

# Chapter One

## Making Choices— Linear Programming

**H**ave you ever watched a Grey Cup football game and wondered how the TV station decides the number of commercials to show during the game? Did you know that it costs more to run commercials in the first half of the game than it does in the second half? Does it surprise you that the likelihood of viewers turning off the game increases with the number of commercials shown?

This chapter will explore how answers to these questions—and others like them—can be used to make decisions that maximize company profits.

*After successfully completing this chapter, you will be expected to:*

1. Find constraints for the variables in problems.
2. Represent these constraints using tables of values, graphs, or equations.
3. Solve systems of linear equations using graphs or algebraic methods.
4. Use linear programming to find the best, or optimal, solution to a problem involving constraints.

# 1.1

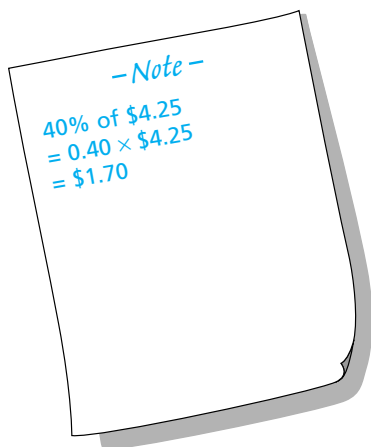
# Exploring an Optimization Problem

Heather cuts fabric for couches and chairs at Spinney Manufacturing. She is paid a flat fee for each bundle of fabric she cuts. A bundle includes all of the pieces that make a chair or couch.

- Heather is paid \$12.00 for each couch bundle she cuts.
- She is paid \$4.25 for each chair bundle she cuts.
- If she cuts only part of a bundle, then she is paid for only that part. For example, she is paid \$1.70 if she cuts 40% of a chair bundle.

Heather is paid every two weeks. What is the greatest amount of money she can earn? Later in the chapter, you will be considering other factors such as:

- time needed to cut a bundle
- number of couches and chairs that are needed
- the maximum number of hours she is allowed to work, and so on



## Investigation 1

### Looking at Heather's Income

#### Purpose

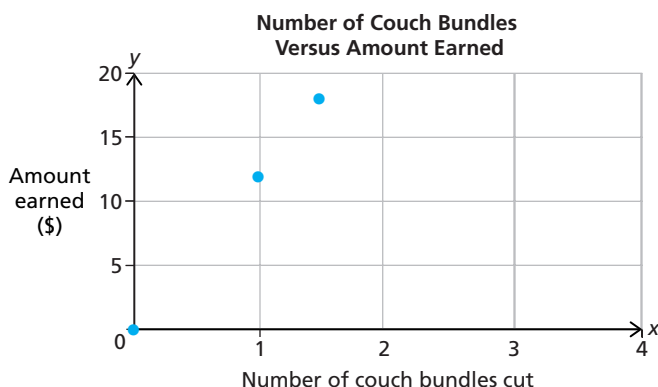
Investigate the amount of money Heather can earn.

#### Procedure

- A. • Copy and complete the table of values to show the relationship between the number of couch bundles Heather cuts and the amount of money she earns. Use at least five more values for the number of bundles.

Number of couch bundles cut	Amount earned (\$)
0	0
1	12.00
1.5	18.00

- Copy and complete the graph. Should you join the points? Why?



- Write the set of all possible values for the number of couch bundles (**domain**) and the set of all possible values for the amount earned (**range**).
  - Which is the **independent variable** and which is the **dependent variable**?
- B.** Repeat Step A for the number of chair bundles.
- C.** Find the **slopes** of the graphs in Steps A and B.
- What does each slope represent?
  - What is the **y-intercept**?
  - What is the **x-intercept**?
  - What do the intercepts represent?
- D.** Describe any patterns you see in the graphs you drew in Steps A and B.
- Write a word equation to represent each graph.
  - Write an equation using variables for each graph.

## Investigation Questions

- At a charity car wash, the student council charged \$3.50 to wash a car.
  - Create a table of values to show the relationship between money earned and the number of cars washed.
  - Create a graph to show any patterns in part (a). Should you join the points? Why?
  - State the domain and range for the relation.

