QUESTION 40 Page 34

The equations involved in the question are

$$3s + 2j + c = 10.54$$
$$s + j + c = 6.67$$
$$c - j = 1.30$$

where s is the cost of a package of spaghetti, j the cost of a jar of sauce and c the cost of a bag of cheese.

Answer

40. The spaghetti costs 79¢ a box, the spaghetti sauce costs $2.29 a jar, and the Parmesan cheese costs $3.59 a container.

QUESTION 41 Page 34

Some students might solve this question with guess-and-test since the numbers are fairly simple. They could deduce from the first two lines of the chart that the cleaning product costs more than the cookies, and from the last line that the cookies aren't too expensive.

The equations involved could be

$$3p + 2c = 13$$
$$2p + 3c = 12$$
$$p + 5c = 13$$

Here p is the cost of a cleaning product and c is the cost of a box of cookies.

Answer

41. $p = 3$ and $c = 2$, so $3p + 3c = 15$; $15

QUESTION 42 Page 35

Some students might struggle with writing the information in the form of equations. That is why the "Think about …" is provided.

If we use a litres of Shade A, we will have $0.5a$ litres of blue and $0.5a$ litres of white.

If we use b litres of Shade B, we will have $0.25b$ litres of blue and $0.75b$ litres of white.

For the first situation, where the paint must be four parts blue, and six parts white, 4 L of blue and 6 L of white and required. Therefore, the equations are

$$0.5a + 0.25b = 4$$
$$0.5a + 0.75b = 6$$

Students could eliminate a to find that $0.5b = 2$, so $b = 4$, and $a = 6$.

For the second situation, where there must be 3 L of blue and 7 L of white, the equations are

$$0.5a + 0.25b = 3$$
$$0.5a + 0.75b = 7$$

By elimination, students could find that $0.5b = 4$, so $b = 8$, and $a = 2$.

Answers

42. (a) 6 L of A and 4 L of B
    (b) 2 L of A and 8 L of B

Management Tip

You might assign Questions 42 and 43 to students in the enhanced course or to others who want the challenge.

Think about …

Question 42

The number of litres of blue paint would be represented by $x \div 4$.

Extension

Ask students to solve a similar problem, but where 8 L of paint instead of 10 L are required.

Suggested Introduction

Throw a ball to observe its parabolic path. Ask students how they know it was not a straight-line path and why it looked parabolic.

Ask whether students have been involved in slow pitch. Discuss the rules.

Ensure that students understand the preamble of the Investigation, that the first position of the ball, in the pitcher's hand, is (0, 1), since the ball is 0 m from the pitcher's plate and 1 m above the ground.

Investigation 9

Applications of Systems and Matrices

[Suggested time: 40 min] ***[Text page 47]***

Purpose

Students will fit a quadratic to data points that describe the path of a ball.

Management Suggestions and Materials

This activity could be done with the **whole class** or with **small groups** or **individuals** working on their own. Students will need grid paper.

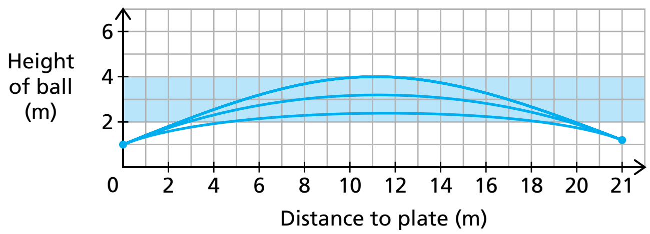
Procedure

Step A

Students should realize that since the ball is 21 m away from the pitcher and is 1.2 m off the ground, the coordinates are (21, 1.2).

Step B

This question helps students to see why we need three rather than two points of data to determine a unique quadratic. There are many different quadratics that go through two points.



An infinite number of parabolas could be drawn through (0, 1) and (21, 1.2) that pass through the zone of the ball's maximun height. Three parabulas are shown.

Management Tip

It would be useful for students to have access to graphing calculators to model the quadratics described in this section. All students will also require grid paper.

Did You Know?

