Chapter One

Making Choices—Linear Programming

30 HOURS

Introduction

This chapter has three main sections:

1. Section 1.1 introduces students to Heather's problem, which will be revisited throughout the chapter. The concepts of linear relationships and linear inequalities are reviewed in this section (partly in the context of the problem), as they are essential to an understanding of linear programming.
2. In the second section, the limitations or constraints on Heather's problem (and other problems) are expressed as inequalities and explored by graphing. Possible solutions are investigated and the concept of a feasible region is introduced.
3. Sections 1.1 and 1.2 set the stage to establish a need to identify the optimal solution, which is found to be a point at a vertex of the feasible region. In Section 1.3, students will learn different methods to find the coordinates of the vertices, which occur at the intersection points of the feasible region's sides. The coordinates can be found algebraically by solving a system of equations. This method will be used to find the exact solution to linear programming problems.

As the chapter progresses, students continue to explore one main problem and the skills needed to solve it.

Traditionally, many textbooks added linear programming on to the work with linear equations and inequalities. The power of linear programming was often not recognized as students were busy learning how to solve systems of equations. By having students explore one problem thoroughly in a separate chapter devoted to linear programming, they should see these skills as necessary tools for solving a real-life problem. The power of linear programming and its connections should be readily understood.

Connections

This chapter reviews and expands upon the work done with linear relations and equations in Chapter 3 of *Constructing Mathematics, Book 1*. This chapter is similar—but not identical—to Chapter 7 in *Mathematical Modeling, Book 1*.

Students are expected to have prior exposure to each of the following. Some review occurs in the chapter.

A. Relations and graphs

- understanding and applying domain, range, and intercepts
- explaining the connection between algebraic and graphical representations of relationships
- reading and constructing tables and graphs to show how one quantity affects a related quantity
- graphing a linear relation using slope and y-intercept
- interpreting graphs that represent linear and non-linear data

B. Equations and inequalities

- graphing and finding the solution set for integer and real-number equations and inequalities in symbols and words
- writing a mathematical expression or equation to represent a real-world situation
- solving and verifying linear equations
- connecting the solution of a linear equation to the x-intercept or to the intersection of the graphs representing each side of the equation
- solving, testing, and graphing solutions (number line) for first-degree linear inequalities

C. Problem solving

- posing and solving problems using rational numbers in meaningful contexts
- in algebraic situations, creating and solving problems using linear equations

1.1 Exploring an Optimization Problem

Suggested instruction time: 5–7 hours

Purpose of the Section

This first section will set the stage for the entire chapter and, in fact, introduce most of what students need to be successful in the chapter. This section will introduce a linear programming problem and give students experience with the following:

- tables of values, graphing, continuous and discrete data
- independent and dependent variables, slope, writing and solving linear equations, interpolation and extrapolation, $y = mx + b$
- solving and graphing linear inequalities
- identifying the variables in a problem
- organizing the information from a problem

CURRICULUM OUTCOMES (SCOs)	RELATED ACTIVITIES	STUDENT BOOK
■ relate sets of numbers to solutions of inequalities A2	■ investigate a maximum-income problem based on known and assumed constraints	p. 2
■ demonstrate an understanding of the relationship between arithmetic operations and operations on equations and inequalities B3	■ graph relationships and express them with equations	p. 3
■ use the calculator correctly and efficiently B4		
■ apply the linear programming process to find optimal solutions C6		
■ express and interpret constraints C11		
■ interpolate and extrapolate to solve problems C18		

ASSUMED PRIOR KNOWLEDGE
■ using SI units of measure ■ solving linear equations ■ independent variables, dependent variables, discrete and continuous data, domain, range, intercepts, slope ■ representing relations using tables of values, ordered pairs, graphs, equations, and words

NEW TERMS AND CONCEPTS	PAGE
■ optimization problem	2

Suggested Introduction

Ask students why they might want a part-time job, and have them brainstorm a list of possible jobs. List the ways in which a worker can be paid: some jobs pay by the hour, some pay commission, and others pay by piecework. Discuss examples of each. In this chapter, Heather's job is piecework and you may want to discuss this prior to beginning the chapter.

Section 1.1 introduces Heather's part-time job and her income. Students are asked to find her maximum income, which will be the central problem on which the chapter is based.

Introduce the concept of linear programming with a definition such as "finding the best solution to a problem when several conditions need to be met." Students should be told that the solution of a linear programming problem requires an understanding of linear relations, and that this will be reviewed in this section.

For your reference, the entire problem is written below. You may want to discuss the entire problem with students and then tell them that you will be examining the problem in small parts. Breaking a problem into parts is a powerful problem solving process and one that students should be using.

Heather wants to know how she can make the most money working at Spinney Manufacturing. She has the following information.

- Heather cuts fabric for couches and chairs at Spinney Manufacturing.
- She is paid a flat fee for each bundle of fabric that she cuts.
- A bundle includes all of the pieces of fabric that are sewn together to make a chair or couch.
- Heather is paid $12.00 for each couch bundle that she cuts.
- She is paid $4.25 for each chair bundle that she cuts.
- If she completes only a part of a bundle, she is paid for only that part. For example, she is paid $6.00 if she cuts half of a couch bundle.
- She can only work a maximum of 36 h every two weeks.
- It takes her 45 min to cut fabric for a chair.
- It takes her two hours to cut fabric for a couch.
- At least eight chair bundles must be cut every two weeks.
- At least ten chair bundles must be cut every two weeks.
- She may use a maximum of 110 m of material to cut in a two-week period.

Management Tip

Discuss what is meant by the term *optimization* in the section heading. Students may want to check the meaning of *optimal* in a dictionary, and discuss what the term means in a problem of this type. They should be able to describe the meaning in their own words.

optimization problem—a problem in which you are trying to maximize or minimize a quantity based on the constraints in a problem. In the case of profit it is maximizing, and in the case of cost it is minimizing. For example, a problem might involve maximizing profit and/or minimizing costs.

Management Tip

The numbers in this problem differ from those in *Mathematical Modeling, Book 1.*

- A chair bundle uses 3 m of fabric.
- A couch bundle uses 5 m of fabric.

Investigation 1

Looking at Heather's Income

[Suggested time: 50–60 min] ***[Text page 2]***

Purpose

Students will investigate various ways for Heather to look at her earning possibilities using tables of values, patterns, graphs, and linear relations. The theory used in this Investigation was covered in Chapter 3 of *Constructing Mathematics, Book 1*. Many students may need to review at least some of the concepts in order to complete the Investigation.

Management Suggestions

Students may complete this investigation **individually**, in **small groups**, or as part of a **whole-class** discussion. As much of this investigation involves the review of work covered in *Constructing Mathematics, Book 1*, the latter two methods are preferable since they allow for more student–student and student–teacher interactions. Some review of concepts, with examples, may be necessary. In every case, all students should plot the graph, write definitions with examples, and answer all questions in their notebooks.

Materials

If you use a **whole-class organization**, you will need:

- overhead transparencies, a graph grid transparency, and overhead markers
- graph paper for each student
- rulers

If you use an **individual** or **small group organization**, you will need:

- graph paper and a ruler for each student
- a completed graph on an overhead to aid in class discussion

Procedure

Have students read about Heather's situation. Encourage them to explain the information in their own words. The Investigation can be completed individually or by students in groups.

Step A

Students explore income possibilities for Heather as they review how to form a table of values and make a graph. Students should see that the relationship is linear and that the data are continuous. An explanation of the differences between *discrete* and *continuous data* may be necessary. The terms *domain* and *range* may also need to be reviewed. The domain of the graph depends on the time available for Heather to cut couch bundles. This is the independent variable. The range of the

Management Tip

Students may benefit from a review of plotting points on the Cartesian plane prior to the Investigation. This Investigation could be used to assess students' knowledge prior to working on Heather's problem. In particular, this Investigation will help you to assess work with domain, range, slope, and intercepts.

Management Tip

Heather's problem will unfold as follows throughout the chapter.

1. Explore the problem.
2. Look at contraints in the problem.
3. Explore feasible regions and multiple constraints.
4. Explore points to maximize income.

Management Tip

The Investigation can be completed individually or by students in groups. Answers will likely be more in-depth if students have the opportunity to discuss their findings with each other.

domain—the set of all possible values for x (the independent variable) in any relation

range—the set of all possible values for y (the dependent variable) in any relation

continuous data—an infinite number of values exist between any two other values in the table of values or on the graph. Data points are joined on the graph.

discrete data—a finite number of data points exist between any two other values. The points on the graph are not joined.

independent variable—a variable that affects another variable in an experiment or relationship

dependent variable—the variable that is affected by other variables in an experiment or relationship

slope—the steepness of a line. Slope, m, can be found using the ratio $\frac{\text{rise}}{\text{run}}$.

It can also be found using the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, if two points on the graph are known. If the equation of the graph is known, slope can be found by comparing this equation to $y = mx + b$.

x-intercept—the point where the graph crosses the x-axis; the point where $y = 0$

y-intercept—the point where the graph crosses the y-axis; the point where $x = 0$

graph depends on the money that Heather earns. Her earnings are the dependent variable, and total earnings depends on the number of couch bundles that are cut. Students should realize that the data are continuous because Heather is paid for cutting part of a bundle, and therefore the points on the graph may be joined.

Step B

Step A is repeated for the number of chair bundles that Heather cuts. Students should realize that the data are again continuous because Heather is paid for cutting part of a chair bundle. The domain is the number of chair bundles that she is able to cut. The range (the money she earns) depends on the number of chair bundles that she cuts. The independent variable is the number of chair bundles; the dependent variable is the amount earned.

Step C

Students have several options for finding the slope of each graph. Most will use the graph and $\frac{\text{rise}}{\text{run}}$. Some may use two points and the slope formula. Very few will use the equation $y = mx + b$. The slope of Graph A is 12, the pay for cutting a couch bundle. The slope of Graph B is 4.25, the pay for cutting a chair bundle. The x- and y-intercepts of both graphs are 0. This is because she does not receive any pay unless she cuts fabric.

Step D

Students should describe the graphs as linear. They should realize that they can read, from the graphs, incomes for cutting different numbers of bundles. They can also investigate the number of bundles that need to be cut to earn a certain income. Some students will see that the y-values in each table increase by the same amount each time. The word and variable equations are as follows.

Graph A
Heather's income = ($12)(Number of couch bundles that she cuts)
$y = 12x$, where y represents her income and x represents the number of couch bundles cut

Graph B
Heather's income = ($4.25)(Number of chair bundles that she cuts)
$y = 4.25x$, where y represents her income and x represents the number of chair bundles cut

Investigation Questions

QUESTION 1 — Page 3

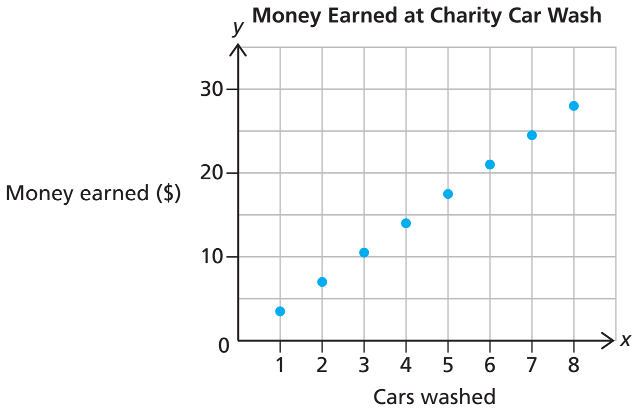
You should work through this question with students to ensure they know how to work with continuous data, domain and range, and how to look for patterns.

Answers

1. (a)

Number of cars washed, *x*	Money earned in dollars, *y*
1	3.50
2	7.00
3	10.50
5	17.50
8	28.00

(b) The points should not be joined because the data are discrete values. It is very unlikely that someone would want only part of a car washed.