**domain**—the set of all possible values for  $x$  (the independent variable) in any relation

**range**—the set of all possible values for  $y$  (the dependent variable) in any relation

**independent variable**—a variable that affects another variable in an experiment or relationship

**dependent variable**—the variable that is affected by another variable in an experiment or relationship

**slope**—the steepness of a line. Slope can be found using the ratio  $\frac{\text{rise}}{\text{run}}$ .

**y-intercept**—the point where the graph crosses the  $y$ -axis; the point where  $x = 0$

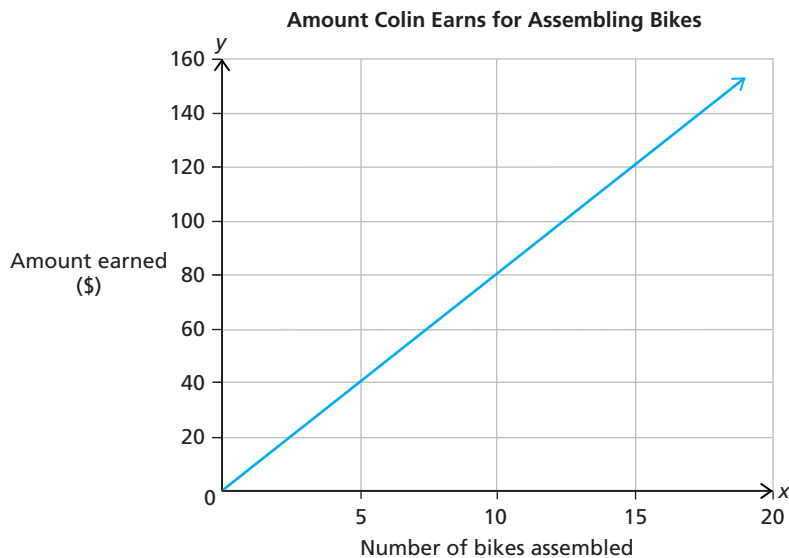
**x-intercept**—the point where the graph crosses the  $x$ -axis; the point where  $y = 0$



2. The student council charged \$5.50 to wash and wax a car.
  - (a) Construct a table of values to show the relationship between money earned and the number of cars washed and waxed.
  - (b) Construct a graph to show any patterns in part (a). Should you join the points? Why?
  - (c) State the domain and range for the relation.
3.
  - (a) Find the slopes of the graphs in Questions 1 and 2.
  - (b) What does each slope represent?
  - (c) What is the  $y$ -intercept of each?
  - (d) What is the  $x$ -intercept of each?
  - (e) What do the intercepts represent?
4. Use the relationships in Questions 1 and 2.
  - (a) Write a word equation to represent each graph.
  - (b) Write an equation using variables for each graph.

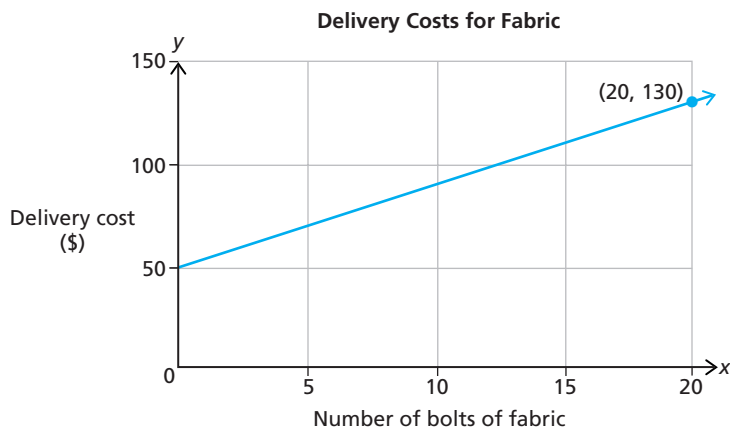
## Check Your Understanding

5. Colin works part time assembling bicycles.



- (a) What does the slope of the graph represent?
- (b) Is Colin paid for putting together part of a bike? Explain.
- (c) If Colin is paid \$68, how many bikes did he put together?
- (d) Write an equation that represents the relationship shown by the graph.

6. (a) The bike shop changes the way Colin is paid. He will receive a base amount of \$25 each week plus \$6 per bike. Make a table of values and plot a graph to show his possible earnings.
- (b) Write an equation to represent the relationship shown by the graph. What do you notice about the slope? the  $y$ -intercept?
7. Spinney Manufacturing pays for delivery of the bolts of fabric for the chairs and couches. The order clerk graphs the data to show his boss. There is a non-refundable \$50 delivery charge.



- (a) Explain the meaning of the slope and the  $y$ -intercept in relation to the delivery cost.
- (b) Use the graph to find the number of bolts of fabric delivered for \$120.
- (c) Do you think it would be reasonable to get a delivery bill of \$2500? \$40? Explain.
- (d) State the domain and range for the relation.
- (e) Write the equation for the graph. Find the cost to deliver 40 bolts of fabric.
8. If Spinney Manufacturing needs a rush delivery, the cost is given by  $y = 10x + 80$ .
- (a) What do the slope and  $y$ -intercept represent?
- (b) Plot the graph of this relationship.
- (c) Use your graph to find the number of bolts delivered for \$200.
- (d) Write and solve an equation to find the number of bolts delivered for \$200. Compare your answer to the answer for part (c). What do you notice?

- Note -

A "bolt" of fabric is a quantity by which the fabric is sold.

**Think about...**

**Question 8**

If you were working in shipping and receiving at Spinney Manufacturing, which would you use to calculate costs: the graph, the equation, or both? Explain.

If you were given only the graph or the equation, could you find the other? Explain.

- Note -

To solve an equation, isolate the term with the variable. This means that you need to get the variable by itself on one side of the equation.

For example:

$$3x + 10 = 15$$

$$3x + 10 - 10 = 15 - 10$$

$$3x = 5$$

$$\frac{3x}{3} = \frac{5}{3}$$

$$x = \frac{5}{3}$$

## Investigation 2

### More Thoughts on Income

– Note –

Before trying Investigation 2, make sure that you answer Question 5 from the previous Check Your Understanding section.

### Think about...



#### The Graph

A straight line is drawn through the points representing this situation. Why do you think this is done?

**inequality**—a mathematical statement that shows that two numerical or variable expressions are not always equal. For example,  $3x \leq 12$  is an inequality.  $x = 2$  satisfies this inequality. Does  $x = 5$ ?

