \( y = -3(x + 3)^2 + 4 \)

3. a) \((0, 4); x = 0\)  b) \((–3, 0); x = –3\)  c) \((0.5, –7.5); x = 0.5\)

4. a) reflection in the x-axis, translation 3 units right and 5 units down
   b) vertical compression by a factor of \( \frac{1}{2} \), translation 6 units left
   and 3 units up

5. \( y = -3(x + 3)^2 + 4 \)

6. a) \( h = -5(t - 5)^2 + 130 \)

7. a) \( y = \frac{1}{2} (x - 4)^2 - 3 \)

8. a) \( y = \frac{1}{3} (x - 1)^2 + 12 \)

9. \( y = -48(x - 15)^2 + 7500 \)

10. \( y = 2(x - 3)^2 - 8 \)

11. \( y = -2(x - 1.5)^2 - 2.5 \)

12. \( a = -2, b = -16 \)

13. a) \( 23.05 \) m  
   b) \( 4.2 \) s
CHAPTER TASK

Human Immunodeficiency Virus (HIV)

Specific Expectations

- Collect data that can be represented as a quadratic relation, from experiments using appropriate equipment and technology (e.g., concrete materials, scientific probes, graphing calculators), or from secondary sources (e.g., the Internet, Statistics Canada); graph the data and draw a curve of best fit, if appropriate, with or without the use of technology.
- Explain the roles of \(a\), \(h\), and \(k\) in \(y = a(x - h)^2 + k\), using the appropriate terminology to describe the transformations, and identify the vertex and the equation of the axis of symmetry.
- Sketch, by hand, the graph of \(y = a(x - h)^2 + k\) by applying transformations to the graph of \(y = x^2\).
- Determine the equation, in the form \(y = a(x - h)^2 + k\), of a given graph of a parabola.
- Determine the zeros and the maximum or minimum value of a quadratic relation from its graph (i.e., using graphing calculators or graphing software) or from its defining equation (i.e., by applying algebraic techniques).
- Solve problems arising from a realistic situation represented by a graph or an equation of a quadratic relation, with and without the use of technology.

Introducing the Chapter Task (Whole Class)

Have students study the table near the beginning of the Chapter Task, on Student Book page 307. Ask them to suggest a trend in the data. (For example, the number of cases first increases and then decreases.) Follow up with questions such as these: Would a linear model be a good fit for the data? Why or why not? What type of model might be a good fit, and why?

Using the Chapter Task

Have students work individually. Ideally, students should be using the vertex form so they can estimate the location of the vertex. Part E is a valuable exercise for recognizing limitations of a model, particularly when extrapolating.

Assessing Students’ Work

Use the Assessment of Learning chart as a guide for assessing students’ work.

Adapting the Task

You can adapt the task in the Student Book to suit the needs of your students. For example:

- Have students work in pairs so they can compare and check each other’s quadratic models and support each other as they think of solutions.
- If students need more guidance, discuss parts D, E, and F as a class.

<table>
<thead>
<tr>
<th>Preparation and Planning</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pacing</strong></td>
</tr>
<tr>
<td>20-25 min</td>
</tr>
<tr>
<td>35-40 min</td>
</tr>
<tr>
<td><strong>Introducing the Chapter Task</strong></td>
</tr>
<tr>
<td>20-25 min</td>
</tr>
<tr>
<td>35-40 min</td>
</tr>
<tr>
<td><strong>Using the Chapter Task</strong></td>
</tr>
<tr>
<td>20-25 min</td>
</tr>
<tr>
<td>35-40 min</td>
</tr>
<tr>
<td><strong>Materials</strong></td>
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</tr>
<tr>
<td>ruler</td>
</tr>
<tr>
<td>graphing calculator (optional)</td>
</tr>
<tr>
<td>spreadsheet program (optional)</td>
</tr>
</tbody>
</table>

Nelson Website
http://www.nelson.com/math
## Assessment of Learning—What to Look for in Student Work...