Remind students that the general form of a parabola is $y = ax^2 + bx + c$, where a, b, and c are unknown. An alternative way of writing a quadratic is in the form $ax^2 + bx + c = d$. There are still only three unknowns, since the equation can be divided by a to obtain the form $x^2 + ex + f = g$.

Management Tip

You may need to help students complete Step B.

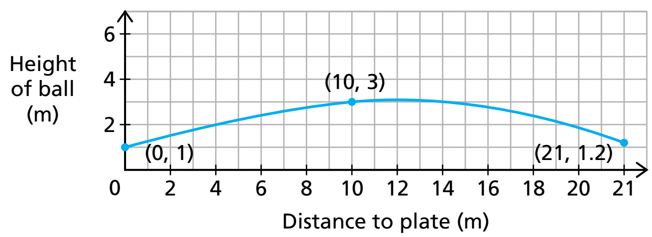
Step C

The point (10, 3) should be added to the graph. Students should note that two points are insufficient for setting up a specific quadratic function; however, three points will determine a unique parabola.

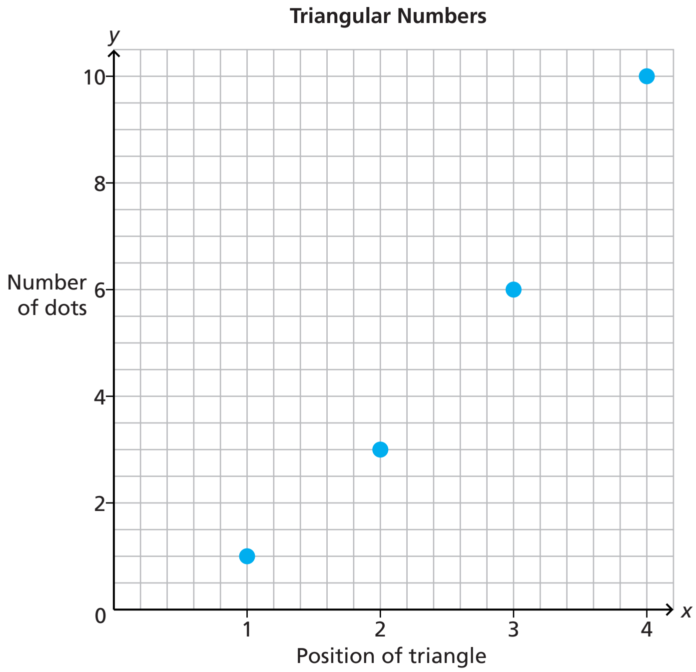
Step D

Students should substitute the values of x and y from the coordinates into the formula in order to find a, b, and c. They will end up with three equations in the three unknowns, a, b, and c.

$$1 = 0a + 0b + c$$
$$1.2 = 441a + 21b + c$$
$$3 = 100a + 10b + c$$



$1 = a + b + c$
$3 = 4a + 2b + c$
$6 = 9a + 3b + c$



Note that the graph is the points (1, 1), (2, 3), (3, 6), (4, 10). There is no line joining the points because $x \in N$.

Answers

4. (a) See graph above.
   (b) parabolic
   (c) This is not a line, so a quadratic might make sense.
   (d) $a = 0.5$, $b = 0.5$, $c = 0$
   (e) 120
   (f) Draw a picture and count the dots.

For Students in the Enhanced Course

These students might want to extend the creation of a formula for triangular numbers to the creation of formulas for pentagonal, hexagonal, and octagonal numbers.

The first four pentagonal numbers are 1, 5, 13 and 25.

The first four hexagonal numbers are 1, 6, 18, and 38.

The first four octagonal numbers are 1, 8, 31, and 80.

QUESTION 5 Page 48

The equations would be:

$897a + 29.95b + c = 100$
$1596a + 39.95b + c = 70$
$2495a + 49.95b + c = 50$

Students find the equation of the quadratic fitting the data, then substitute the value \$34.95 for p.