– Note –

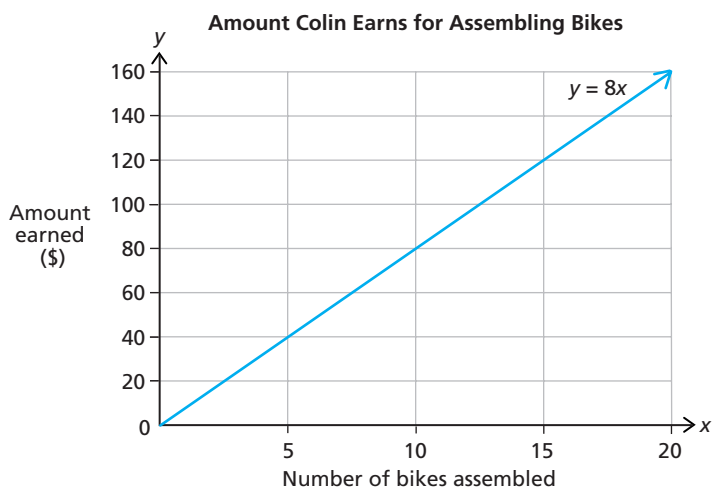
“Greater than” is shown using the symbol  $>$  and “less than” is shown using the symbol  $<$ .

### Purpose

In Question 5, you explored how Colin could earn money by assembling bicycles. Now, you will explore how Colin might earn different amounts of money.

### Procedure

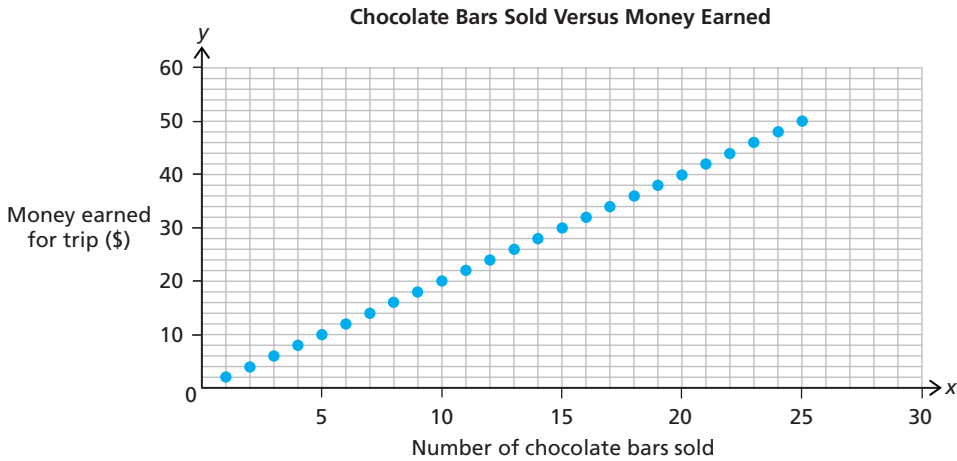
The graph below shows the amount of money Colin can earn.



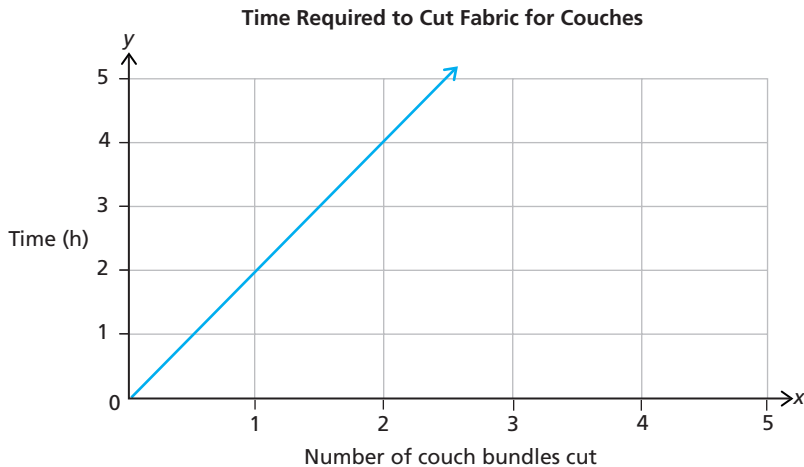
- A. List four points on the line  $y = 8x$ . What do the points on the line represent?
- B.
  - (i) Points such as (10, 90) and (12, 100) lie “above” the line. What does each point represent?
  - (ii) List four other points that lie above the line. What do all points that lie above the line represent? Discuss your reasons with other students.
- C.
  - (i) Points such as (10, 60) and (5, 22) lie “below” the line. What does each point represent?
  - (ii) List four other points that lie below the line. What do all points that lie below the line represent? Discuss your reasons with other students.
- D. Compare the points you chose with points chosen by others in the class. Write an **inequality** to represent the points in Step B. Then, write an inequality to represent the points in Step C.

## Investigation Questions

9. To raise money for a school trip, Rhonda and her friends sold chocolate bars. The number of chocolate bars and the amount of money earned for the trip are shown in the graph below.



- Write an equation to describe the points on the line.
  - Write an inequality to describe the points above the line. What do these points represent in terms of chocolate bars sold and money earned?
  - Write an inequality to describe the points below the line. What do these points represent?
10. It takes Heather two hours to cut the fabric for a couch. She graphed the time that it would take to cut fabric for a couch and the number of couch bundles cut.