(c) The domain of the graph is all possible values for the number of cars washed. The minimum value of this domain would be 0. The maximum value would depend on the time, the number of students working, and the number of customers. Students in the class may have varying estimates for this number. The range of the graph is all possible values for the amount of money earned. The maximum value of the range will depend on the maximum number of cars washed.

QUESTION 2 Page 4

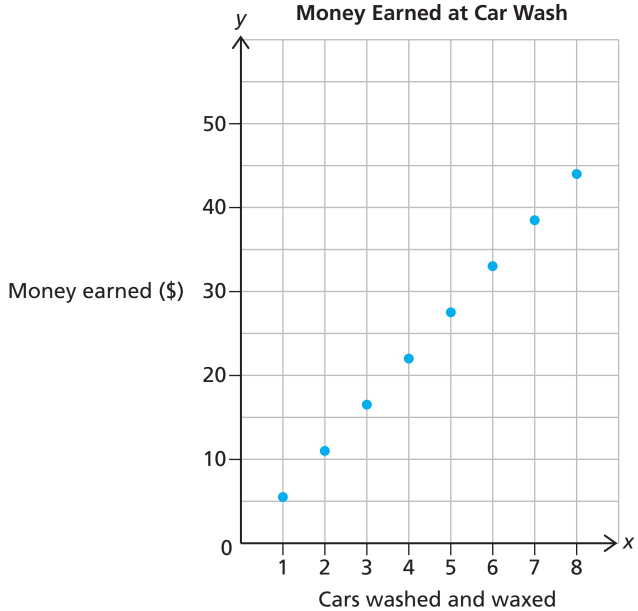
To save time, this question could be completed for homework. It parallels Question 1. If students have successfully completed Question 1, they should have little difficulty completing this one. If done for homework, solutions could be presented and discussed the next day.

Answers

2. (a)

Number of cars washed, *x*	Money earned in dollars, *y*
1	5.50
2	11.00
3	16.50
5	27.50
10	55.00

(b) The points should not be joined. Only whole numbers of cars will be washed.



(c) The domain is all posssible values for the number of cars that could be washed and waxed. The range is all possible values for the money earned. The maximum value of the range will depend on the number of cars that are washed and waxed.

QUESTION 3 Page 4

Students could work in pairs to complete this question and Question 4. Assign one part of the question to each pair of students until all parts have been assigned. This could be assigned for the next day. Then, have one student in each pair "stay at their desk" while the other student in the group goes around and finds the answers to the questions that were not assigned to that pair. The student who "stays at their desk" is providing the answer to anyone who comes around. All students then go back to their partners and report what was discovered. This is a form of jigsaw grouping.

Answers

3. Graph from Question 1: The slope is 3.5 and represents the charge for a car wash. The *x*-intercept and *y*-intercept are both zero. The *x*-intercept represents the number of cars washed if no money is made. The *y*-intercept represents the money earned if no cars are washed.

   Graph from Question 2: The slope is 5.5 and represents the charge for a car wash and wax. The *x*-intercept and *y*-intercept are both zero. The *x*-intercept represents the number of cars washed and waxed if no money is made. The *y*-intercept represents the money earned if no cars are washed and waxed.

QUESTION 4 Page 4

Answers

4. Equations for Question 1:

   money earned = \$3.50 (the number of cars washed)
   $y = 3.50x$, where y represents the money earned in dollars and x represents the number of cars that are washed.

   Equations for Question 2:
   money earned = \$5.50 (the number of cars washed and waxed)
   $y = 5.50x$, where y represents the money earned in dollars and x represents the number of cars that are washed and waxed.

Check Your Understanding

[Completion and discussion: 50 min]

Questions 5 through 8 form the basis of a review of the algebra that was explored in *Constructing Mathematics, Book 1*. As much class time as needed should be used to have students complete and discuss these questions. The rest could be assigned for homework to be taken up the next day. Have students work together to complete these questions and intervene only when difficulty arises. If the majority of students are having difficulty in one particular area, then a more extensive review of the material from *Constructing Mathematics, Book 1* may be needed.

QUESTION 5 Page 4

This question reviews work with slope, interpolation, extrapolation, and writing an equation of a straight line. Make sure students have a good understanding of this question before moving on to Questions 6 to 8.

Answers

5. (a) The slope of the graph represents the amount that Colin is paid to assemble one bike.
   (b) The points on the graph are joined. The data are continuous. Therefore, he is paid for assembling part of a bike.
   (c) He put together approximately seven bikes. Some students may see that he is paid \$8 per bike and do a calculation. The exact answer is 8.5 bikes.
   (d) $y = 8x$, where y represents Colin's earnings and x represents the number of bikes he assembles.

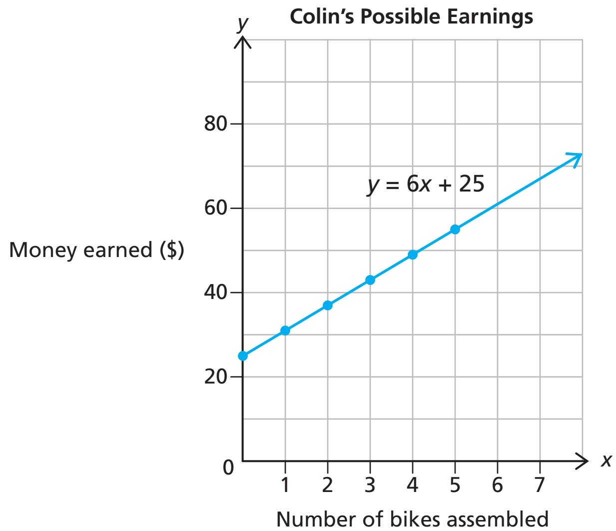
QUESTION 6 Page 5

Students should interpret the new graph and equation and compare it to those in Question 4. Also, the equation of the graph should be compared to $y = mx + b$.

Answers

6. (a)

Number of bikes assembled, *x*	Amount earned in dollars, *y*
0	25
1	31
2	37
3	43
4	49
5	55



(b) $y = 6x + 25$, where y represents the money earned and x represents the number of bikes. The slope of the graph is 6, which represents the amount earned to assemble a bike. The graph has a y-intercept of 25, which represents the initial amount paid before any bikes are assembled.

QUESTION 7 Page 5

This question is similar to Question 5, except this time the y-intercept is not zero. Students are still expected to write the domain and range as well as interpolate from the graph, extrapolate from the graph, and write an equation for the straight line. An extra step is to use the equation to make a prediction. To save time, this question could be assigned for homework after Question 5 has been taken up.

Answers

7. (a) The slope, 4, is the amount charged to deliver each bolt of material. The y-intercept, 50, represents the initial delivery cost.

(b) approximately 17.5 bolts

(c) If Spinney manufacturing orders a large number of bolts of material, a bill of \$2500 is possible. It is impossible to get a bill of \$40 because the initial delivery cost is \$50.

(d) domain: $x \geq 0$; range: $y \geq 50$

(e) $y = 4x + 50$, where x represents the number of bolts of cloth and y represents the delivery costs in dollars.

Substituting into the equation will give $y = 4(40) + 50$, or $210.

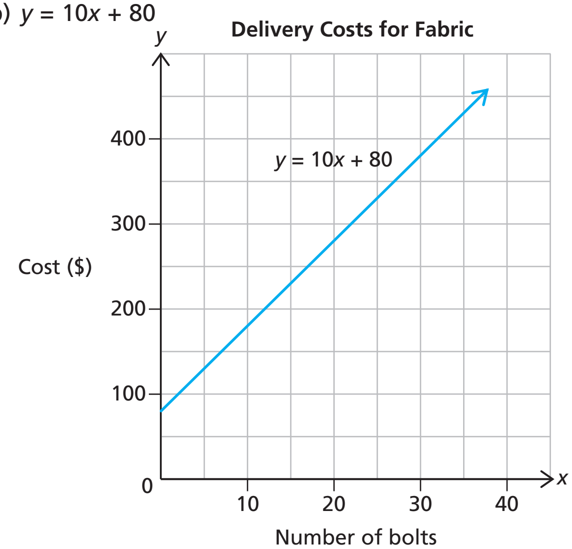
QUESTION 8 Page 5

This question parallels Question 7. It is recommended that this question be completed only after taking up Question 7. If Question 7 is assigned as homework, then this question could be assigned the next day to ensure that students have an understanding of the appropriate work.

Answers

8. (a) The slope, 10, represents the delivery charge for each bolt of fabric. The y-intercept, 80, represents the initial delivery charge for a rush order.

(b) $y = 10x + 80$



(c) 12 bolts

(d) $10x + 80 = 200$

$x = 12$

In this case, the estimated answer from the graph and the value from the equation may be equal because the answer is an integer.

Investigation 2

More Thoughts on Income

[Suggested time: 45–50 min] ***[Text page 6]***

Purpose

The purpose of this Investigation is to identify the variables in a real-life problem, investigate how each could affect the solution, and identify any connections between the variables. Solutions that lie above or below the line will be examined, which will lead to an initial understanding of the concept of an inequality. Students should be able to complete the investigation and answer the questions that follow with minimal intervention.

Think about …

Question 8

This question is open-ended and students are likely to tell you that they need either the graph or an equation. Students would be able to use the graph to find the equation and vice versa.

Management Tip

Some students will find the concept of an inequality difficult to understand, making a discussion more necessary.

Management Suggestions

This Investigation can be completed **individually**, **in groups**, or as part of a **whole-class discussion**. More in-depth answers will likely be given, however, if students have the chance for discussion with each other or with the teacher. In all cases, all students should plot their own graphs and answer all questions in their notebooks.

Materials

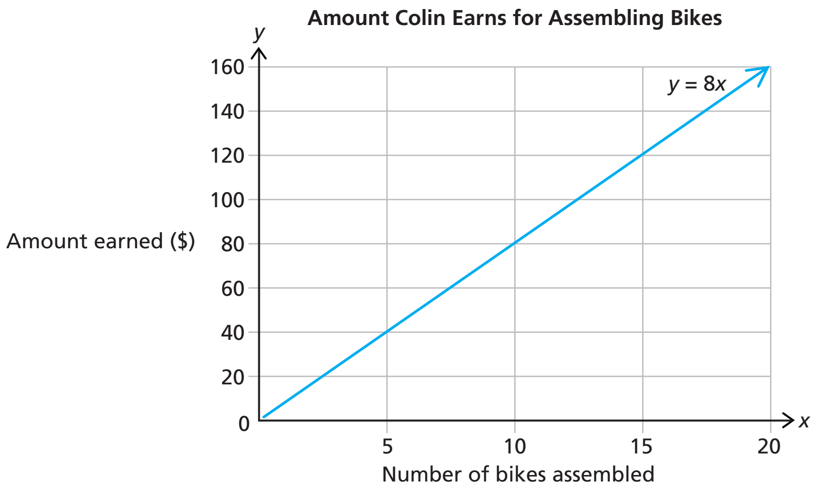
If you use a **whole-class organization**, you will need:

- overhead transparencies, a graph grid transparency, and overhead markers
- graph paper for each student
- rulers

If you use an **individual** or **small group organization**, you will need:

- graph paper and a ruler for each student
- a completed graph on an overhead to aid in class discussion

Procedure



Step A

The points (x, y) on the line represent the number of bikes assembled and the amount earned. Students may determine points from the graph or by using the equation. These answers may vary. Some sample points are: (2, 16), (3, 24), (5, 40), (6, 48), (8, 64), (12, 96), (15, 120), (18, 144), (20, 160).

Step B

(i) The point (10, 90) represents the fact that 10 bikes have been assembled, and that the owner of the store overpaid. The point (12, 100) represents that 12 bikes have been assembled, and that again the owner of the store overpaid.