Some students might want to write p in terms of S:

$$29.95 = 10\,000a + 100b + c$$
$$39.95 = 4900a + 70b + c$$
$$49.95 = 2500a + 50b + c$$

The equation would be $p = 0.0037S^2 - 0.95S + 88.5$. The problem is that it would be much more difficult now for students to solve for S, knowing p, than it is in the first situation, where students find the equation for S in terms of p.

Answers

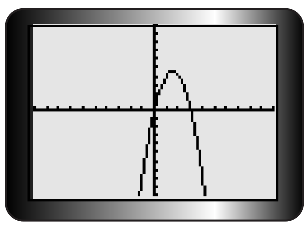
5. (a) $S = 0.05p^2 - 6.50p + 249.68$; predicted number of sales is 84
   (b) There were three variables.

QUESTION 6 Page 49

Students should probably observe, from the data, that the function will not be linear. A rocket that moves in a linear fashion will continue on forever and will never come back to the earth. Students should be encouraged, however, to enter the data into their calculators and perform regression analysis techniques on the data to verify which type of function, linear or quadratic, best fits the data. This will help your students see that what they felt was intuitively obvious was in fact true.

Answers

6. (a)



   (b) By entering the data into their calculator, students can see that the linear regression model has an r^2 value of 0.006 while the quadratic regression model has an r^2 value or 0.9998. This shows that the quadratic function fits the data better than the linear function.
   (c) Using their calculator, students can see that a reasonable function that can be used to model the data is about $y = -5x^2 + 30x - 0.5$. This graph is shown in the screen of part (a).
   (d) Using the function in part (c), the rocket will be 33.5 m above the earth after 4.5 s. The value can be found more accurately if the exact values in the function defined by the calculator is used.

For Students in the Enhanced Course

Encourage students to find other data in a global almanac or through the Internet. They could use both linear and quadratic regression on their TI-83 calculators to decide which model best fits the data.

To use quadratic regression, students use the statistics menu. They input the data in two lists, L1 and L2.

Press [STAT].
Scroll over to CALC.
Scroll down to Quadratic regression.
Press [ENTER] [ENTER].

Students should see the coefficients of the quadratic. They could plot the quadratic and compare the fit to the linear regression.

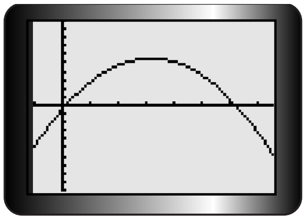
They could look for data where a quadratic fit is likelier to be a better fit.

QUESTION 7 Page 49

Encourage students to think about whether a linear function or a quadratic function best describes these data. Students should be able to determine that the quadratic function will be more accurate because the *distance driven per kilometre* increases and then decreases which will cause a "curve" when the data is graphed. If students are having difficulty seeing this pattern, encourage them to graph the data and use their calculator to analyse and sketch a curve of best fit.

Answers

7. (a) This is a quadratic function. The graph is shown below.



(b) A function that models the data is $y = -0.003x^2 + 0.378x - 0.84$. The r^2 value is only 0.84 however indicating that the fit is not as strong as one might like.

(c) 10 km/h: 2.64 km/L 120 km/h: 1.32 km/L

(d) This could be an interesting class discussion. Restrictions can invoke some interesting and unique responses from students. Whenever a member of your class mentions a restriction, be sure to ask them why that restriction might be necessary. For example, average speed should not be more that 200 km/h because not only could that speed not be maintained, it would imply that there is a negative distance that can be driven. Make a list of the restrictions at which students arrive and post them around the room.

Notebook Entry

Have students write a paragraph describing the number of points it takes to define a line, and the number of points it takes to define a quadratic, to describe the relationship between two variables. Ask them to consider when or whether it is possible to have too many or too few points to define the relationship.

