Name: _____ Date: _____

Appendix A

Cake Choices	Total Calories in One Serving
Chocolate	320
Pound	225
Banana	355
Angel Food	180
Lemon Chiffon	295

Icing	Total Calories in One Serving
Butter cream – Vanilla (including coloured decorative icing)	165

Ice Cream	Total Calories in One Serving
Vanilla	265

Teaching Notes: Sharing a Cake

Components

• *Application Question: Circular Sharing*
  • To develop Competency 2: Uses mathematical reasoning
  • To develop Competency 3: Communicates by using mathematical language
  • Can be used after completing *Nelson Mathematics Secondary Year Two, Cycle One* Chapters 2, 5, and 8
• *Situational Problem: Cutting Cake and Calories*
  • To develop Competency 1: Solves a situational problem
  • Can be used after completing *Nelson Mathematics Secondary Year Two, Cycle One* Chapters 2, 8, and 9

Broad Area of Learning: Citizenship and Community Life, Health and Well-Being

Educational Aim

- To enable students to use mathematical reasoning when making rules governing life in society.
- To provide students with the ability to communicate using mathematical language to share information relating to a healthy lifestyle.

Focus of Development

- Students will learn to take part in the democratic life of their classroom and develop respect for diversity.
- To encourage students to adopt a self-monitoring procedure in developing healthy lifestyle habits.

Cross-Curricular Competencies

- Uses information
- Solves problems
- Exercises critical judgment

Concepts

Arithmetic: Number Sense With Regard to Decimal and Fractional Notation and Operation Sense
– Reading, writing, various representations, patterns, properties
Arithmetic: Operations Involving Numbers Written in Decimal and Fractional Notation
– Use of a calculator: operations and sequences of operations performed in the proper order
Arithmetic: Understanding Proportionality
– Proportion
  • Ratio and proportionality coefficient
– Direct or inverse variation
Algebra: Understanding Algebraic Expressions
– Algebraic expression
  • Variable
– Equality, equation and unknown
Geometry: Geometric Figures and Spatial Sense
– Plane figures
  • Circle and sector
    – Radius, diameter, chord, arc

Processes

Arithmetic: Working With a Proportional Situation
– Recognizing a proportional situation by referring to the context, a table of values or a graph
– Solving a proportional situation
Algebra
– Constructing an algebraic expression
– Numerical evaluation of an algebraic expression

Geometry
– Finding unknown measurements
- Lengths
  – Circumference of a circle and arc length
- Areas
  – Area of circles and sectors

Application Question: Circular Sharing

Preparation and Planning	
Pacing	5–10 min Introduction 40–50 min Individual work
Materials	• a ruler • a compass • a protractor • a calculator
Masters	• Circular Sharing
Can be done after completing	*Nelson Mathematics Secondary Year Two, Cycle One* Chapters 2, 5, and 8

Introduction (Whole Class) 10–15 min

Discuss with your class the importance of giving everyone a fair share in various situations.

Review with students the parts of circle, such as radius and diameter and how they relate to each other. Discuss how to draw a circle. Also review the topics of central angle, arc length, and sector.