- Write an equation to describe the points on the line.
- Write an inequality to describe the points above the line. What do these points represent in terms of time spent and fabric cut?



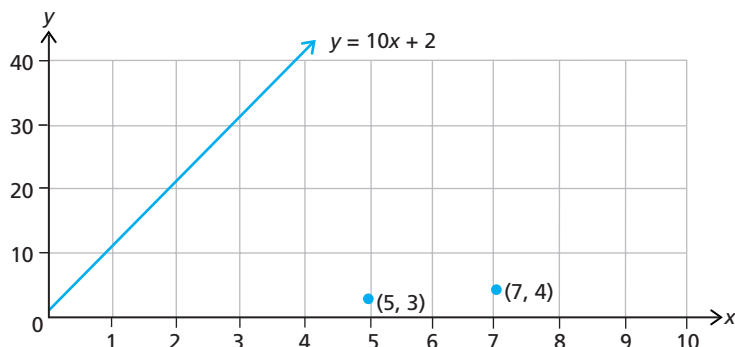
- (c) Write an inequality to describe the points below the line. What do these points represent?



## A Further Look at Inequalities

You can show that  $(5, 3)$  and  $(7, 4)$  are solutions to  $y \leq 10x + 2$  by substituting the values into the inequality. They will make the inequality true.

$$\begin{array}{ll} y \leq 10x + 2 & y \leq 10x + 2 \\ 3 \leq 10(5) + 2 & 4 \leq 10(7) + 2 \\ 3 \leq 52 & 4 \leq 72 \end{array}$$



### Think about...



$$y \leq 10x + 2$$

In Step B, did you list any points on the line shown by  $y = 10x + 2$ ? Should you have done so? Why?

### Focus Questions

11. Determine whether  $y \leq 10x + 2$  has any other solutions and what a graph of these solutions might look like.
- Graph the line  $y = 10x + 2$ .
  - Complete the table to find four points that are solutions to the inequality  $y \leq 10x + 2$ . Plot these points.

$x$	$y$	$y \leq 10x + 2$ (yes or no)
-1	-8	yes
2	22	yes

- Compare your points with the points of at least three other students in the class. Mark their points on your graph. What do you notice about the points?

### Think about...



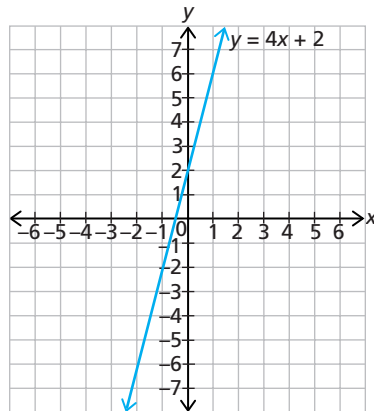
#### Questions 11 (c)

Where will all of the points that satisfy  $y \leq 10x + 2$  be found?

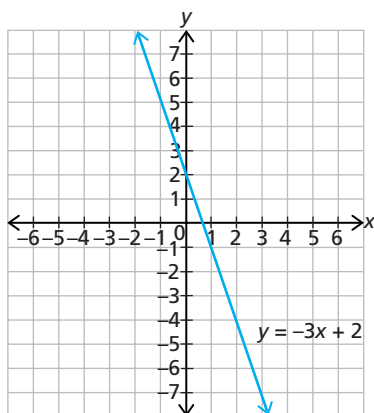
12. Do you think that  $(15, 20)$  is a solution to the inequality  $y \leq 10x + 2$ ?  
Give reasons for your answer.
13. Do you think that  $(2, 22)$  is a solution to the inequality  $y \leq 10x + 2$ ?  
Give reasons for your answer.
14. Do you think that  $(1, 25)$  is a solution to the inequality  $y \leq 10x + 2$ ?  
Give reasons for your answer.
15. Graph the line  $y = 4x - 7$ .
- Find four points that are solutions to the inequality  $y \geq 4x - 7$ .
  - Compare your points with the points of at least three other students in the class. Mark the points that are different from yours on your graph. What do you notice about the points?
  - Where will all of the points that satisfy  $y \leq 4x - 7$  be found?

## Check Your Understanding

16. Place a  $<$  or  $>$  sign between each pair of numbers to make a true statement. Write the statement in words.  
Example: Given 2, 5  
Statement:  $2 < 5$ , two is less than five
- (a) 7, 4                      (b)  $-5, -3$                       (c)  $-2, 4$                       (d)  $\frac{1}{2}, \frac{2}{3}$
17. (a) If  $x \leq 2$ , state some possible values of  $x$ . Explain.  
(b) If  $y \geq 4$ , state some possible values of  $y$ . Explain.
18. Using the graph  $y = 4x + 2$ , find the coordinates of three points that are solutions to  $y \leq 4x + 2$ .