(ii) Four other possible points are (5, 50), (8, 70), (12, 150), (15, 130). Answers will vary. Points (x, y) that lie above the line represent the number of bikes assembled and an overpaid amount.

Step C

(i) The point (10, 60) represents the fact that 10 bikes have been assembled and that the owner of the store underpaid. The point (5, 30) represents that 5 bikes have been assembled and that again the owner of the store underpaid.

Think about …

The Graph

A straight line is drawn through the points representing this situation because the relationship is linear. We also use the line so that we can discuss points above or below the line and the appropriate relationship these points represent.

Management Tip

A discussion of the accuracy of answers obtained from the graph compared to those from the equation would be useful.

inequality—a mathematical statement which shows that two numerical or variable expressions are not always equal. For example, $3x \leq 12$ is an inequality. $x = 2$ satisfies this inequality. Does $x = 5$? (No)

(ii) Four other possible points below the line are (2, 10), (8, 60), (12, 90), (15, 110). Student answers will vary. Points (x, y) that lie below the line represent (number of bikes assembled, an underpaid amount).

Step D

By comparing answers with others in the class, students should realize that there are many correct answers for points that are above and below the line. An inequality to represent points above the line would be $y > 8x$, where y represents the money earned and x represents the number of bikes assembled. An inequality to represent points below the line would be $y < 8x$, where y represents the money earned and x represents the number of bikes assembled.

Notebook Entry

"Greater than" is shown using the symbol > and "less than" is shown using the symbol <.

Investigation Questions

QUESTION 9 Page 7

This question is beginning the consolidation of the work on inequalities that students have just explored. Students should be able to complete this question readily after completing the Investigation. Have students present their results before moving on.

Management Tip

Students may need to list points from the graph before they are able to write an equation.

Answers

9. (a) $y = 2x$, where x represents the number of chocolate bars sold and y represents the money earned.
   (b) $y > 2x$, where x represents the number of chocolate bars sold and y represents the money earned. These points represent the chocolate bars that are sold and an amount of money greater than the amount made by selling these bars.
   (c) $y < 2x$, where x represents the number of chocolate bars sold and y represents the money earned. These points represent the chocolate bars that are sold and an amount of money less than the amount made by selling these bars.

QUESTION 10 Page 7

This question parallels Question 9 and could be assigned for homework. If students have successfully completed Question 9, this question will pose little difficulty. Make a point, in part (a), that a solid line is used because Heather gets paid for part bundles cut.

Answers

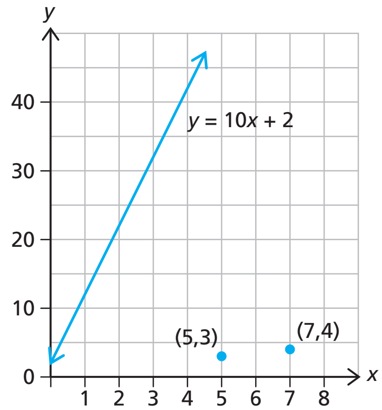
10. (a) $y = 2x$, where x represents the number of couches, and y represents the time to cut the fabric for these couches.
   (b) $y > 2x$, where x represents the number of couches, and y represents the time to cut the fabric for these couches. These points represent the number of couches and an amount of time greater than the time spent.
   (c) $y < 2x$, where x represents the number of couches, and y represents the time to cut the fabric for these couches. These points represent the number of couches and an amount of time less than the time spent.

![FOCUS A]

A Further Look at Inequalities

[Suggested time: 25–30 min] ***[Text page 8]***

In the previous Investigation, students used the graph of a linear relation to find points that would satisfy the corresponding inequalities. In this Focus, they will find points that satisfy an inequality by substitution into the inequality. These points will then be plotted on the graph.



Management Tip

The use of the ≤ symbol may confuse some students. The meaning should be discussed with the class and compared to the < symbol. Reference to the graph will help with this discussion

Think about …

$y \le 10x + 2$

Students should see that points can also be on the line. This is an important concept for students to understand.

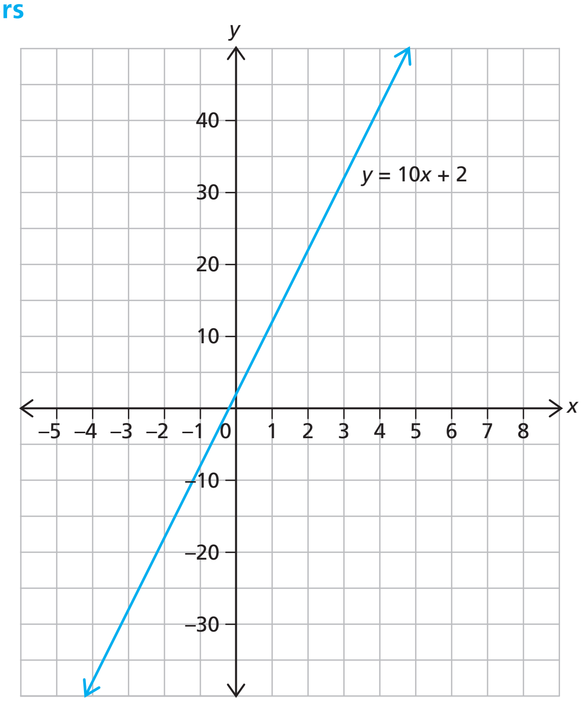
Focus Questions

QUESTION 11 Page 8

Two solutions are shown, verified in the inequality, and plotted. Students are asked to determine other solutions. Although this is a Focus, students are asked to complete the question; however, they are led through the correct process.

Answers

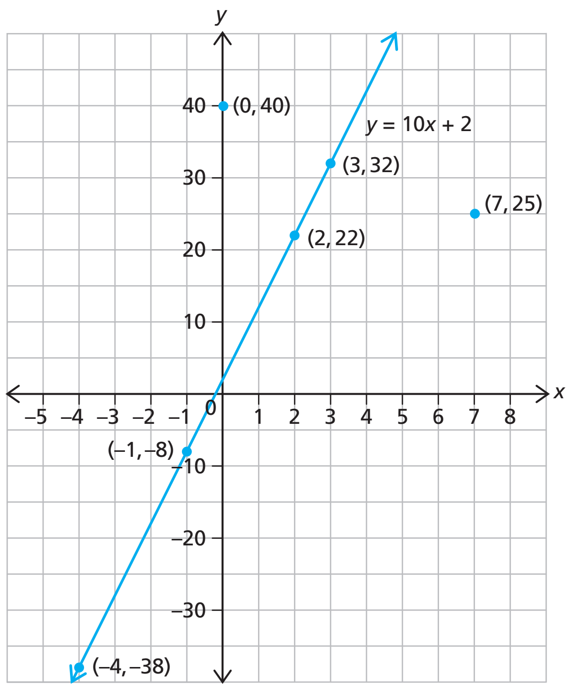
11. (a)



(b) Possible values:

x	y	$y \le 10x + 2$ (yes or no)
–1	–8	yes
2	22	yes
3	32	yes
–4	–38	yes
4	40	no
7	25	yes

(c)



All points in the region are below or on the line.

Management Tip

A random check of students' work would be useful to ensure they are substituting correctly and evaluating whether the inequality is true.

Think about …

Quesstion 11 (c)

Encourage students to use their own language to answer this question. Make sure the idea of "on or below the given line" is evident in their answers.

QUESTIONS 12 TO 14 Page 9

Questions 12 through 14 examine specific points in a region and how those points are connected to the equation of the line. This will help give students a "feel" for an inequality, and what it means to have an inequality expressed in equation form. Having students discuss as a class the reasons why they made the decisions they made will help ensure that they are beginning to understand inequalities.

Answers

12. The point (15, 20) is a solution to the inequality. This can be shown in two ways. If the point is plotted on the graph, it will be below the line. Substitution into the inequality gives a true statement.
13. The point (2, 22) is a solution to the inequality. This can be shown in two ways. If the point is plotted on the graph, it will be on the line. Substitution into the inequality gives a statement that is true.
14. The point (1, 25) is not a solution to the inequality. This can be shown in two ways. If the point is plotted on the graph, it will be above the line. Substitution into the inequality gives a statement that is not true.

QUESTION 15 Page 9

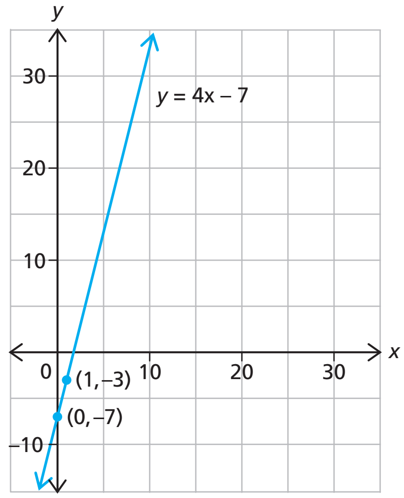
Students will need to make a table of values to plot this graph or use the slope and y-intercept.

x	y
0	–7
1	–3
2	1
3	5

Using the slope–y-intercept form of an equation, $y = mx + b$, the slope of the graph is 4 and the y-intercept is –7. Students can plot the point (0, –7) and then count over 1 and up 4 to represent a slope of 4. The next point will be (1, –3).

Answers

15\.



(a) *examples*: (1, –1), (2, 3), (3, 8), (4, 12). Answers will vary, and can be verified by plotting the points on the coordinate grid.

(b) All points are on or above the line.

(c) The points that satisfy the less than or equal to inequality would be on or below the line.

Check Your Understanding

[Completion and discussion: 20–30 min]

QUESTION 16 Page 9

This question could be assigned for homework after completing the Focus. It is designed to give students some additional basic understanding of inequalities and their meaning. Students should record their findings in their notebooks. This question should only be assigned to students who completed Question 15.

Answers

16\. (a) $7 > 4$ seven is greater than 4

(b) $-5 < -3$ negative five is less than negative three

(c) $-2 < 4$ negative two is less than 4

(d) $\frac{1}{2} < \frac{2}{3}$ one-half is less than two-thirds

QUESTION 17 Page 9

Have students compare their answers and make sure they discuss any differences they see. Students should have little difficulty with this question, but it will help to reinforce their understanding of an inequality.

Management Tip

It may be necessary to review why points can be on the line as part of the inequality.

Answers

17. (a) Some possible answers for x are 2, 1, 0, –1, –2, and so on. The statement says that x is less than or equal to two.

    (b) Some possible answers for y are 4, 5, 6, 7, 8, and so on. The statement says that y is greater than or equal to four.

QUESTION 18 Page 9

This question parallels the Focus, and students should be able to complete it quite readily. Students should be given time in class to do this so that you can ensure that they understand how to work with inequalities. This question and Question 19 are setting the stage for the Focus on shading that follows.

Answers

18. *examples*: (–2, –8), (–1, –3), (0, 0), (0, 2), (1, 4), (1, 6), (2, 5). Answers will vary, and can be verified by plotting the points on the same coordinate grid as the graph.

QUESTION 19 Page 10

This question could be assigned for homework once Question 18 has been completed successfully. This question parallels Question 18 with the inequality reversed.

Answers

19. *examples*: (–2, 10), (–1, 7), (–1, 5), (0, 4), (0, 2), (1, 0), (2, –2). Answers will vary, and can be verified by plotting the points on the same coordinate grid as the graph.

QUESTION 20 Page 10

This question will be challenging for students. They are now working backward from the graph to the inequality. You may need to assign only part (a) and then take it up with students. The rest of the question could be assigned as a homework assignment to be is taken up the next day. Encourage students to work together here, even when they are completing the rest of the question for homework.

Discuss why both solutions to each inequality make sense. There could also be other inequalities that students want to use and these could be discussed as a class.

