Surface Area of Irregular Solids

**GOAL**
Calculate surface area and lateral area of figures created by combining right prisms, right pyramids, and right cylinders.

Learn about the Math

Throughout this chapter you have been learning about solid figures, including right prisms, right pyramids, and right cylinders. You have learned how to sketch them and how to calculate their surface area and lateral area.

In the previous lesson you saw how these solids can be combined to form new solids. In this lesson we will sketch nets of these irregular solids and learn how to calculate their total surface area and lateral area.

A net is a two-dimensional drawing of a three-dimensional figure. By drawing a net, you are able to see the shapes and dimensions of each of the object’s faces.

Raven created a sculpture for the school art show. The drawing shown here is a sketch of her project.
Raven is completing a questionnaire for the judges about her project and needs to calculate the total surface area of her sculpture.

How can Raven calculate the surface area of this object?

A. How many faces does this figure contain?
B. Calculate the area of each face.
C. Add the area of the faces together to determine the total surface area of the object.
D. Now try a different method for calculating the surface area of this object. What solid shapes do you see in Raven’s model?
E. Calculate the surface area of each of these shapes, using the formulas learned previously.
F. Which faces of the individual solid shapes that you found in this figure would not be included in the surface area?
G. Adjust the formulas used to calculate surface area to account for leaving out these sides and calculate the surface area again.
H. Add the surface areas of these solid figures (minus the faces not included as surface area here) together to determine the total surface area of this object.
I. Compare your answer for step H to your answer for step C.
J. Which method did you prefer for calculating the surface area of this irregular solid? Why?

Reflecting

1. Describe two methods for calculating the surface area of an irregular figure formed by combining right prisms, right cylinders, and right pyramids.

2. When calculating the area of an irregular figure formed by joining together right prisms and cylinders, why can you not just use the formulas for calculating the area of the shapes and then add them together?
3. What is the net of a three-dimensional figure?

4. How can sketching the net of a three-dimensional object be useful?

**Work with the Math**

**Example 1: Calculating the surface area and lateral area of an irregular solid**

Calculate the surface area and lateral area of this figure.

In this figure, the solid contains a rectangular prism and a cylinder.

**Jorge’s Solution**

I can determine the surface area of this irregular solid by determining what solids are combined to form it. This solid contains a rectangular prism and a cylinder.

The formula for the surface area of a rectangular prism is $SA = 2B + Ph$, where $B$ is the area of the base, $P$ is the perimeter of the base, and $h$ is the height of the prism.

In this formula the area of the base is multiplied by 2, because the top and bottom faces on a rectangular prism are equal. In this irregular solid, however, part of the top of the rectangular prism is covered by the cylinder. Therefore, the area of the part of the top of the prism covered by the cylinder will have to be subtracted.
The surface area of a cylinder is $2\pi r^2 + \pi dh$. However, the bottom base of the cylinder is covered. The area of the bottom base must be subtracted. And this is the same as the area of the top face of the prism that is being covered. This means that $2\pi r^2$ will be subtracted. Therefore, we can just add $2\pi r^2 + \pi dh - 2\pi r^2 = \pi dh$ to the surface area of the prism to determine the total surface area.

The surface area of the entire irregular solid is $2(3 \text{ cm})^2 + 12 \text{ cm}(4 \text{ cm}) + \pi(3 \text{ cm})(2 \text{ cm})$. Substitute 3.14 for $\pi$ and evaluate.

$$2(3 \text{ cm})^2 + 12 \text{ cm}(4 \text{ cm}) + 3.14(3 \text{ cm})(2 \text{ cm}) \approx 84.84 \text{ cm}^2$$

The surface area of the solid is about 85 cm$^2$.

To determine the lateral area of the solid, I add the areas of the four lateral faces of the prism and the area of the curved surface of the cylinder.

The surface area is $4(3 \text{ cm})(4 \text{ cm}) + \pi(3 \text{ cm})(2 \text{ cm})$. Substitute 3.14 for $\pi$ and evaluate.

$$4(3 \text{ cm})(4 \text{ cm}) + 3.14(3 \text{ cm})(2 \text{ cm}) \approx 66.84 \text{ cm}^2$$

The lateral area of the solid is about 67 cm$^2$.

**Checking**

5. Determine the surface area of this figure.

6. Calculate the surface area of this irregular solid.

7. List the solids that are combined to form this object; then calculate the surface area.
8. Calculate the surface area of this figure.

9. Calculate the lateral area of this solid.

10. Determine the lateral area of the object shown here.

11. Sketch the net of the object shown.

12. Draw your own irregular solid containing right prisms, right pyramids, or right cylinders, and then draw its net.

**Extending**