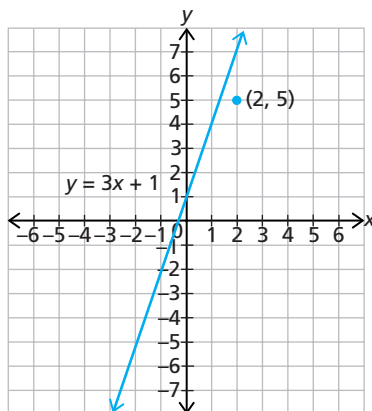


19. Using the graph  $y = -3x + 2$ , find the coordinates of three points that are solutions to  $y \geq -3x + 2$ .

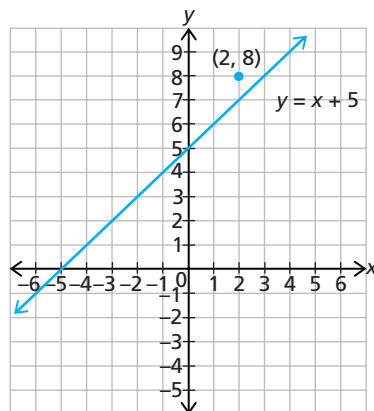


20. A point is marked on each graph. Write an inequality related to the line given that the point would satisfy.

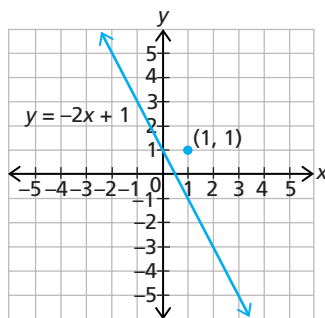
(a)



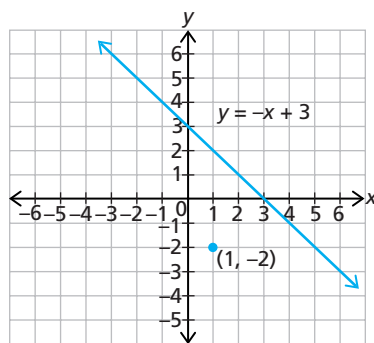
(b)



(c)

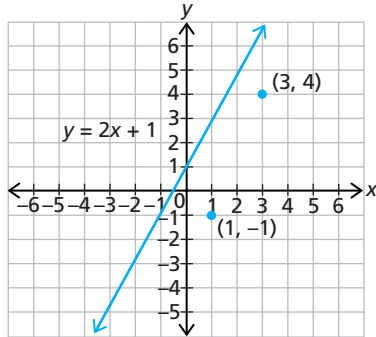


(d)



## Shading

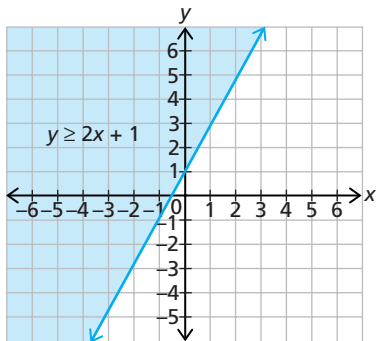
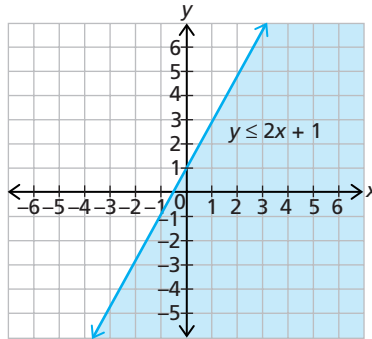
Points that are solutions to an inequality lie either above or below the graph of the equation or on the line.



$$y \leq 2x + 1$$

Solutions are points on or below the line, such as (1, -1) and (3, 4).

To show all possible solutions, the graph can be shaded below the line. All possible solutions will be in the shaded region.



Similarly, solutions to  $y \geq 2x + 1$  can be shown by shading above the line. Because the line is solid, points on the line are also included.

## Check Your Understanding

21. (a) Make a table of values and plot the graph of  $y = 4x - 3$ .
- (b) Locate a point on the graph that is a solution to  $y \geq 4x - 3$ . Explain how you chose the point.
- (c) Shade the region of the graph that represents all possible solutions to  $y \geq 4x - 3$ .



22. John claimed that  $(1, 8)$  is a possible solution to  $y \leq 2x + 3$ . Is John correct? Explain.
23. Plot the graph of the boundary line for each inequality and shade the graph to represent the solution.
- (a)  $y \leq x + 2$                       (b)  $y \geq 3x - 2$                       (c)  $y \leq -2x + 4$
24. Wendy's Internet bill is the lowest of all her friends. Her company charges a monthly fee of \$10 plus \$2 per hour of use.
- (a) Plot the graph that represents Wendy's Internet charges.  
 (b) Give examples of amounts that her friends might pay.  
 (c) Draw the graph to represent all of the possible amounts that her friends might pay.
25. One of Colin's friends is hired to work with him to assemble bicycles. The owner of the store tells her that, since she has less experience than Colin, her earnings will be less than or equal to Colin's earnings.
- (a) Colin's income is represented by the equation  $I = 25 + 6b$ . State an inequality that represents his friend's income.  
 (b) Plot a graph that represents his friend's income.



## Think About Inequalities

Heather worked with her brother Colin to investigate inequalities.