Answers

20. (a) $y \le 3x + 1$; $y < 3x + 1$

    (b) $y \ge x + 5$; $y > x + 5$

    (c) $y \ge -2x + 1$; $y > -2x + 1$

    (d) $y \le -x + 3$; $y < x + 3$

FOCUS B Shading

[Suggested time: 5–10 min] ***[Text page 11]***

In this Focus, students are shown that an efficient way to show all of the points that satisfy an inequality is to shade the region of the graph that contains the points. If students have successfully completed the Check Your Understanding questions, little time will be needed to discuss this Focus. In fact, these students could move directly to Questions 21 to 25. This will leave you more time to work individually with students who may be struggling with this concept, to help them stay on task, and to understand inequalities.

You may have noticed by now that a fair amount of time is devoted to inequalities in this section. It is an important concept, and one that students at this level sometimes find challenging. There is ample time in this curriculum for you to meet this challenge and many opportunities in this book to help you ensure that students have a grasp of this concept. It is vital to an understanding of linear programming.

Management Tip

Students should discuss the concept of an infinite number of solutions to an inequality.

Check Your Understanding

[Completion and discussion: 20–30 min]

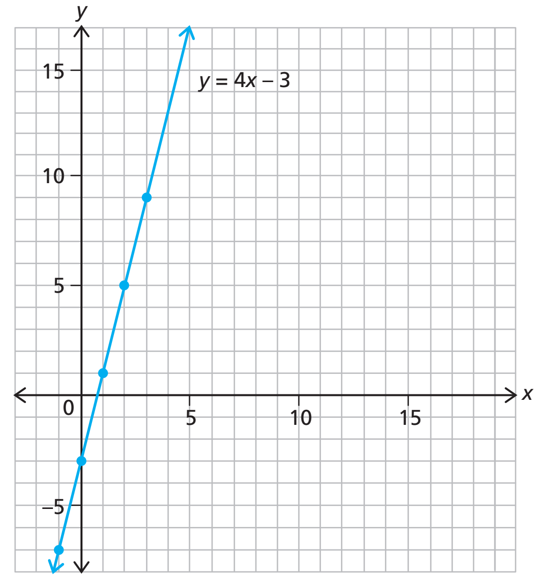
QUESTION 21 Page 11

The goal of this question is to have students develop and use a method to find the region described by an inequality. Part of a method is given here through the location of a point. If students effectively use this point in part (c), then they will have developed one method for finding the shaded region. There are other methods, like thinking "above" or "below" the line or even moving a ruler from the line toward a point, and once students have completed this question, the other methods should be discussed and a list created. This list could be posted in the classroom for students to refer to whenever they wish. After discussing this question, students should use whatever method they discover to find the appropriate region of an inequality. *This is an important question for all students to complete and discuss*.

Answers

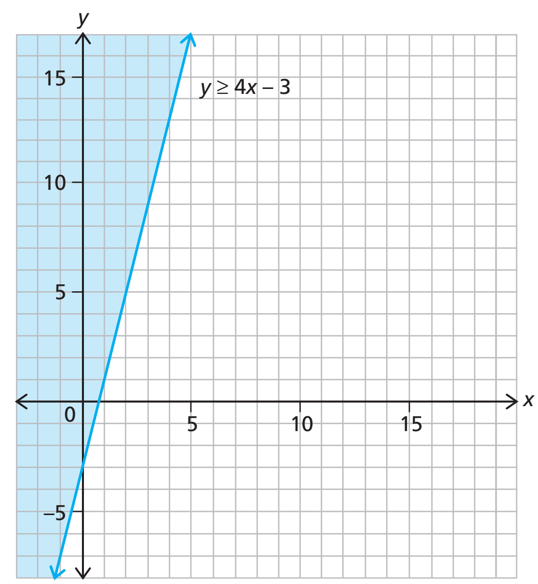
21. (a)

x	y
–1	–7
0	–3
1	1
2	5
3	9



(b) *examples*: (–1, –2), (0, 0), (1, 4), (2, 7), (2, 5), (3, 11). Answers will vary. However, any point plotted above or on the line is correct.

(c)

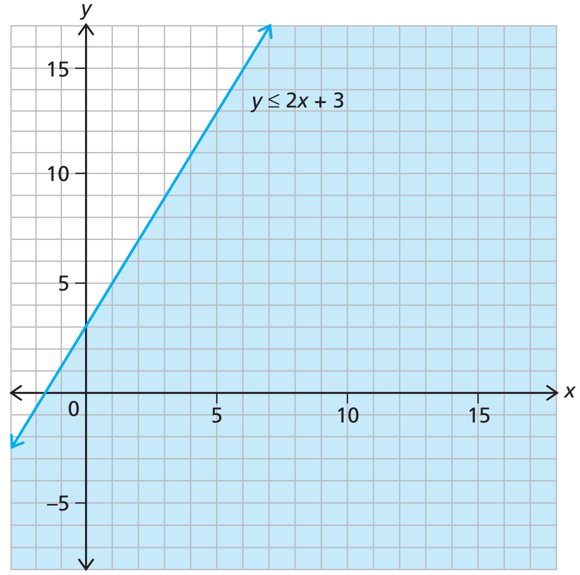


QUESTION 22 Page 12

This question is similar to Question 21 and could be assigned for homework after completing and discussing Question 21. In fact, having students hand this in to you for evaluation will help you identify the ones who understand how to find the region defined by an inequality, and those that are still struggling. Question 23 could be used to help you work with those students who still need help understanding the concept.

Answers

22. The graph shows that (1, 8) is not a possible solution to $y \leq 2x + 3$.

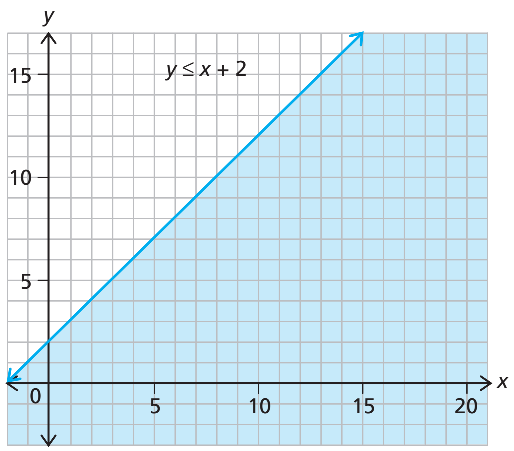


QUESTION 23 Page 12

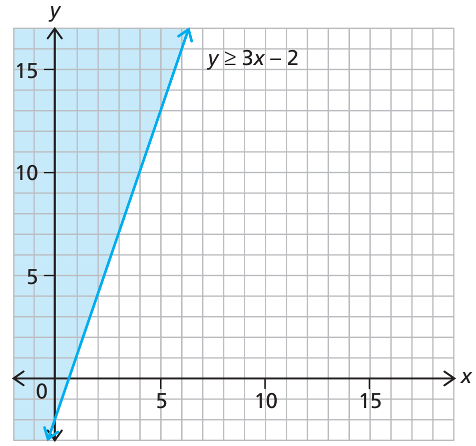
This question will have students find the region defined by inequalities. This could also be a homework question. You may have noticed at this point that all the inequalities are "less than or equal to" or "greater than or equal to." Inequalities of "less than" and "greater than" will be explored later in the chapter.

Answers

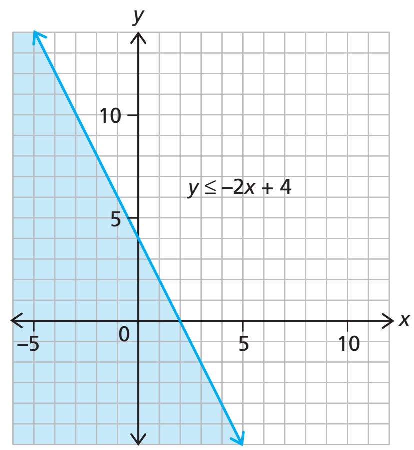
23. (a)



(b)



(c)



QUESTION 24 Page 12

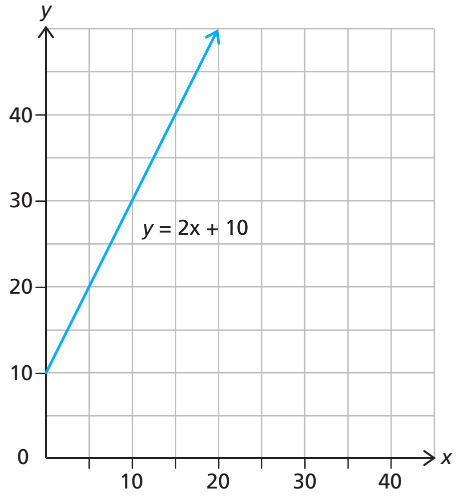
This question and Question 25 are applications. Students who have successfully completed the Check Your Understanding questions to this point will have little difficulty with these applications.

For part (c), discuss why the line is dotted. Students need to be aware that points on a dotted line are not part of the region. All values are *greater than* Wendy's and not *greater than or equal to*. In other words, Wendy's friends all pay more than Wendy. Some students may write an equation to represent the situation described in the problem and then use it to create a table of values or to plot the graph using $y = mx + b$. Others may simply use the description in the problem to create the table of values. The equation will be $y = 2x + 10$, where x represents the hours of Internet use, and y represents the amount of the bill.

Answers

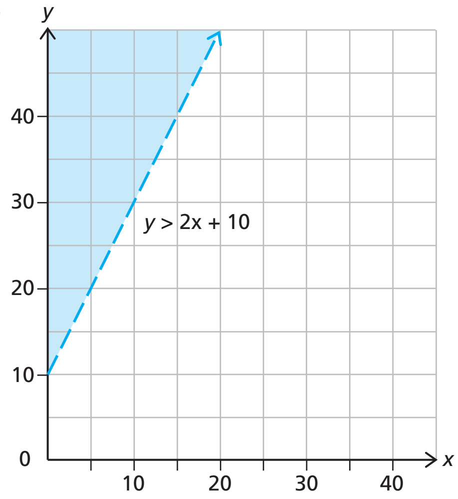
24. (a)

x	y
0	10
1	12
3	16
4	18
5	20



(b) Wendy's friends all pay more for using the Internet. *Examples*: (1, 15), (0, 20), (2, 18), (3, 25). Answers will vary. All correct answers will plot above the line.

(c)

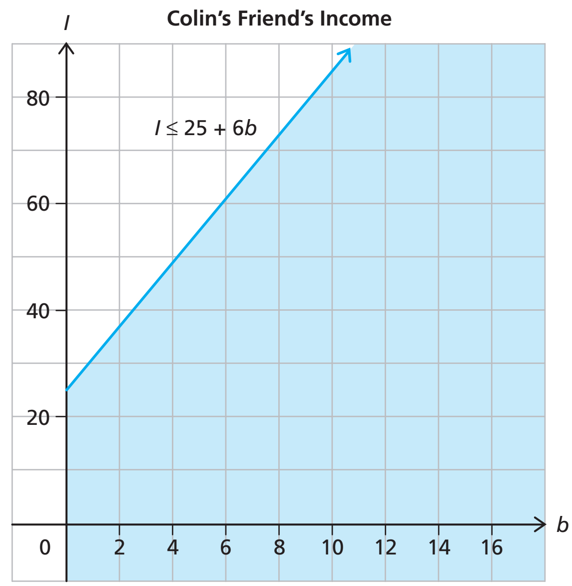


QUESTION 25 Page 12

Answers

25. (a) $l \leq 25 + 6b$

(b)



FOCUS C

Think About Inequalities

[Suggested time: 10–15 min] ***[Text page 12]***

Students are shown how to rearrange an inequality in the form y is less than or equal to ($y \leq$) or the form y is greater than or equal to ($y \geq$). If the equation is written in this form, students will be in a better position to decide whether to shade above or below the line. It may also be easier to verify that a point is above or below the line by substitution. Finally, if students want to use graphing technology, the inequality will have to be written in this form. Students should realize that rearranging an inequality is similar to rearranging a linear equation to the form $y = mx + b$.