FOCUS A

Modeling With Graphs

[Suggested time: 20 min] ***[Text page 90]***

Students will review how graphs can be used to model real-world relationships. Have students read the Focus on their own and then try to sketch a curve that models the situation given. The students should arrive at a graph that is similar to the one in the Student Book. Students will probably tell you that sales got close to zero and, hence, the graph levels off.

independent variable—a factor that affects another factor in an experiment or relationship

dependent variable—the factor that is affected by other factors in an experiment or relationship

function—a relation in which each value of the independent variable (*x*) is paired with one and only one value of the dependent variable (*y*)

Focus Questions

QUESTION 1 Page 90

Review the terms *dependent variable*, *independent variable*, and *function*. Definitions that were used in *Mathematical Modeling, Book 1* are shown. Have students record these in their notebooks for future reference.

Answer

1. The cumulative sales (in number of units or dollar value) of the game is the dependent variable and time is the independent variable.

QUESTION 2 Page 90

Encourage students to draw a sketch of the curve based solely on the description of the situation without referring to the graphs given. After they have done this, they should refer to the given graphs and match the situation with the appropriate graph. Ask students to provide reasons for their selections and to discuss them in groups.

Answers

2. (a) B (b) F (c) E (d) A (e) H (f) D

Think about …

Graphs

Graphs (a) and (c) show functions. For each value of the independent variable, there is only one value of the dependent variable.

Graphs (b) and (d) do not show functions. For some values of the independent variable, there is more than one value of the dependent variable.

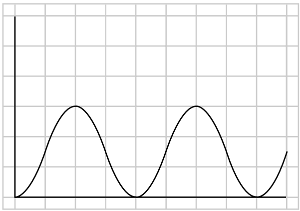
QUESTION 7 Page 93

Record and keep these graphs on the wall for future use. This demonstrates a vertical stretch and will be referred to later. When you come back to it later, you could discuss the amplitude change, what caused it on the can, and how it showed up on the graph.

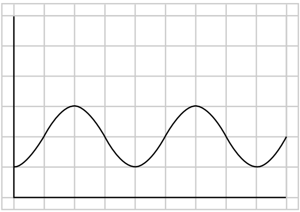
Answer

7.

Original graph (tape on edge)



New graph (tape 1 cm from edge)



Check Your Understanding

[Completion and discussion: 20 min]

QUESTION 8 Page 93

Note that Anne's speed can be expressed in revolutions per minute or metres per second. This is a good homework question that could be done independently after Questions 10, 11, and 12 have been done in pairs during class time. In this question, students will refer to the graph to find the answers, and this is good preparation for the work on solving trigonometric equations in the next chapter. Part (f) of this question will reinforce students' understanding of the repetitive nature of a periodic function. In part (g), students will use the repetitive nature of a periodic function to predict a value that is not on the graph. They must also note that for Anne the ride cycle begins at the lowest height, say 24 s not at 0 s.

Think about …

Question 8

The wave represents the gradual change in height over time as Anne approaches and then descends from the top of the Ferris wheel.

Answers

8. (a) 7 m (b) 6 m (c) 32 s (d) 1 m
(e) 1.18 m/s (f) four, two (g) 11 m, 3 m, 12 m

QUESTION 9 Page 94

Encourage students to discuss the examples used so far and to explain why they may be examples of periodic behaviour. They could also be asked to read the Chapter Project for homework after completing this question, and be ready to discuss it the next day. This question leads into Question 10.

Notebook Entry

Students should describe periodic behaviour in their own terms.

Answers

9. (a) *example:* periodic behaviour: a phenomenon that occurs again and again at regular intervals
(b) Focus A, Question 2 (a), depth of water in the Bay of Fundy

Check Your Understanding

[Completion and discussion: 50 min]

QUESTION 14 Page 99

Students should work in groups to complete this question during class time. It is a direct application of the material presented in the Focus and will give you the opportunity to ensure that students have grasped the basic concepts. The rest of the questions can be done for homework.

Answers

14. (a) period = 24 s
    (b) The period represents the time for the wheel to complete one revolution.
    (c) equation of sinusoidal axis: $y = 10$
    (d) The sinusoidal axis represents the height of the axle.
    (e) amplitude = 8 m
    (f) The amplitude represents the radius of the wheel.
    (g) height intercept = 10 m
    (h) The height intercept represents Derrick's initial height when timing starts.
    (i) There is no time intercept because at a height of zero, the wheel would be striking the ground.