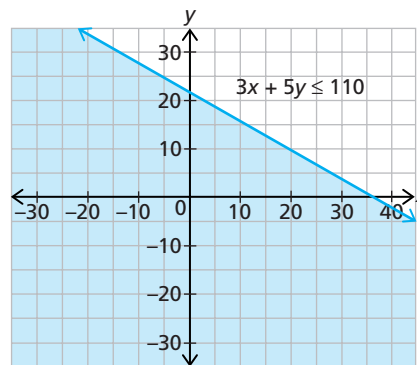
At one point, Heather turned to Colin and said, "You know, the inequality  $y \leq 10x + 2$  was given to us. What if we had to graph an inequality in a different form? For example, what if we had to graph  $3x + 5y \leq 110$ ? We would have to rearrange the inequality. How would we do that?"

### Example

What inequality would Heather and Colin enter into their graphing calculator to graph  $3x + 5y \leq 110$ ?

### Solution

$$\begin{aligned}
 3x + 5y &\leq 110 \\
 3x - 3x + 5y &\leq 110 - 3x \\
 5y &\leq 110 - 3x \\
 \frac{5y}{5} &\leq \frac{110 - 3x}{5} \\
 y &\leq \frac{110}{5} - \frac{3x}{5} \\
 y &\leq 22 - 0.6x
 \end{aligned}$$



### -Note-

Notice how the slope of the graph can be written as a fraction or as a decimal number. Which do you find easier to relate to in the context of the graph?

## Check Your Understanding

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26. Write each inequality in the form  $y \leq mx + b$ .

(a)  $6x + 3y \leq 30$

(b)  $15x + 5y \leq 25$

27. Write each inequality in the form  $y \geq mx + b$ .

(a)  $4x + 2y \geq 7$

(b)  $12x + 4y \geq 20$

28. Graph each inequality.

(a)  $y \leq 2x - 5$

(b)  $y \geq 3x - 4$

(c)  $y \leq -5x + 2$

(d)  $y \geq -2x + 6$

29. Graph each inequality.

(a)  $2x + y \geq 6$

(b)  $3x + 6y \geq 9$

(c)  $10x + 5y \geq 8$

(d)  $x + 2y \geq 4$

30. Mark walks or jogs a minimum of 20 km per week. He usually walks at a speed of 5 km/h and jogs at a speed of 10 km/h.

(a) Write an inequality to describe this relationship.

(b) Graph this relationship.



# 1.2

# Exploring Possible Solutions

**constraint**—any condition that must be met

*-Note-*  
You will need to use your graph from Part 1 as you complete Part 2.

## Reflections

Remember Heather's original problem. Heather cuts fabric for couches and chairs at Spinney Manufacturing. She is paid a flat fee for each bundle of fabric she cuts. A bundle includes all of the pieces that make a chair or couch.

- Heather is paid \$12.00 for each couch bundle.
- She is paid \$4.25 for each chair bundle.
- If she completes only part of a bundle, she is paid for only that part. For example, she is paid \$1.70 if she cuts 40% of a chair bundle.

Heather is paid every two weeks.

## Investigation 3

### Part 1: Look at the Constraints on Chairs

Heather's manager at Spinney Manufacturing gave her the following **constraint**:

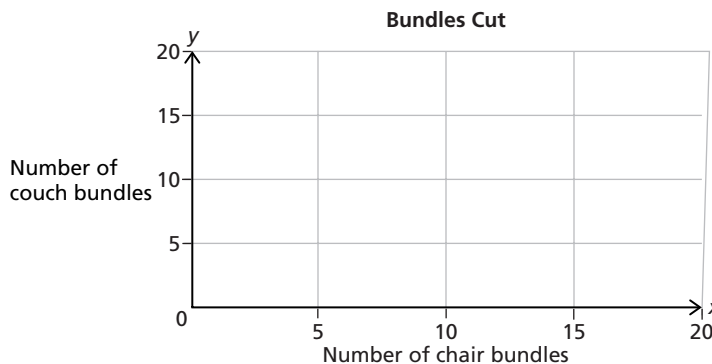
- She must cut at least eight chair bundles every two weeks.

### Purpose

Represent Heather's new constraint visually.

### Procedure

- A. Copy this coordinate grid onto graph paper.



- B. Heather has to cut at least eight chair bundles every two weeks.
- Rewrite this constraint in a sentence using the language of inequalities, such as "greater than," "less than," and so on. Explain.
  - Write an inequality to represent the constraint. Use  $x$  to represent the number of chair bundles cut.
  - Identify at least ten points that satisfy your inequality. Organize the values in a table. Explain how you chose these  $y$ -values.
  - Plot these points on your coordinate grid.
  - Add points from three other students to your table and graph.
  - Describe the region of the graph where the points are found. Where is the "boundary" to the region? Describe how you found it. Write an inequality to represent this region.

## Investigation 3

### Part 2: Look at the Constraints on Couches

Heather's manager also mentioned the following constraint:

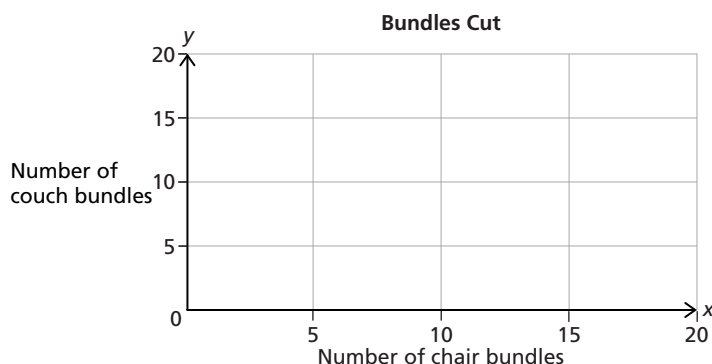
- At least ten couch bundles must be cut every two weeks.