Management Tip

Every inequality that is rearranged in this Check Your Understanding will only require division by a positive coefficient. This is because of the context of linear programming, which very seldom has information that gives constraints with negative values. You may decide to introduce division by a negative coefficient to some classes.

Check Your Understanding

[Completion and discussion: 30–35 min]

QUESTIONS 26 TO 30 Page 13

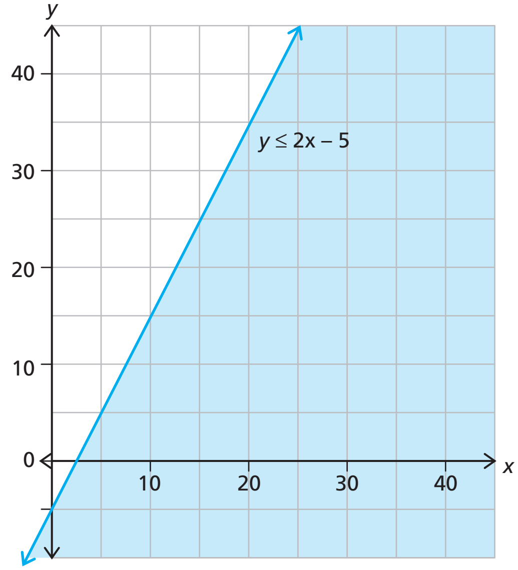
Question 26 could be assigned to all students, with the results being taken up after they have had a chance to complete it. The rest of the questions could be assigned for homework. Questions 27 through 29 could be used as remediation for those students having difficulty.

Answers

26. (a) $y \leq -2x + 10$ (b) $y \leq -3x + 5$

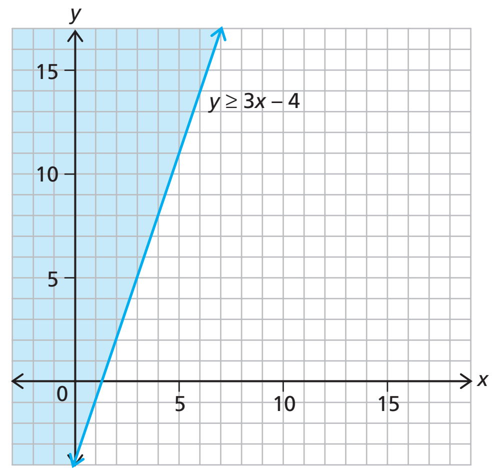
27. (a) $y \geq -2x + \frac{7}{2}$ (b) $y \geq -3x + 5$

28. (a)



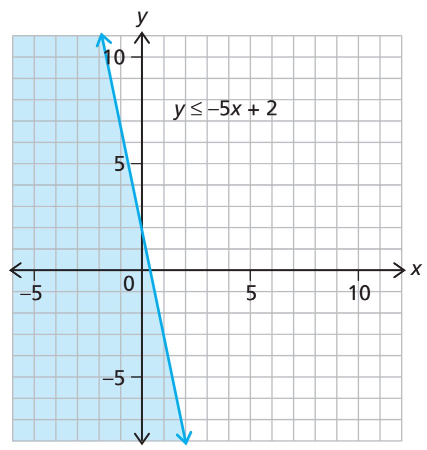
y
40
30
20
10
0
10
20
30
40
x
y ≤ 2x – 5

(b)



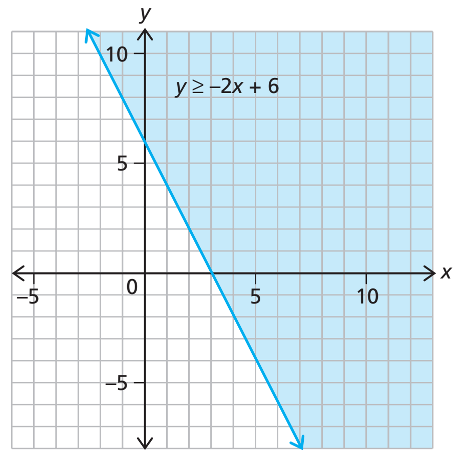
y
15
10
5
0
5
10
15
x
y ≥ 3x – 4

(c)

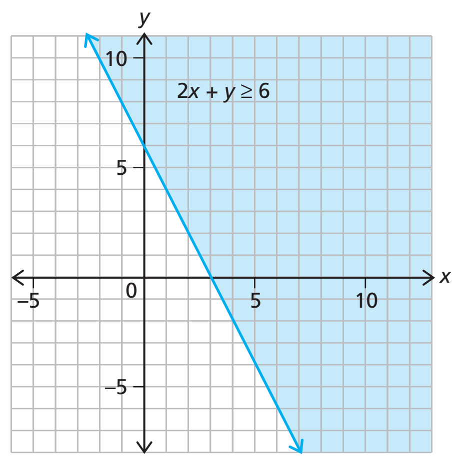


y
10
5
–5
0
5
10
–5
x
y ≤ –5x + 2

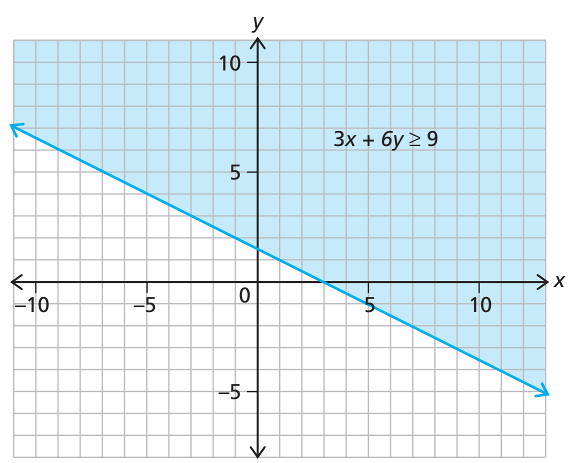
(d)



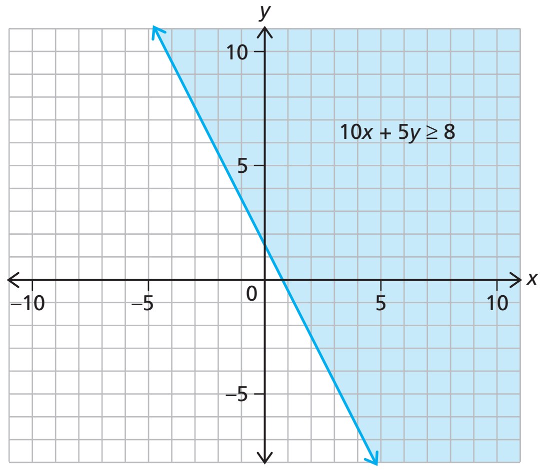
29. (a) $y \geq -2x + 6$



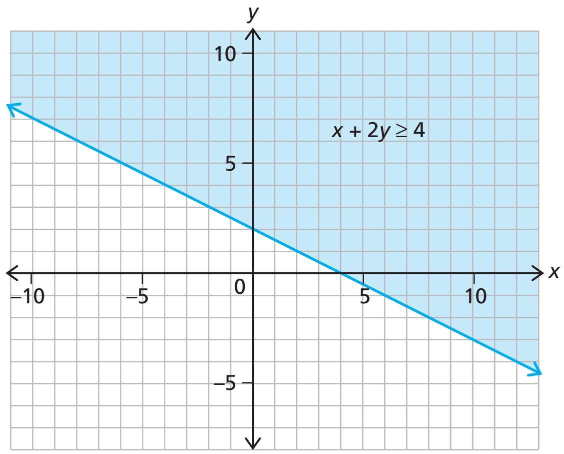
(b) $y \geq -\frac{1}{2}x + \frac{3}{2}$



(c) $y \geq -2x + \frac{8}{5}$

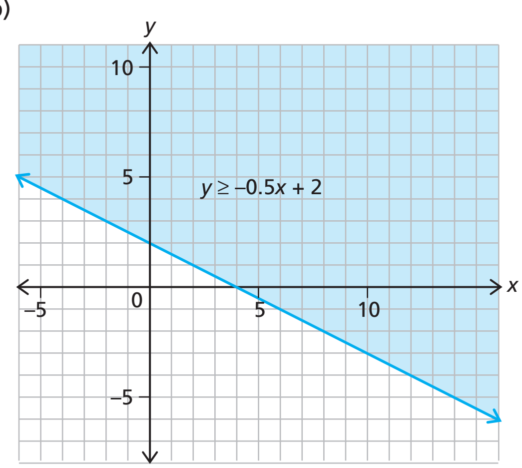


(d) $y \geq -\frac{1}{2}x + 2$



30. (a) $5x + 10y \geq 20$; the inequality can be arranged as $y \geq -0.5x + 2$

(b)



1.2 *Exploring Possible Solutions*

Suggested instruction time: 5–7 hours

Purpose of the Section

Students investigate possible solutions to a maximization and/or minimization problem that involves linear relationships. They will develop an understanding of the term *feasible region* in the context of various problems. Students will associate points in the feasible region with possible solutions to the problem by relating them to the constraints on the variables. Graphing of linear inequalities will be investigated and practised as a start for showing a feasible region on a coordinate grid. The main emphasis in this section is on the understanding of the meaning of the concept of a feasible region.

CURRICULUM OUTCOMES (SCOs)	RELATED ACTIVITIES	STUDENT BOOK
■ relate sets of numbers to solutions of inequalities A2	■ investigate a maximum-income problem based on known and assumed constraints	p. 14
■ apply the linear programming process to find optimal solutions C6	■ graph relationships and express them with equations	p. 14
■ demonstrate an understanding of real-world relationships by translating between graphs, tables, and written descriptions C8	■ identify constraints and their effects	p. 14
■ express and interpret constraints C11		
■ interpolate and extrapolate to solve problems C18		
■ solve systems of equations and inequalities both with and without technology C20		
■ represent systems of inequalities as feasible regions E3		
■ represent linear programming problems using the Cartesian coordinate system E4		

ASSUMED PRIOR KNOWLEDGE
■ linear equations in one variable
■ inequality
■ solving equations
■ substitution
■ graphing linear equations $y = mx + b$

NEW TERMS AND CONCEPTS	PAGE
■ constraint	14
■ feasible region	18

Suggested Introduction

Refer to Heather's problem from Section 1.1 and discuss some possible constraints on her income that could be caused by her personal life. Discuss why the manufacturer will also impose constraints that have an impact on Heather's situation. In any situation like this, the wants and needs of the individual may have to be adjusted so that the company's priorities can be met. This problem presents an opportunity for students to discuss the concepts of give and take and conciliation.

Investigation 3

Part 1: Look at the Constraints on Chairs

[Suggested time: 10–15 min] ***[Text page 14]***

Purpose

Students will consider the constraint that Spinney Manufacturing puts on the number of chair bundles that Heather must cut every two weeks.

constraint—any condition that must be met

Materials

- grid paper

Procedure

Step A

Students are asked to copy the given coordinate grid on to graph paper.

Step B

- The number of chair bundles cut must be greater than or equal to eight.
- $x \geq 8$, where x represents the number of chair bundles. The number of chair bundles must be greater than or equal to 8 and the number of couch bundles can be represented by any value.

Management Tip

Remind students to label the x- and y-axes.

- Possible points: (Answers will vary.)