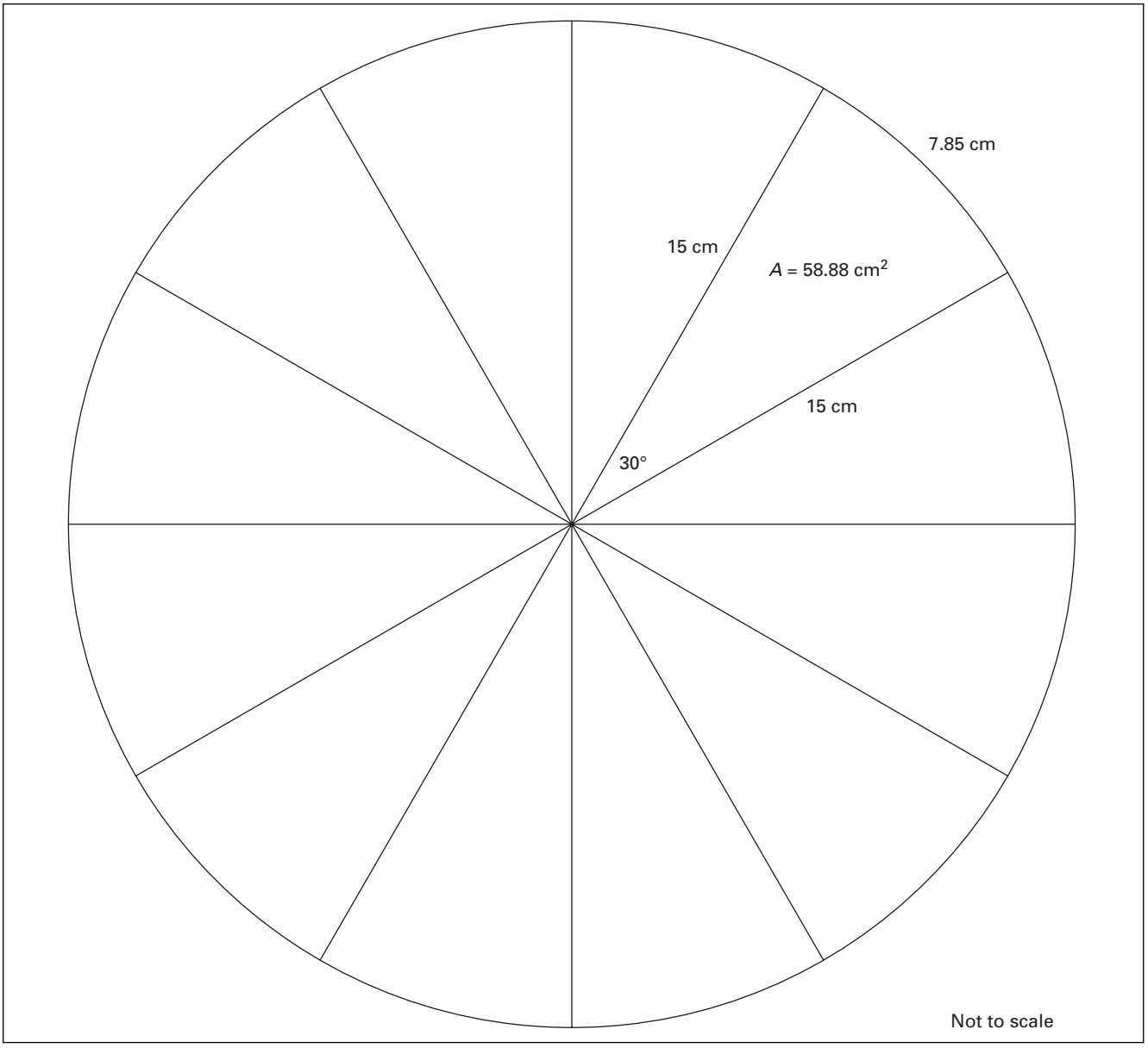
Using the Application Question (Individual) 30–45 min

As a class, read the introduction and the criteria for Circular Sharing.

If students are having difficulty. . .	**What you can do to help**
Students may have difficulty determining the measurements for one piece of cake.	• Suggest that students first determine all relevant measurements for the entire cake.

Sample Solution (Thorough)

A. For example,



The diameter is given as 30 cm, so the radius is half of that, or 15 cm. There are 360º in a circle. The cake is being divided into 12 equal pieces, so the central angle for each piece must be $\frac{360°}{12}$ or 30º. The circumference of the cake is given by the formula $C = 2\pi r$, so the circumference of the cake is about $2 \times 3.14 \times 15$ or 94.2 cm. The arc length for each of the 12 pieces of cake is $\frac{1}{12}$ of the circumference or 7.85 cm. The area of the top of one piece of cake is the area of the entire top of the cake, $\frac{1}{12} \times 3.14 \times 15^2$. The area is about 58.88 cm^2.