#### Purpose

Represent this new constraint and the constraint from Investigation 3, Part 1 visually.

#### Procedure

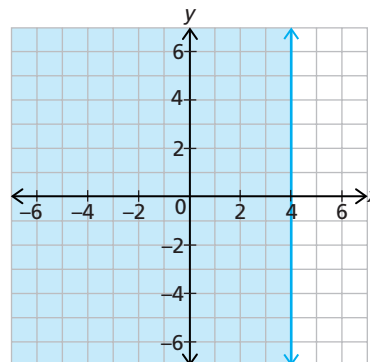
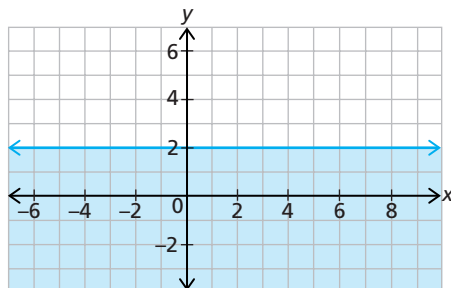
- A. Copy this coordinate grid on graph paper.



- B. Heather has to cut at least ten couch bundles.
- Rewrite this constraint using the language of inequalities.
  - Write an inequality to represent this constraint, using  $y$  to represent the number of couch bundles cut.
  - Identify at least ten points that satisfy your inequality.
  - Plot these points on the coordinate grid from Step A.
  - Add points from three other students to your table and graph.
  - Describe the region of the graph where the points are found. Where is the "boundary" to the region? Describe how you found it. Write an inequality to represent this region.
- C. Refer to the two graphs you now have from Parts 1 and 2. Place one graph on top of the other.
- Where is the common region of the two graphs? What does the overlap of the two regions represent?
  - Draw a new graph of the common region.

## Check Your Understanding

1. (a) State the inequalities represented by the shaded regions on the following graphs.



- (b) Create the graph that represents both inequalities on the same set of axes.
- (c) Suppose these were the graphs of Heather's constraints. What might the constraints imposed by the manager be?
2. Write an inequality to represent the information in each part below. Graph the inequality on a coordinate grid.
- Ostrich farming has become popular in Manitoba. One ostrich can lay up to 70 eggs in one lifetime.
  - For many farmers in the Tai Lake Valley of China, silk production provides the main source of income. To form a cocoon, each silk worm produces a single strand up to 1.5 km long in a lifetime. These threads are used to make silk fabric.
3. Write an inequality that represents each constraint, then graph the region.
- Heather must cut a minimum of six chair bundles and a minimum of nine couch bundles.
  - Sam works at a clothing store. The manager tells him that he gets a bonus if he sells at least 10 pairs of jeans and at least 15 T-shirts.
  - According to Canada's food guide, teenagers should eat between five and ten servings of vegetables and fruit each day.



4. Jorge must write a term paper on the writings of Canadian author Margaret Laurence. His bibliography must include at least ten sources. At least three sources must be original works by Margaret Laurence and the rest can be secondary sources.
- Write inequalities to represent the number of primary sources,  $p$ , and the number of secondary sources,  $s$ , that Jorge must use.
  - Graph the inequalities on the same axes.
  - Could Jorge have used 15 sources in his bibliography? Why?
5. The accounting department at Spinney Manufacturing spends a maximum of 24 h a week sending bills and accepting payments. It takes 5 min to prepare a bill and 8 min to process a payment.
- Represent these constraints with an inequality.
  - Graph the region.
  - Is it possible that the accounting department could process 200 bills in one week? Give reasons for your answer.

*– Note –*  
A secondary source is information written about a subject that is not original data; for example, a newspaper article about the federal budget rather than a copy of the budget.

*– Note –*  
In Question 5, all times must have the same units. To convert hours to minutes, multiply by 60.



## Groups of Inequalities

Because backpackers need large amounts of energy and must carry their food, they must plan their meals carefully. Sadie carries a mixture of peanuts and chocolate chips as she backpacks through the Gros Morne Trail in Newfoundland and Labrador. The mixture provides quick energy from carbohydrates.

### Example

Sadie wants the mixture to contain at least 150 g of carbohydrates and have a mass of less than 500 g. One kilogram of chocolate chips contains 737 g of carbohydrates. One kilogram of peanuts contains 275 g of carbohydrates. Write and graph two inequalities to show how much of each ingredient she should use.

### Solution

Let  $p$  represent the number of grams of peanuts.  
Let  $c$  represent the number of grams of chocolate chips.