QUESTION 15 Page 99

You might introduce the term *horizontal stretch* for part (d). You many want to come back to these questions after the discussion on transformations. You could express these graphs as equations and describe their transformations.

Answers

15. (a) Start the movement in the opposite direction.
    (b) Start farther away from the detector.
    (c) Move the paddle back and forth but not as far.
    (d) Move the paddle back and forth a little more quickly.

QUESTION 16 Page 101

Distribute **Blackline Master 3.1.3**, Hours of Sunlight. This will give students the table to keep in their notebooks for future use and will ensure that students do not write the answers in the Student Book. This question will allow students to apply their understanding of periodic phenomena, and would make a good homework question.

Assessment

After all groups have completed Question 14, discuss the solutions. Ask questions such as:

- Are all sinusoidal functions periodic? (yes)
- Are all periodic functions sinusoidal? (no)

Management Tip

For Question 15, consider using a motion detector and a ping pong paddle to conduct this experiment. You will need to have one paddle and motion detector per group and try to match the graph in the Student Book. Students should describe how they modeled each situation. This might be a good time to team up with the science department.

Management Tip

For Question 16, consider entering the data from the table on a spreadsheet. Not only will the data be kept in its entirety for use throughout the question, but there is a graphing feature of the spreadsheet program that will help in part (b).

Answer

8. From the equation, the y-value is found by examining the vertical translation of 1 and, the x-value is found by examining the horizontal translation of –3. Then the vertex (0,0) becomes $(-3,1)$.

QUESTION 9 Page 105

From the new graph, the vertex is at (–3, 1). This was found in Question 8. Starting at the vertex, going over 1 and up 2 results in the new points—in this case, either (–4, 3) or (–2, 3). Continuing from the vertex, going over 2 and up 8 results in other points on the graph—in this case, (–5, 9) or (–1, 9). Ensure that students are aware that the pattern always begins from the vertex, which, in a quadratic function, is an easily identifiable point.

Answer

9. From the equation, the vertical stretch is 2. Applying this to the pattern over 1 up 1, over 2 up 4 for $y = x^2$ gives the new pattern for $y = 2x^2$ which must then be translated left 3 and up 2.

Investigation 3

Transformations Through the Water Wheel Problem

[Suggested time: 60 min] ***[Text page 105]***

Purpose

Students will determine how various transformations affect the graphs of sinusoidal functions. Parameter changes to the equation will also be addressed. Students will be expected to communicate these changes in terms of transformations and to incorporate the terminology specific to sinusoidal functions. For example, vertical stretches affect the amplitude of a sinusoidal function.

Management Suggestions

Students should work in **groups** of three or four so that they can share ideas and discuss concepts.

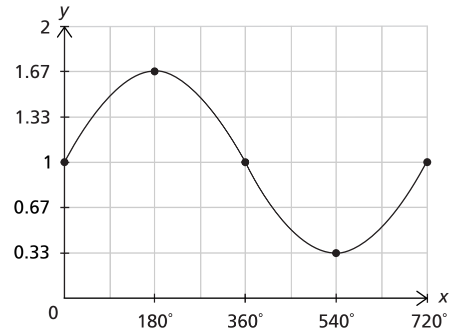
A simple warm-up activity is the "Strike a Pose" activity. Ask students to flex their arms in the shape of a parabola and to reposition them based on an equation written on the board. In an "Arnold Schwarzenegger" voice ask them if it is:

- upside down or right-side up (arms flexed downward or arms flexed upward)
- shimmied to the left, shimmied to the right, or normal (slide to left, right, or stay where they are)
- shimmied up, shimmied down, or normal (on tiptoes, crouching, or stand with normal posture)

(f) $\frac{3}{2}(y - 1) = \sin \frac{1}{2}x$

$(x, y) \rightarrow \left(2x, \frac{2}{3}y + 1\right)$

x	y
0°	1
180°	$1\frac{2}{3}$
360°	1
540°	$\frac{1}{3}$
720°	1



QUESTION 15 — Page 111

Students will describe transformations in different ways.