To determine the amount of each of the colours of decorative icing, write equations that represent the known relationships between the three amounts of icing. Then solve for one of the colours and use it to determine the other amounts.

B. The area of the top of each piece of cake when diameter is 30 cm (the radius is 15 cm) is $3.14 \times 15^2 \div 12$, or 58.88 cm^2. The area of the top of each piece will be more than double when the diameter is doubled. When the diameter is doubled to 60 cm, the radius is doubled to 30 cm, and the area of the top of each of the 12 pieces of cake is $3.14 \times 30^2 \times 12$, or 235.5 cm^2. As long as the cakes have the same height, each person gets 4 times as much cake from the larger cake than they did from the smaller cake.

The guest's claim that the central angle for one piece of cake is doubled if the diameter is doubled is false. Regardless of the length of the diameter, each piece is $\frac{1}{12}$ of the entire cake, so the central angle for each piece must be $\frac{360^\circ}{12}$ or 30º.

C. To determine an algebraic expression for the area of the top of each piece of cake that each of n guests would receive if a cake with a diameter of d centimetres is divided evenly among them, first determine the area of the entire circle that is the top of the cake. Since diameter is d centimetres, the radius is centimetres, and the area of the entire top of the cake is $3.14 \times \left(\frac{d}{2}\right)^2$, or $\frac{3.14d^2}{4}$ To determine the area of the top of the piece that each of the n guests receives, divide the entire area by n. The area of the top of each of the n pieces is $\frac{3.14d^2}{4n}$. When evaluated for $d = 30$ cm and $n = 12$, the expression produces a value of $\frac{3.14 \times 30^2}{4 \times 12}$ or 58.88 cm^2.

Assessment of Learning: Circular Sharing

Level	Competency	Overall judgment at end of cycle
5	Advanced	The student's competency **exceeds** the requirements.
4	Thorough	The student's competency **clearly meets** the requirements.
3	Acceptable	The student's competency **barely meets** the requirements.
2	Partial	The student's competency **fails to meet** the requirements.
1	Minimal	The student's competency **clearly fails to meet** the requirements.

Competency 2: Uses mathematical reasoning	
Evaluation criteria for the competency: **Uses mathematical reasoning**	**Observable elements** **The student . . .**
CR 3- Proper application of mathematical reasoning suited to the situation	• determines measurements for one piece of cake by applying proportional reasoning • proves or disproves each of the guests' claims • provides justification for their choice(s)/statement
CR 2- Correct use of concepts and processes appropriate to the situation	• determines a correct algebraic expression for the area of the top of a piece of cake when the diameter and number of guests are unknown • determines all appropriate measurements for one piece of cake
CR 4- Proper organization of the steps in an appropriate procedure shows his/her work in a clear and organized manner	• correctly draws and labels all relevant parts of the divided cake • shows his/her work for the measurements of one piece of cake in a clear and organized manner
CR 5- Correct justification of the steps in an appropriate procedure	• makes statements that justify his/her conclusions about the guests' claims • uses solid mathematical arguments and calculations in explanation of determining the measurements for one piece of cake
CR 1- Formulation of a conjecture	