### Assessment Strategy: Interview/Observation and Product Marking

<table>
<thead>
<tr>
<th>Level of Performance</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Knowledge and Understanding</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Knowledge of content</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>demonstrates limited knowledge of content</td>
<td>demonstrates some knowledge of content</td>
<td>demonstrates considerable knowledge of content</td>
<td>demonstrates thorough knowledge of content</td>
<td></td>
</tr>
<tr>
<td>Understanding of mathematical concepts</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>demonstrates limited understanding of concepts (e.g., is unable to identify transformations of ( y = x^2 ) to obtain a quadratic model)</td>
<td>demonstrates some understanding of concepts (e.g., is able to identify transformations of ( y = x^2 ) to obtain a quadratic model, but is unsure about the order of the transformations)</td>
<td>demonstrates considerable understanding of concepts (e.g., correctly identifies transformations of ( y = x^2 ) to obtain a quadratic model, including a correct order of the transformations)</td>
<td>demonstrates thorough understanding of concepts (e.g., correctly identifies transformations of ( y = x^2 ) to obtain a quadratic model, including a correct order of the transformations; can discuss the combination of translations and other possible orders)</td>
<td></td>
</tr>
<tr>
<td><strong>Thinking</strong></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Use of planning skills</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>- understanding the problem</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>- making a plan for solving the problem</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>uses planning skills with limited effectiveness</td>
<td>uses planning skills with some effectiveness</td>
<td>uses planning skills with considerable effectiveness</td>
<td>uses planning skills with a high degree of effectiveness</td>
<td></td>
</tr>
<tr>
<td>Use of processing skills</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- carrying out a plan</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- looking back at the solution</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>uses processing skills with limited effectiveness</td>
<td>uses processing skills with some effectiveness</td>
<td>uses processing skills with considerable effectiveness</td>
<td>uses processing skills with a high degree of effectiveness</td>
<td></td>
</tr>
<tr>
<td>Use of critical/creative thinking processes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>uses critical/creative skills with limited effectiveness</td>
<td>uses critical/creative skills with some effectiveness</td>
<td>uses critical/creative skills with considerable effectiveness</td>
<td>uses critical/creative skills with a high degree of effectiveness</td>
<td></td>
</tr>
</tbody>
</table>
### Assessment Strategy: Interview/Observation and Product Marking

<table>
<thead>
<tr>
<th>Level of Performance</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Communication</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expression and organization of ideas and mathematical thinking. Using oral, visual, and written forms</td>
<td>expresses and organizes mathematical thinking with <strong>limited</strong> effectiveness</td>
<td>expresses and organizes mathematical thinking with <strong>some</strong> effectiveness</td>
<td>expresses and organizes mathematical thinking with <strong>considerable</strong> effectiveness</td>
<td>expresses and organizes mathematical thinking with a <strong>high degree</strong> of effectiveness</td>
</tr>
<tr>
<td>Communication for different audiences and purposes in oral, visual, and written forms</td>
<td>communicates for different audiences and purposes with <strong>limited</strong> effectiveness</td>
<td>communicates for different audiences and purposes with <strong>some</strong> effectiveness</td>
<td>communicates for different audiences and purposes with <strong>considerable</strong> effectiveness</td>
<td>communicates for different audiences and purposes with a <strong>high degree</strong> of effectiveness</td>
</tr>
<tr>
<td>Use of conventions, vocabulary, and terminology of the discipline in oral, visual, and written forms</td>
<td>uses conventions, vocabulary, and terminology of the discipline with <strong>limited</strong> effectiveness</td>
<td>uses conventions, vocabulary, and terminology of the discipline with <strong>some</strong> effectiveness</td>
<td>uses conventions, vocabulary, and terminology of the discipline with <strong>considerable</strong> effectiveness</td>
<td>uses conventions, vocabulary, and terminology of the discipline with a <strong>high degree</strong> of effectiveness</td>
</tr>
<tr>
<td><strong>Application</strong></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Application of knowledge and skills in familiar contexts</td>
<td>applies knowledge and skills in familiar contexts with <strong>limited</strong> effectiveness (e.g., is unable to determine the equation of a quadratic model fitted to a scatter plot)</td>
<td>applies knowledge and skills in familiar contexts with <strong>some</strong> effectiveness (e.g., determines the equation of a quadratic model fitted to a scatter plot, with some errors)</td>
<td>applies knowledge and skills in familiar contexts with <strong>considerable</strong> effectiveness (e.g., determines the equation of a quadratic model that is a reasonable fit to the data in a scatter plot, with few errors)</td>
<td>applies knowledge and skills in familiar contexts with a <strong>high degree</strong> of effectiveness (e.g., determines the equation of a quadratic model that is an excellent fit to the data in a scatter plot, with no errors)</td>
</tr>
<tr>
<td>Transfer of knowledge and skills to new contexts</td>
<td>transfers knowledge and skills to new contexts with <strong>limited</strong> effectiveness</td>
<td>transfers knowledge and skills to new contexts with <strong>some</strong> effectiveness</td>
<td>transfers knowledge and skills to new contexts with <strong>considerable</strong> effectiveness</td>
<td>transfers knowledge and skills to new contexts with a <strong>high degree</strong> of effectiveness</td>
</tr>
<tr>
<td>Making connections within and between various contexts</td>
<td>makes connections within and between various contexts with <strong>limited</strong> effectiveness</td>
<td>makes connections within and between various contexts with <strong>some</strong> effectiveness</td>
<td>makes connections within and between various contexts with <strong>considerable</strong> effectiveness</td>
<td>makes connections within and between various contexts with a <strong>high degree</strong> of effectiveness</td>
</tr>
</tbody>
</table>