Number of chair bundles	Number of couch bundles
x	y
8	5
8	10
9	10
9	11
9	12
10	7
10	10
10	11
10	12
11	12

- Students will add the points from three other students to their table and graph.
- The points are found in a region of the graph where x equals eight, or to the right of x equals eight. The boundary of the region is the vertical line $x = 8$.The region can be represented by the inequality, $x \geq 8$.

Investigation 3

Part 2: Look at the Constraints on Couches

[Suggested time: 15–20 min] ***[Text page 15]***

Purpose

Spinney Manufacturing also places a constraint on the number of couch bundles that must be cut every two weeks. Students will investigate this constraint and combine it with the constraint on chair bundles.

Materials

Students will need grid paper and the graph they drew in Investigation 3, Part 1.

Step A

Students are asked to copy the given coordinate grid on to graph paper.

Step B

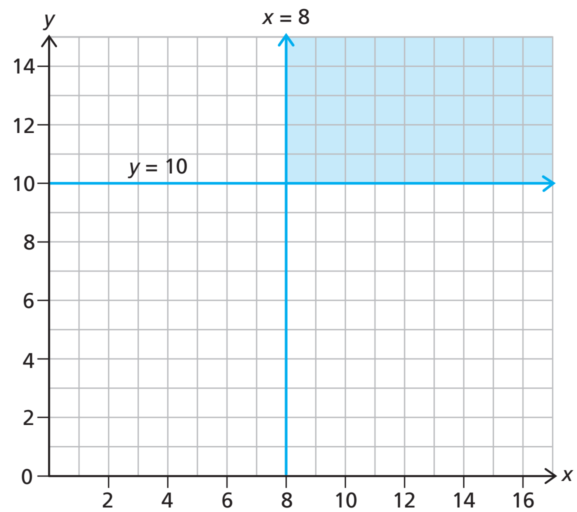
- The number of couch bundles cut must be greater than or equal to 10.
- $y \geq 10$, where y represents the number of couch bundles.
- Answers will vary. Every value chosen for y must be greater than or equal to 10. If students take the inequality in Part 1, Step A into account, they will choose values for x that are greater than or equal to 8. However, this question does not require that they do this. The point (7, 10) is an acceptable answer at this time.

Number of chair bundles	Number of couch bundles
x	y
7	10
8	10
8	11
9	10
9	11
9	12
10	10
10	11
10	12
11	12

- Students will add the points of three other students to more firmly establish the location of the region of the graph where all points will be found.
- The region of the graph where the points are found is on the line $y = 10$ or above this line. The boundary to the region is the horizontal line, $y = 10$. The region can be represented by the inequality, $y \geq 10$.

Step C

- When the graphs are placed on top of each other, the common region occurs to the right of $x = 8$ and above $y = 10$ and includes the line. Discuss with students what the region now represents. This will be an important discussion for students as it is the first time they have seen a feasible region, or at least the beginning of a feasible region, and it is also the beginning of the development of the feasible region for Heather's problem.



Management Tip

Students may have to hold their graphs up to the light, place them against a window, or place them on the overhead projector to see the overlap region.

Management Tip

It will be helpful to have previously prepared examples of the two graphs on overhead transparencies so that the overlap region can be displayed for discussion.

Check Your Understanding

[Completion and discussion: 35–40 min]

Questions 1 to 3 will give borders that are horizontal and/or vertical. Most students should be able to progress through these questions quickly.

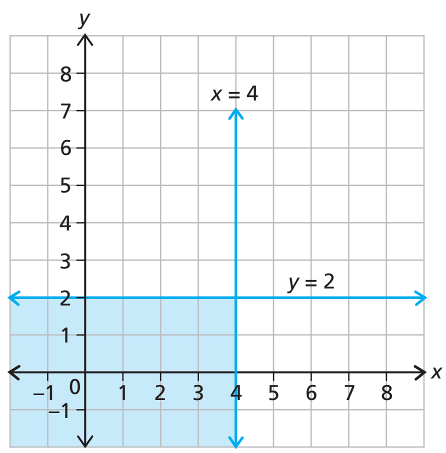
QUESTION 1 Page 16

Although students did a question like this in the previous section, that was simply setting the stage for the work to be done here. Students who successfully completed Question 20 of the previous section should have no difficulty with this question. Other students will have little difficulty with it. This question will also set the stage to return to Heather's problem and write the inequalities describing her assistance in the production of couches and chairs.

Answers

1. (a) $y \leq 2$; $x \leq 4$

(b)

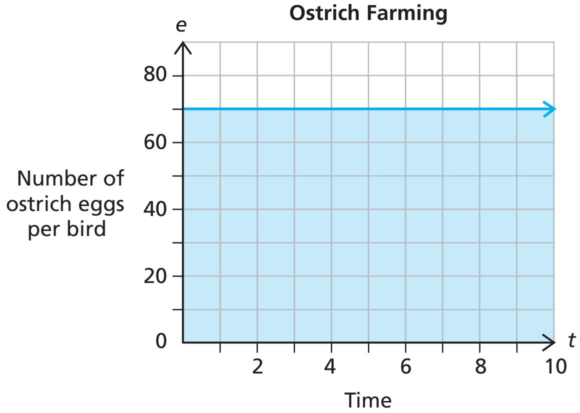


(c) Answers may vary. Students may tell you that she can cut only a maximum of four couch bundles and a maximum of two chair bundles. As an extension, you could have students modify their shaded regions to better show their context.

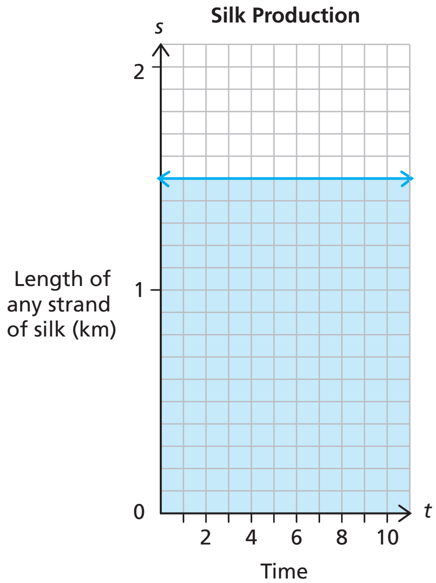
QUESTION 2 Page 16

Answers

2. (a) $e \leq 70$, where e represents the number of ostrich eggs



(b) $s \le 1.5$, where s represents the length of silk produced by each worm

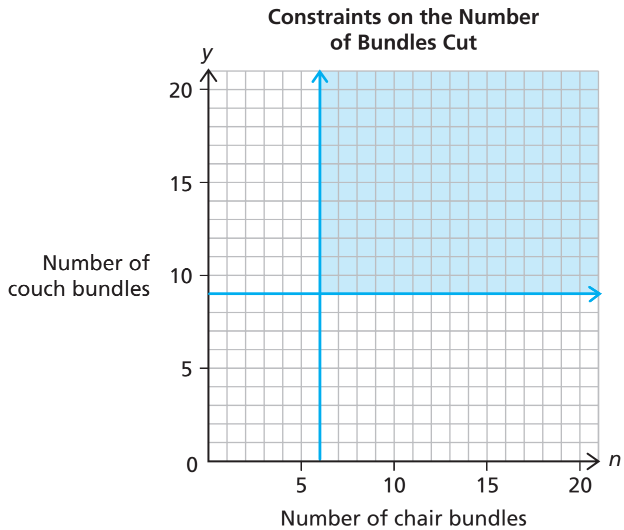


QUESTION 3 Page 16

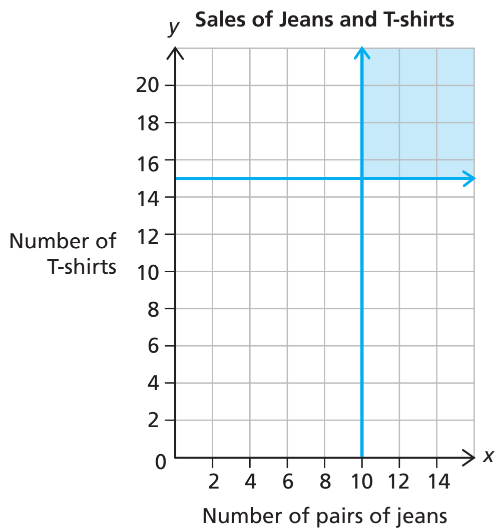
This question parallels Heather's problem and asks a "What If ...?" question. Students will be working with an application that gives an inequality whose border can be represented by a horizontal and/or a vertical line.

Answers

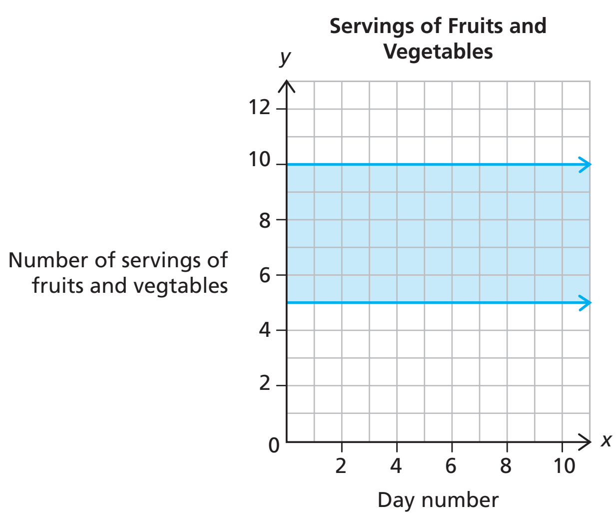
3. (a) $x \ge 6$, x represents the number of chair bundles
   $y \ge 9$, y represents the number of couch bundles



(b) $x \geq 10$, x represents number of pairs of jeans
$y \geq 15$, y represents the number of T-shirts



(c) $y \geq 5$ and $y \leq 10$, where y represents a serving of fruit or vegetables



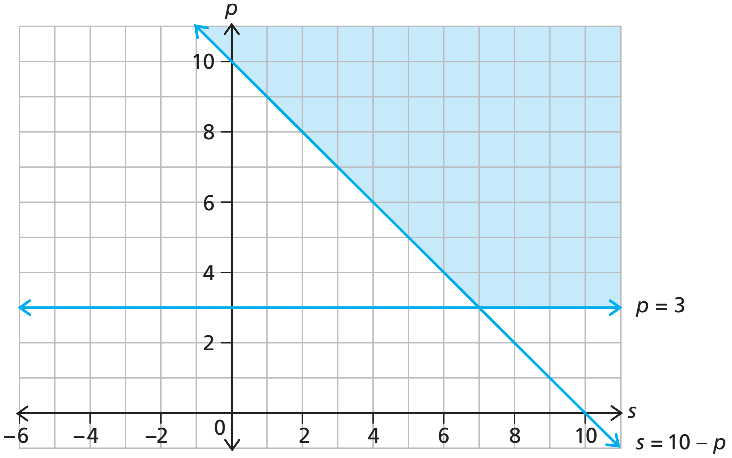
QUESTION 4 Page 17

This is the first question that requires students to work with a pair of inequalities that does not produce only horizontal and vertical lines. Point this out to students before having them begin the question so that they are not questioning their results.

Answers

4. (a) $p \geq 3$; $s \geq 10 - p$

   (b) $s \geq 10 - p$



   (c) Yes. Students should see that the $s = 15$ will be in the feasible region.