Competency 2: Uses mathematical reasoning					
Evaluation Criteria	**Advanced**	**Thorough**	**Acceptable**	**Partial**	**Minimal**
Formulation of a conjecture appropriate to the situation					
Correct use of the concepts and processes appropriate to the situation	• chose appropriate mathematical concepts and processes and applied them correctly	• chose appropriate mathematical concepts and processes and applied them appropriately but makes minor errors	• chose some mathematical concepts or processes that are not appropriate; applied the chosen mathematical concepts and processes appropriately but made some procedural errors	• chose some mathematical concepts or processes that are not appropriate; applied the chosen mathematical concepts and processes but made several procedural errors	• chose several mathematical concepts or processes that are not appropriate; applied mathematical concepts and processes inappropriately
Proper application of mathematical reasoning suited to the situation	• took every aspect of the given situation into account	• took the main aspects of the given situation into account	• took some aspects of the given situation into account	• took few aspects of the given situation into account	• took almost no aspect of the given situation into account
Proper organization of the steps in an appropriate procedure	• presented a complete and organized procedure for determining the algebraic expression	• presented a complete procedure for determining the algebraic expression, even though some of the steps are not explained	• presented a procedure for determining the algebraic expression that does not make it clear what was done or how it was done, because the work is unclear or not well organized	• presented a procedure for determining the algebraic expression that consists of only isolated elements of the given criteria and contains little or no explanation for how it was done	• presented a procedure for determining the algebraic expression that is completely unrelated to the given criteria
Correct justification of the steps in an appropriate procedure	• rigorously observed the rules and conventions of mathematical language in his/her explanations	• observed the rules and conventions of mathematical language in his/her explanations	• made some errors or was sometimes inaccurate in using the rules and conventions of mathematical language in his/her explanations	• made several errors related to the rules and conventions of mathematical language in his/her explanations	• showed little or no concern for the rules and conventions of mathematical language in his/her explanations

Competency 3: Communicates using mathematical language	
Evaluation criteria for the competency: Communicates by using mathematical language	**Observable elements The student . . .**
CR 1- Correct interpretation of a message involving at least one type of mathematical representation suited to the situation	• recognizes the purpose of the message • states valid explanation or counterexample for each of the claims made by guests
CR 2- Production of a message suited to the context, using appropriate mathematical terminology and following mathematical rules and conventions	• accurately draws the divided cake and explains how to determine the measurements for one piece of the cake • provides accurate measurements and calculations for one piece of the cake

Competency 3: Communicates by using mathematical language					
Evaluation Criteria	**Advanced**	**Thorough**	**Acceptable**	**Partial**	**Minimal**
Correct interpretation of a message involving at least one type of mathematical representation suited to the situation	• used the elements of mathematical language and of everyday language to efficiently explain how to determine the effect of doubling the radius	• used the elements of mathematical language and of everyday language to appropriately explain how to determine the effect of doubling the radius	• used some elements of mathematical language and of everyday language in his/her explanations about how to determine the effect of doubling the radius	• used few appropriate elements of mathematical language and of everyday language in his/her explanations about how to determine the effect of doubling the radius	• used inappropriate elements of mathematical language and of everyday language in his/her explanations about how to determine the effect of doubling the radius
Production of a message suited to the context, using appropriate mathematical terminology and following mathematical rules and conventions	• produced an articulate, coherent, message that includes all relevant information	• produced a clear, well-organized message that includes the relevant information	• produced a message that includes elementary, ambiguous, or repetitive information	• produced a message that includes confusing and unconnected information	• produced an explanation that includes erroneous or unrelated information

Situational Problem: Cutting Cake and Calories

Preparation and Planning	
Pacing	15–20 min Introduction 45–60 min Individual work
Materials	• a calculator
Masters	• Cutting Cake and Calories • Appendix A: Cake Descriptions
Can be done after completing	*Nelson Mathematics Secondary Year Two, Cycle One* Chapters 2 and 9

Introduction (Whole Class) 15–20 min

Discuss the importance of healthy eating and how calorie intake affects your overall health. Discuss the variety of cake types available and how depending on the ingredients the overall healthiness of the cake may vary.

Using the Situational Problem (Individual) 45–60 min

As a class, read Cutting Cake and Calories and review the guidelines given for each guest. Remind students to use the Evaluation Criteria Checklist to help them complete the activity.