Answers

15.

	Amplitude	Period	Equation of sinusoidal axis	Mapping rule
(a)	2	360°	$y = 1$	$(x, y) \rightarrow (x, 2y + 1)$
(b)	1	180°	$y = -3$	$(x, y) \rightarrow (\frac{1}{2}x, y - 3)$
(c)	$\frac{1}{3}$	720°	$y = 0$	$(x, y) \rightarrow (2x, -\frac{1}{3}y)$
(d)	2	360°	$y = -5$	$(x, y) \rightarrow (x - 10°, -2y - 5)$

QUESTION 16 — Page 111

Students will match graphs and equations. This question is designed to give students a more intuitive understanding of sinusoidal functions and transformations, rather than relying on the mapping rule and the table of values method for determining the appropriate matches.

Answers

16. (a) C (b) H (c) I (d) A (e) E (f) D

QUESTION 17 — Page 112

So far, students have graphed functions, determined the amplitude, period, and equation of the sinusoidal axis of functions, and matched graphs with their appropriate functions. Students also need practice with finding the equation of a function given the graph of the function.

Answers

17. (a) $\frac{1}{3}y = \sin 0.5x$ (b) $\frac{1}{3}(y + 4) = \sin 2x$

(c) $-2(y - 2) = \sin x$ (d) $-(y + 2) = \sin 3x$

Extension

Musical chords are a practical application of combinations of functions [$h(x) = f(x) + g(x)$]. After completing this extension, students should be able to determine, by using graphing technology, which combinations of notes result in simple, pleasant sounds (consonance), as opposed to more shocking sounds (dissonance).

Management

If students have some apprehension about not being familiar with the musical concepts presented, assure them that the principles involved are fairly simple and will be clearly explained.

It is a good idea to complete this extension before assigning it to students. You will have to be familiar with the locations of the commands on your graphing calculator so that you can assist students.

It is probably best to run through the procedure for graphing the A Major chord with your students, but maybe not with students in the enhanced course. All students should work in groups.

When students graph the A Major chord, check that they do the following:

- Check that the calculator is in DEGREE mode.
- Adjust the WINDOW setting to the appropriate values.
- Enter the equation Y_1 in the graphing calculator.
- Find the commands for Y_1, Y_2, Y_3, and Y_4 on their graphing calculators. On the TI-83 they can be found by pressing VARS, moving the cursor to Y-VARS, and selecting 1: Function.

After students have graphed the A Major chord, ask them to address the Think about … question. They should be able to determine that the graph for each note has a sinusoidal axis at $y = 0$ and an amplitude of 1. Therefore, the range is $\{y \mid -1 \le y \le 1, y \in \mathrm{R}\}$. If they must add four equations together, all having the same range between 1 and –1, then the largest possible y-value for the local maximum of the resulting function would have to be 4, and the smallest possible y-value for the local minimum would have to be –4. The WINDOW setting on the calculator should read:

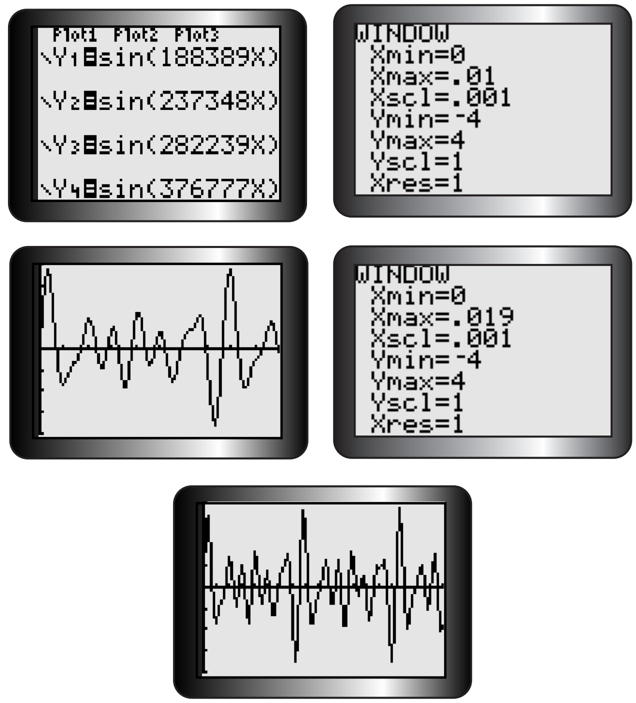
$$Y_{min} = -4 \qquad Y_{max} = 4$$

Answers

(a) periodic

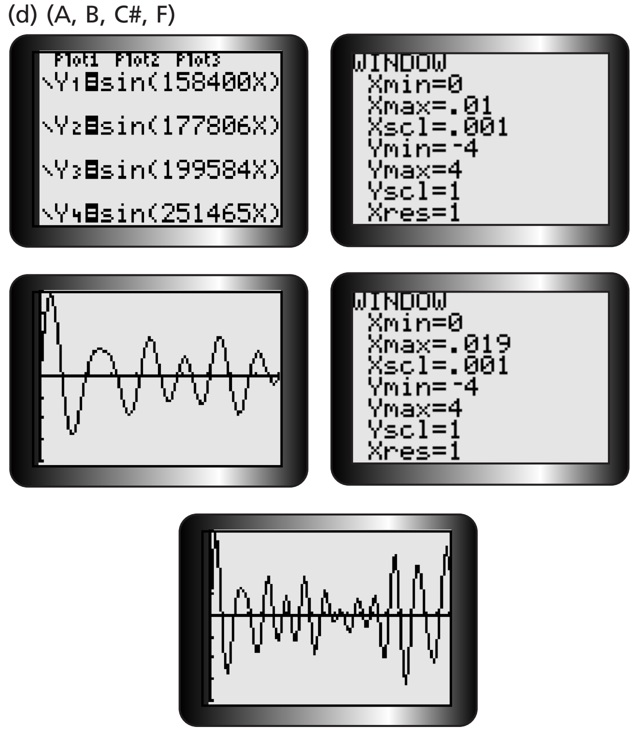
(b) not sinusoidal

(c) • C Major Chord (C, E, G, C n.o.)



- It is periodic like the A Major chord and has the same general shape.
- It only differs in its period. The period is shorter for the C Major chord.

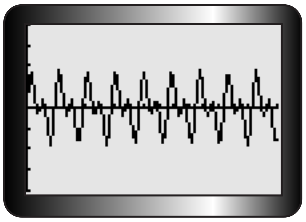
(d) (A, B, C#, F)



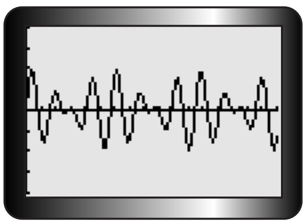
It does not appear to be periodic at this scale and in no way resembles the other major chords dealt with previously.

(e) Graphs for CC and CF display consonance because their periods are short, whereas the periods for CD and CB are much longer.

- CC—consonance

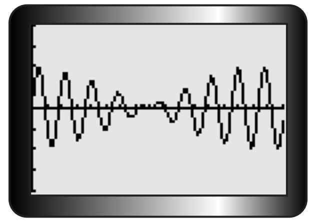


- CF—consonance

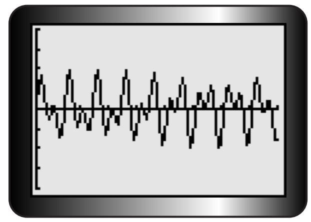


A detailed explanation for CC uses a WINDOW setting Xmax = 0.004. This helps zoom in on the smallest wave lengths needed to complete one cycle of CC.