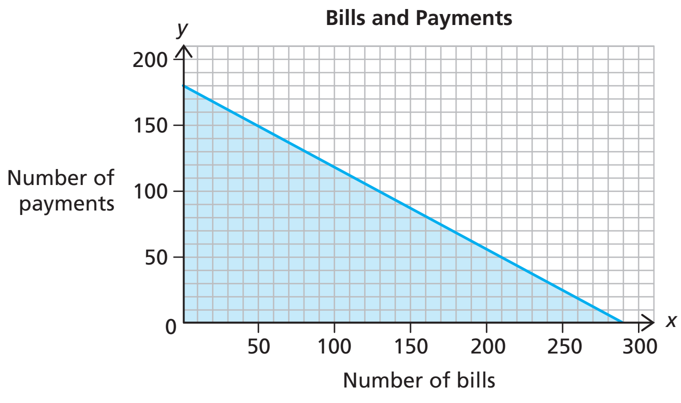
QUESTION 5 Page 17

This question extends the concept of an inequality a bit further. At first glance, students may think that there is only one equation that is defining a border; however, this is not the case. There are the "invisible" inequalities that suggest that real objects cannot be less than zero. As a result, $x \geq 0$ and $y \geq 0$. This idea could be discussed with students, and you could have them summarize the conversation in their notebooks. Also, have students use examples of their own to support their summary. This question is best done using graphing technology.

Answers

5. (a) $5x + 8y \leq 1440$, where x represents the number of bills and y represents the number of payments (1440 min = 24 h)

   (b)



secondary source—a source written *about* the subject, not *by* the subject

(c) Yes. There are combinations that sum to 200 that will allow 200 bills to be processed. They cannot, however, all be payments that are processed.

FOCUS D Groups of Inequalities

[Suggested time: 30–40 min] *[Text page 18]*

New problem solving situations are introduced where the overlap region of the graphs of inequalities will contain possible solutions to the problem. This will help students to solidify their understanding of a feasible region and how its formation can lead to determining possible solutions to the problem. Students should also see that they have not determined a single solution to the problem; they have only found a region of the graph where all possible solutions can occur. Allow students time to work through the Focus by working together before taking up the results.

Management Tip

These steps are to make it easier for students to follow the solution.

feasible region—a shaded region on a graph indicating that all points within the region are possible solutions to the problem

Management Tip

When you are discussing first-quadrant solutions, you can introduce the added constraints on the problem that $x \geq 0$ and $y \geq 0$.

Example

Step A

The inequalities that represent the amount of carbohydrate in the peanuts and the chocolate chips are given, as is the inequality that shows the total amount of carbohydrate in the mixture.

Step B

The total mass of peanuts and chocolate chips must be less than or equal to 500 g.
$p + c \leq 500$

Step C

The feasible region will be formed by the p- and c-axes and the lines $0.275p + 0.737c = 150$, and $p + c = 500$. There are no solution points outside of the first quadrant because you cannot have a negative amount of peanuts or chocolate chips. Also, students should be aware that feasible solutions could occur on the boundary lines or at the vertices or corners of the feasible region. This should be discussed with students.

Step D

Answers will vary. All possible solutions must be in the feasible region or on its boundaries or vertices.

Focus Question

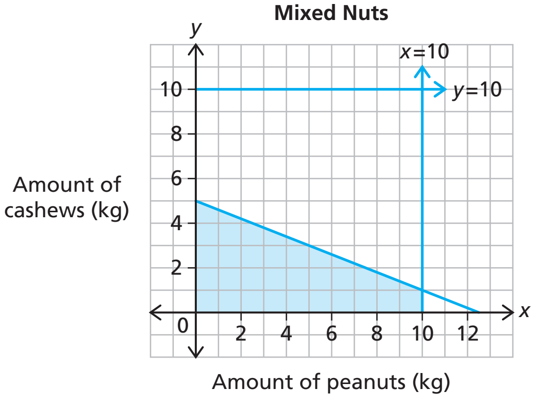
QUESTION 6 Page 19

The inequality $x \leq 10$ represents the fact that there can be a total of 10 kg of nuts.

If peanuts cost \$4/kg and there are x kg of peanuts, then $4x$ represents the cost of the peanuts. If cashews cost \$10/kg and there are y kg of cashews, then $10y$ represents the cost of the cashews. The total cost must be equal to or less than \$50. The inequality $4x + 10y \leq 50$ represents the total cost constraint.

Answers

6.



The shading stops at the x- and y-axes because you cannot have a negative amount of peanuts or cashews. The minimum amount of each is zero. These additional constraints can be represented by $x \geq 0$ and $y \geq 0$.

Some sample solutions are (3, 3), (4, 2), (5, 2.5), (6, 2), and (8, 1). Answers will vary, but each answer should be a point in the feasible region. There is no one correct answer but many possible solutions.

Management Tip

Discuss with students if points on the lines could be solutions to the problem.

Check Your Understanding

[Completion and discussion: 35–45 min]

QUESTION 7 Page 19

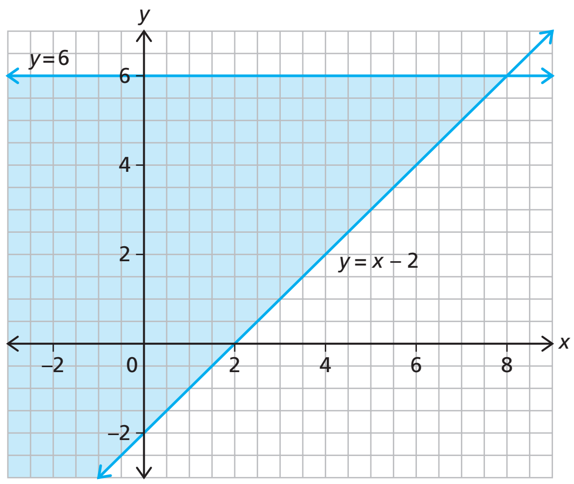
This is a vital question for students to understand. If students cannot graph a system of two inequalities, then they will not be able to complete the process of linear programming. Spend as much time as you can on this question to ensure student understanding. If necessary, rearrange the inequalities into the form $y = mx + b$ for students so that the arrangement of the inequality doesn't confuse them. In other words, it is the graphing that is important here, not the rearrangement of inequalities.

Management Tip

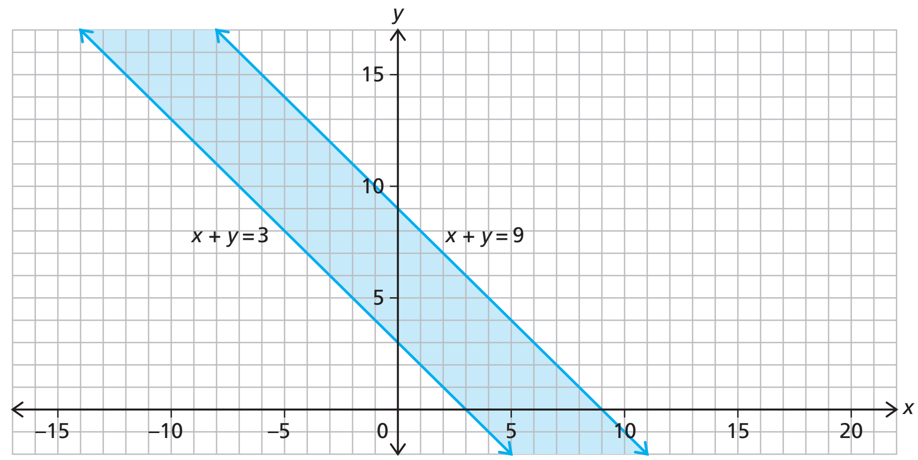
For Question 7, students could be encouraged to complete only half of the questions and then share their results with others in the class. As long as students record all the others' work and ask any necessary questions, they could have success and save time when completing this exercise.

Answers

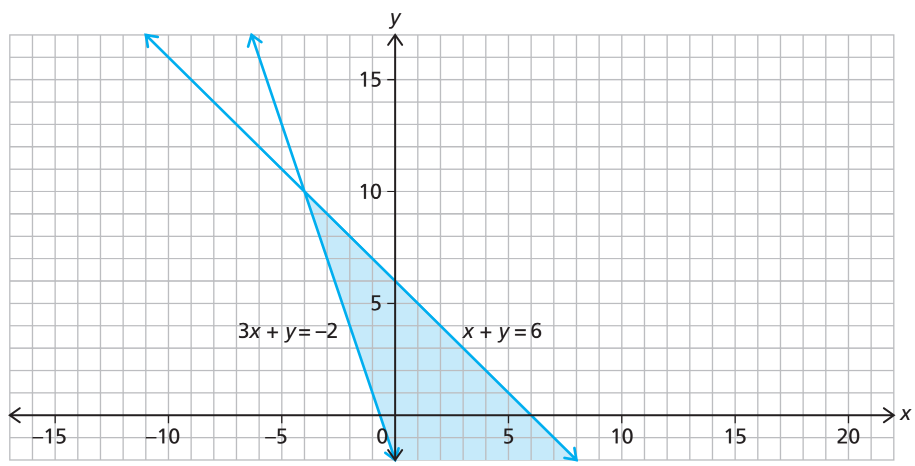
7. (a) $y \leq 6$ and $y \geq x - 2$



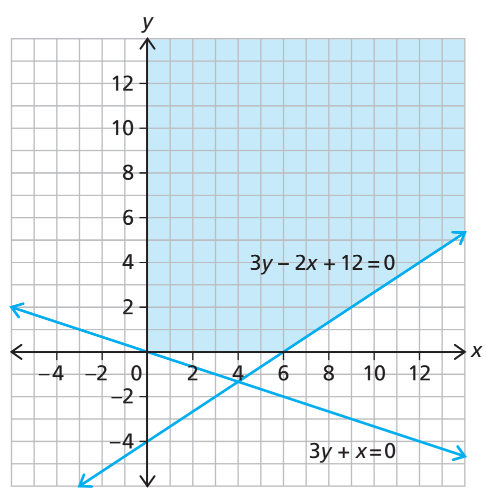
(b) $y \le 9 - x$ and $y \ge -x + 3$



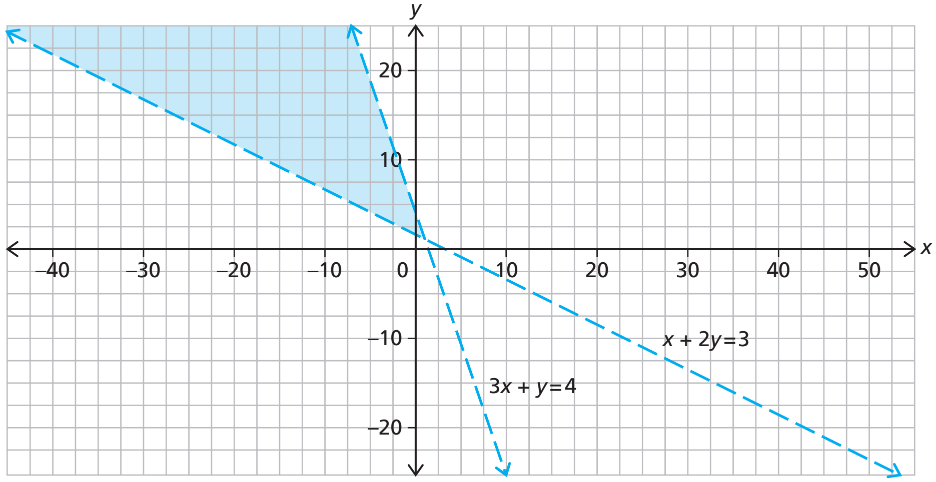
(c) $3x + y \ge -2$ and $x + y \le 6$



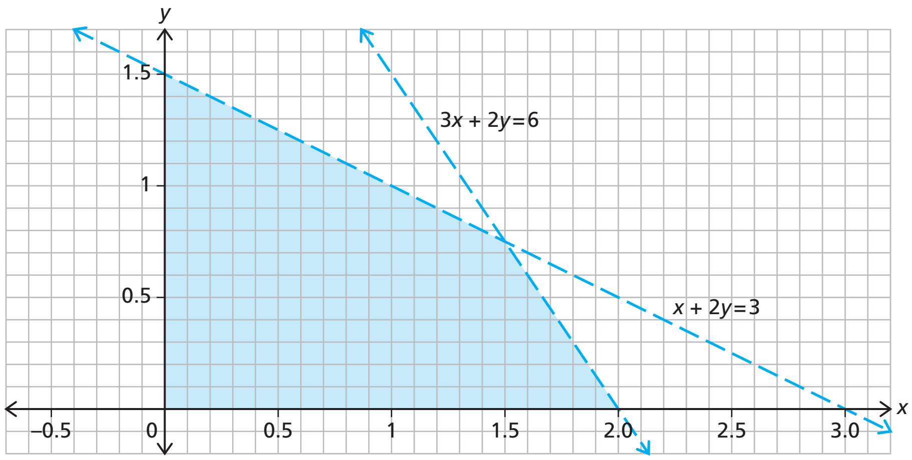
(d) $3y - 2x + 12 \ge 0$ and $3y + x \ge 0$ and $x \ge 0$ and $y \ge 0$