If students are having difficulty . . .	**What you can do to help**
Students may have difficulty determining a variety of possibilities for each guest.	• Remind students that the guests have options, such as taking ice cream or not taking ice cream, having cake with icing or without. This may broaden the number of choices they see available for each guest.

Sample Solution (Thorough)

A. For example, I will look at the guests' preferences and mark an X for any cakes that I know do not fit the criteria. For instance, I will mark off chocolate cake if the guest is allergic to chocolate. Then, if the guest wants ice cream, I will subtract 265 calories from their goal to account for the calories in the ice cream. Next, if the guest wants icing, I will add 165 calories to the calories found in one serving of each type of cake. If that total is more than the remaining calories allowed for the guest, then I will begin dividing by 2 (to see if $\frac{1}{2}$ fits the criteria), 3 (to check $\frac{1}{3}$), or 4 (to check $\frac{1}{4}$).

B. For example,

Guest	Chocolate	Pound	Banana	Angel Food	Lemon-Chiffon
1	X	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$
2	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{4}$
3	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{2}$	1	$\frac{1}{2}$
4	$\frac{3}{4}$	1	$\frac{3}{4}$	X	1
5	$\frac{1}{4}$	X	X	X	X
6	1	1	$\frac{3}{4}$	1	1
7	1	1	$\frac{3}{4}$	1	1
8	$\frac{3}{4}$	1	$\frac{3}{4}$	1	$\frac{3}{4}$
9	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	X	$\frac{1}{3}$
10	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{3}$

I first write an equation by expressing the amount of red icing and yellow icing in terms of the amount of blue icing. Then I solve for the amount of blue icing.

$$\frac{1}{2}b + b + 2b + 10 = 500$$
$$\frac{7}{2}b + 10 = 500$$
$$\frac{7}{2}b = 490$$
$$7b = 980$$
$$b = 140$$

There are 140 mL of blue icing. There is half as much red as blue, or 70 mL. The amount of yellow icing is $500 - 140 - 70$, or 290 mL.

C. For example, I followed my plan, except it had to be altered in one way. After comparing the calorie requirements of each guest to the calories in a regular serving of cake, I found that sometimes a fraction greater than $\frac{1}{2}$, but less than 1 was needed. I also needed to check $\frac{2}{3}$ of the total calorie amount, as well as $\frac{3}{4}$. I could have determined the exact fraction of a particular kind of cake a guest could have, but I used simple fractions that are easy to work with to make estimating easier and also because it would be more practical to cut $\frac{1}{4}$ of a standard serving of cake than $\frac{19}{80}$, for example. For example, Guest 7 could have $\frac{350}{355}$ of a standard serving of banana cake, but accurately producing a piece this size is not practical. Estimates for this particular situation should be close to 1.

D. For example, I checked all of the criteria requested by each guest with each type of cake. If the regular serving sizes did not fit the criteria, I used a fraction of the regular pieces instead. Subtracting the calories for those who wanted ice cream first seemed helpful. I used a calculator to aid with the computations.

To determine if the amounts of icing are correct, check that they satisfy all of the given criteria. 70 is half of 140; 140 is five less than half of 290; and 140 + 70 + 290 = 500. All of the criteria are satisfied, so the amounts are correct.

Assessment of Learning: Cutting Cake and Calories

Competency 1: Solves a situational problem					
Evaluation Criteria	**Advanced**	**Thorough**	**Acceptable**	**Partial**	**Minimal**
Oral or written explanation showing that the student understands the situational problem	• took into account all of the given criteria about each guest's preferences, and was able to distinguish between the relevant and irrelevant	• took into account most of the given criteria about each guest's preferences, but was unable to distinguish between the relevant and irrelevant	• took into account some of the given criteria about each guest's preferences	• took into account few of the given criteria about each guest's preferences	• identified some of the given criteria about each guest's preferences, but was unable to distinguish between the relevant and irrelevant
Mobilization of mathematical knowledge appropriate to the situational problem	• presented a correct solution or one containing only very minor errors	• presented a solution containing few errors	• presented a solution containing some errors	• presented a partial solution of only the easiest choices with several errors	• presented only a partial solution with several major errors or no solution
Development of a solution (i.e. a procedure and a final answer) appropriate to the situational problem	• checked his/her solution and corrected it, if necessary	• checked the main steps in his/her solution, and corrected them if necessary	• checked some of the steps in his/her solution	• made little attempt to question his/her solution	• did not evaluate his/her solution