- CD—dissonance



- CB—dissonance



The y-coordinate of point P is $\frac{1}{2}$.

$$a^2 + b^2 = c^2$$
$$x^2 + y^2 = 1^2$$
$$x^2 + \left(\frac{1}{2}\right)^2 = 1^2$$
$$x^2 + \frac{1}{4} = 1$$
$$x^2 = \frac{3}{4}$$
$$x = \sqrt{\frac{3}{4}} \text{ or } -\sqrt{\frac{3}{4}}$$

Only one of these solutions is correct because a rotation of 30° has its image point in the *first* quadrant, where x-coordinates are *positive*.

$$x = \frac{\sqrt{3}}{2}$$

$$R_{30°}(1, 0) \rightarrow (\cos 30°, \sin 30°)$$
$$\rightarrow \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

Step C

It is always important for students to verify that their answers and conclusions are correct. Once this step has been completed, and students are comfortable with their conclusions about the "special" triangles, they should be ready to move on to the Investigation Questions.

$\cos 45° \approx 0.71$	$\sin 45° \approx 0.71$
$\frac{1}{\sqrt{2}} \approx 0.71$	$\frac{1}{\sqrt{2}} \approx 0.71$
$\cos 30° \approx 0.87$	$\sin 30° = 0.50$
$\frac{\sqrt{3}}{2} \approx 0.87$	$\frac{1}{2} = 0.50$

Note: When expressing the value of a trigonometric ratio in decimal form the symbol for "approximately equal to" should be used whenever the value is not exact or is not requested to a specific number of decimal places.

Investigation Questions

QUESTION 5 — Page 132

Here is an opportunity to discuss what happens when the line is reflected across the line $y = x$. Discuss how to get the 60° angle from the 30° angle. Have students record in their notebooks what happens to the 30° angle and how it generates a 60° angle. This is a connection that can help students work with trigonometric ratios and the unit circle as the chapter continues.

Answer

5. $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

(e) $x = 13.3° + 60°k, k \in I$

(f) $x = \begin{cases} 27.8° + 180°k, k \in I \\ 135.2° + 180°k, k \in I \end{cases}$

(g) $x = \begin{cases} -227.5° + 1440°k, k \in I \\ 787.5° + 1440°k, k \in I \end{cases}$

(h) $x = \begin{cases} 37.1° + 120°k, k \in I \\ 74.9° + 120°k, k \in I \end{cases}$

QUESTION 38 — Page 146

This question is similar to Question 37. Students are asked to find the equation of a situation from small pieces of information. This would be a good question to assign for homework and to have students hand in the next day for marking.

Answer

38. $\frac{1}{21\,000}(d - 386\,000) = \cos(13.14(t - 13.7))°$

$\cos(13.14(t - 13.7))° \approx 0.1905$

$$t \approx \begin{cases} 19.7 + 27.4k, k \in I \\ 35.1 + 27.4k, k \in I \end{cases}$$

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These equations extend the students' understanding of solving trigonometric equations and combine their work with factoring. Students may need to see you complete part (a) in order to have a guide from which to work.

Answers

39. (a) $\sin^2 x + 2\sin x - 3 = 0$

$(\sin x + 3)(\sin x - 1) = 0$

$\sin x + 3 = 0$ or $\sin x - 1 = 0$

$\sin x = -3$ or $\sin x = 1$

no solution — $x = 90° + 360°k, k \in I$

(b) $\cos x = -\frac{1}{2}$ or $\cos x = \frac{1}{2}$

$x = \begin{cases} 120° + 360°k, k \in I \\ 240° + 360°k, k \in I \end{cases}$ $\quad x = \begin{cases} 60° + 360°k, k \in I \\ 300° + 360°k, k \in I \end{cases}$

Simplified: $x = \begin{cases} 60° + 180°k, k \in I \\ 120° + 180°k, k \in I \end{cases}$

(c) $x = \begin{cases} 60° + 180°k, k \in I \\ 120° + 180°k, k \in I \end{cases}$

(d) $x = \begin{cases} 70.5° + 360°k, k \in I \\ 289.5° + 360°k, k \in I \end{cases}$

(e) $x = 180° + 360°k, k \in I$

(f) $x = \begin{cases} 45.6° + 180°k, k \in I \\ 134.4° + 180°k, k \in I \end{cases}$

(g) $x = \begin{cases} -19.5° + 360°k, k \in I \\ 199.5° + 360°k, k \in I \end{cases}$

(h) $x = 0° + 180°k, k \in I$