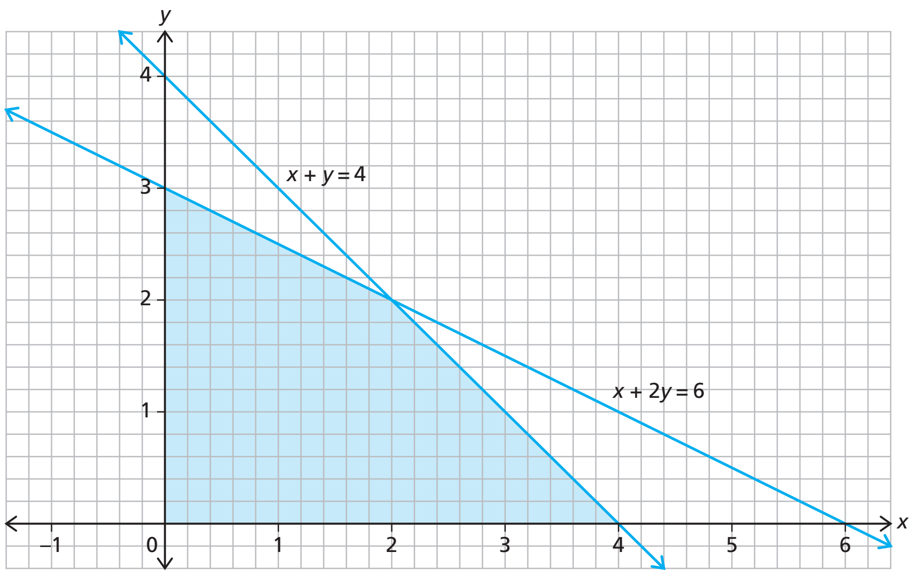
(e) $y < 4 - 3x$ and $y > 1.5 - 0.5x$



(f) $3x + 2y < 6$ and $x + 2y < 3$ and $x \geq 0$ and $y \geq 0$



(g) $x + y \leq 4$ and $x + 2y \leq 6$ and $x \geq 0$ and $y \geq 0$



Management Tip

It will likely be necessary to review with students how to find the y-intercept and the slope from a graph. Extra examples should be given.

Management Tip

Students will have to extend the lines $y \le -\frac{4}{5}x + 16$ and $y \le -\frac{2}{5}x + 12$ to find the intercepts.

QUESTION 8 Page 19

Remind students how to find the equation of a line from a graph using $y = mx + b$. The slope and y-intercept can be read from the graph and substituted into $y = mx + b$.

Answers

8. (a) You need four inequalities because the feasible region is in the first quadrant. The x- and y-axes are also boundaries of the region.
   (b) The inequalities are $x \ge 0$, $y \ge 0$, $y \le -\frac{2}{5}x + 12$, and $y \le -\frac{4}{5}x + 16$.
   (c) Answers will vary. Many will use the ideas of Examples 1 and 2 . The context of the problem should be reasonable.

QUESTION 9 Page 20

This is the first of four straight word problems. The problems have been broken into parts for ease of reading and completion. Allow students to work together so that reading the problem will not interfere with their ability to solve the problem and demonstrate good understanding of the mathematics.

Answers

9. (a) Sue can plant 2000 ha of vegetables. She can plant a maximum of 1500 ha of lettuce and a maximum of 1000 ha of corn. Any other constraints caused by market demand, quality of the soil, available hours of sunlight, etc., are not mentioned in the problem.
   (b) *examples:*

Lettuce	Corn
1500 ha	500 ha
1400 ha	600 ha
1300 ha	700 ha
1000 ha	1000 ha

   (c) *example:* 1100 ha of corn and 900 ha of lettuce; exceeds maximum for corn
   (d) *examples:* the profit that can be made on each vegetable, the cost of care for the plants, the time required to plant and care for the plants, etc.

QUESTION 10 Page 20

To help students solve this problem, encourage them to set up and use a chart. This will help them summarize the information given in the problem and see a solution more readily. A chart that students could use is shown in the following solution.

Answers

10. (a) 22 Berry Patch gift packs could be made; profit $88
    (b) 12 Morning Glory gift packs could be made; profit $36

(c)

Possible Morning Glory Pack			Possible Berry Patch Pack			Totals	
Morning Glory	Jam	Marmalade	Berry Patch	Jam	Marmalade	Total Jam	Total Marmalade
7	7	28	20	80	20	87	48
10	10	40	10	40	10	50	50
8	8	32	15	60	15	68	47

(d) *example*: 20 Berry Patch and 7 Morning Glory would use 87 bottles of jam and 48 jars of marmalade; Profit: $101

Not all of the available jam and marmalade would be used.

Management Tip

Students will likely discover a variety of answers for Question 10 (d). In most cases, they are likely getting close to the optimal answer.

QUESTION 11 — Page 21

Up to this point in the section, the Student Book has focused on the feasible region. It was important that students continued to develop their understanding of a region in order to continue to develop their understanding of linear programming. It is important, throughout this chapter, that students do not lose sight of the ultimate goal, and that is to solve Heather's problem. As a result, this question and Question 12 will have students refer to Heather's original problem and continue to solve it. By this time, students should have a good grasp of the concepts presented and the mathematics should not get in the way of being able to work further on the original chapter problem.

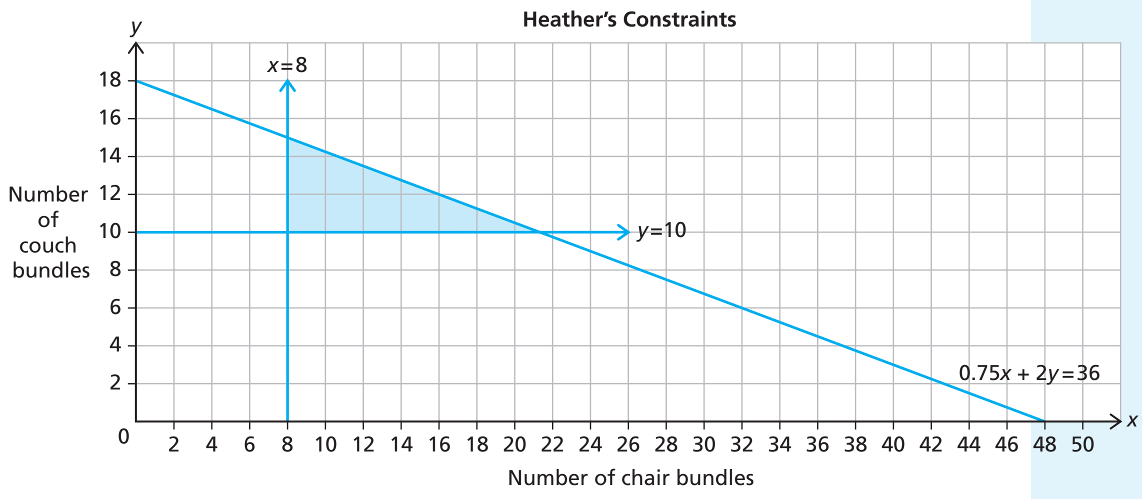
The original problem is summarized here for students so that they are not searching the Student Book for the original information and can dive quickly back into the problem.

Make sure that students complete Questions 11 and 12 as this will help them discover the feasible region for Heather's original problem. The region will also be used in Section 1.3.

Answers

11. (a) It takes 0.75 hours to cut a chair bundle and 2 hours to cut a couch bundle. The total time that Heather works in 2 weeks is equal to or less than 36 hours. The time constraint can be expressed as $0.75x + 2y \leq 36$.

(b) Graph of $x \geq 8$, $y \geq 10$, and $0.75x + 2y \leq 36$



(c) The feasible region will be smaller. In order to satisfy all three constraints, the overlap region becomes smaller. The number of solutions in the feasible region decreases.

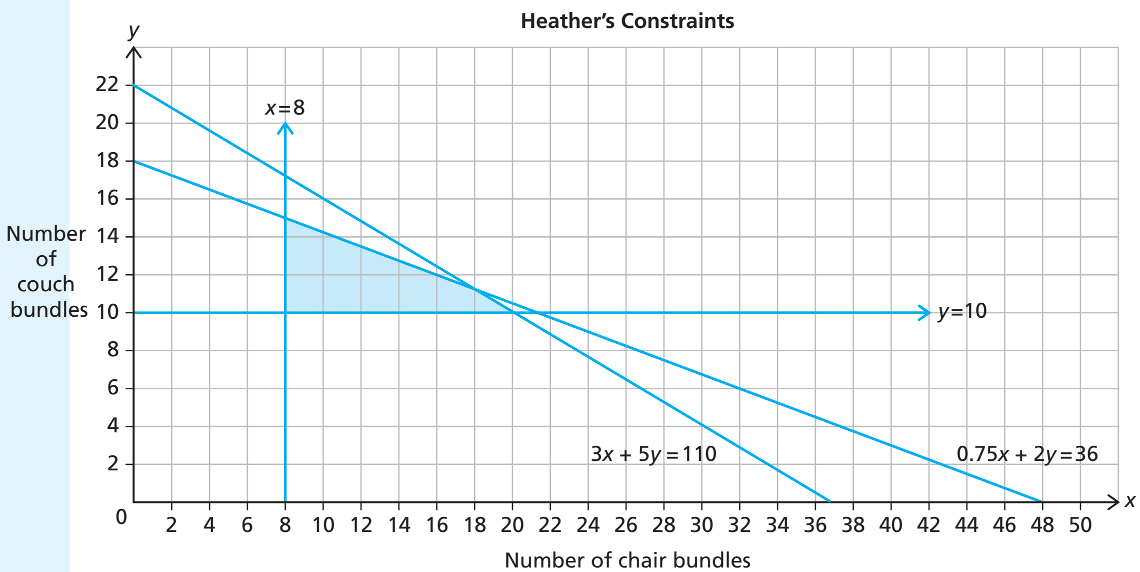
QUESTION 12 Page 21

This question can only be completed when Question 11 has been completed. Make sure students keep all their work from Question 11 to use here and in the next section. Once students have completed this question, they will have sketched the feasible region for Heather's problem. Because students will need an accurate graph of the feasible region to complete the next section, **Blackline Master 1.2.1** is provided at the end of the chapter.

Answers

12. (a) It takes 3 m of material to cut a chair bundle, and 5 m of material to cut a couch bundle. Heather can cut an amount of material less than or equal to 110 m in two weeks. The material constraint can be expressed as $3x + 5y \leq 110$.

(b) Graph of $x \geq 8$, $y \geq 10$, $0.75x + 2y \leq 36$, and $3x + 5y \leq 110$



(c) The feasible region becomes slightly smaller when the material constraint is added.

(d) *examples*: (15, 10), (16, 10), (10, 11), and (8, 10). Answers will vary. All must be contained in the feasible region.