Cross-Curricular Competencies					
Evaluation Criteria	**Advanced**	**Thorough**	**Acceptable**	**Partial**	**Minimal**
Cross-Curricular Competency 1: Uses information					
Critical analysis of information	• a clear explanation is provided as to how the cake selections were made	• an explanation is provided as to how the cake selections were made	• some explanation is provided as to how the cake selections were made	• little explanation is provided as to how the cake selections were made	• no explanation is provided as to how the cake selections were made
Logical organization of information	• explanations are clearly structured and organized	• explanations are structured and organized	• explanations are somewhat clear and organized and some errors were made	• explanations have some clarity and organization	• explanations are not organized and were difficult to understand

Evaluation Criteria	Advanced	Thorough	Acceptable	Partial	Minimal
Cross-Curricular Competency 2: Solves problems					
Accurate definition of the problem	• student shows a clear understanding of the problem presented and explained this in his or her answer	• student shows a clear understanding of the problem presented	• student shows a somewhat clear understanding of the problem presented	• student shows a lack of complete understanding of the problem presented	• student does not show any understanding of the problem presented
Variety and relevance of solutions proposed	• an accurate solution is proposed for the problem presented with an clear explanation provided	• an accurate solution is proposed for the problem presented	• a solution is proposed for the problem presented containing some errors	• an inaccurate solution is proposed for the problem presented	• no solution is proposed for the problem presented
Evaluation of possible strategies	• an accurate evaluation of the plan is provided and alternative strategies and steps were discussed	• an accurate evaluation of the plan is provided	• an evaluation of the plan is provided but it lacked several details	• an unclear evaluation of a plan is provided	• no evaluation of a plan is provided
Scope of the analysis	• an accurate analysis of the plan was performed and the explanation details specific uses of this information	• an accurate analysis of the plan was performed	• an analysis of the plan was performed but contained some errors	• an inaccurate analysis of the plan was performed	• no analysis of the plan was performed
Application of strategies developed in other situations	• there is clear evidence of the use of strategies developed previously with a detailed explanation of their possible use	• there is evidence of the use of strategies developed previously	• there is some evidence of the use of strategies developed previously	• there is a small amount of evidence of the use of strategies developed previously	• there is no evidence of the use of strategies developed previously

Evaluation Criteria	Advanced	Thorough	Acceptable	Partial	Minimal
Cross-Curricular Competency 3: Exercises critical judgment					
Openness to questioning of the judgment	• checked his/her solution and corrects it, if necessary	• checked the main steps in his/her solution, and corrects them if necessary	• checked some of the steps in his/her solution	• made little attempt to question his/her solution	• did not question his/her solution
Appropriateness of the criteria used	• used all given criteria in forming a solution	• used the most important criteria in forming a solution	• made some attempt to distinguish relevant criteria from irrelevant criteria in forming a solution	• made little attempt to distinguish relevant criteria from irrelevant criteria in forming a solution	• was not able to distinguish relevant criteria from irrelevant criteria
Well-reasoned justification of the judgment	• rigorously observed the rules and conventions of mathematical language in his/her justification of the solution	• observed the rules and conventions of mathematical language in his/her justification of the solution	• made some errors or is sometimes inaccurate in using the rules and conventions of mathematical language in his/her justification of the solution	• made several errors related to the rules and conventions of mathematical language in his/her justification of the solution	• showed little or no concern for the rules and conventions of mathematical language in his/her justification of the solution