A. You know the following:

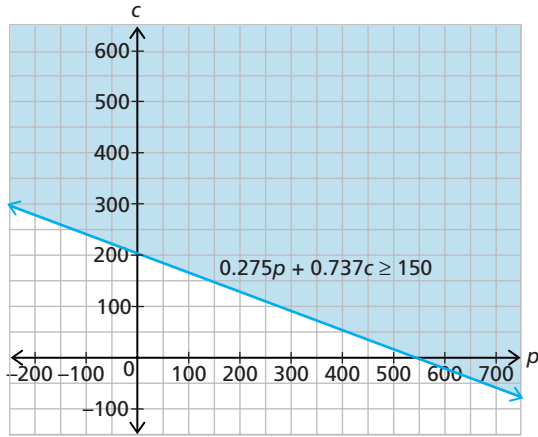
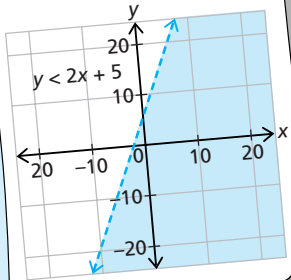
- The carbohydrate content of peanuts is 275 g/kg times  $0.001p$ .
- The carbohydrate content of chocolate chips is 737 g/kg times  $0.001c$ .
- The total mass of carbohydrates in the mixture will be greater than or equal to 150 g.

This means that  $0.275p + 0.737c \geq 150$ . This is graphed on the next page.

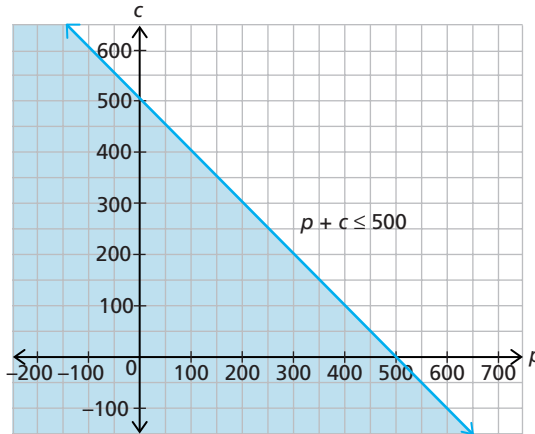


*- Note -*

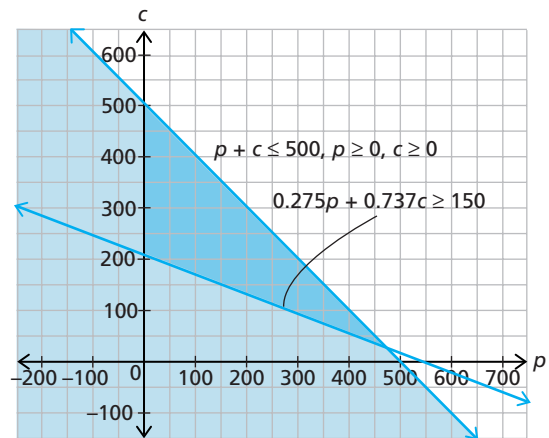
When graphing inequalities, mathematicians use a dotted line to represent " $<$ " or " $>$ ". For example, the graph of  $y < 2x + 5$  is shown below. The dotted line means that points on the line are not included in the region as part of the solution.



- B. Since the total mass is less than or equal to 500 g,  $p + c \leq 500$ . This is graphed below.



- C. Since both of these constraints happen at the same time, the graphs are placed on top of each other to show the overlap region. The overlap region shows the points that satisfy both inequalities. This is called the **feasible region** because this area represents all possible solutions.



**Did You Know?**

Backpackers often eat high-fat diets when hiking because one gram of fat provides more than twice as much energy as one gram of carbohydrates.

**feasible region**—a shaded region on a graph indicating that all points within the region are possible solutions to the problem

- D. Some of the masses for peanuts and chocolate chips that Sadie could pack are (100, 300), (150, 200), (200, 300), and (300, 100). Any combination that sits in the shaded region will be a solution to the problem.

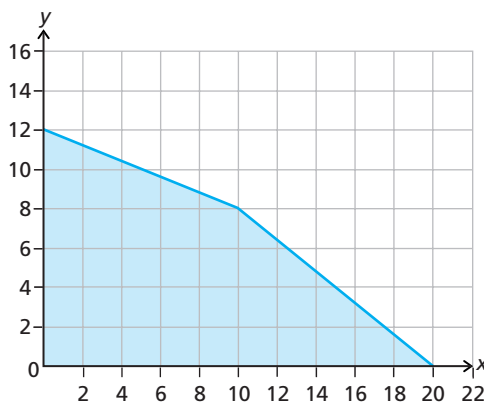
### Focus Question

6. Gina and Eric have been asked to buy peanuts and cashews for the party after the Lomond Trail hike. They will need a maximum of 10 kg of nuts and they have a maximum of \$50 to spend. Peanuts cost \$4/kg and cashews cost \$10/kg. Identify the feasible region and list possible solutions.

## Check Your Understanding

7. Sketch the feasible region that corresponds to each set of inequalities.
- $y \leq 6$  and  $y \geq x - 2$
  - $y \leq 9 - x$  and  $y \geq -x + 3$
  - $3x + y \geq -2$  and  $x + y \leq 6$
  - $3y - 2x + 12 \geq 0$  and  $3y + x \geq 0$  and  $x \geq 0$  and  $y \geq 0$
  - $y < 4 - 3x$  and  $y > 1.5 - 0.5x$
  - $3x + 2y < 6$  and  $x + 2y < 3$  and  $x \geq 0$  and  $y \geq 0$
  - $x + y \leq 4$  and  $x + 2y \leq 6$  and  $x \geq 0$  and  $y \geq 0$

8. (a) How many inequalities would you need to define the feasible region shown on the graph?
- (b) Write the inequalities that define this region.
- (c) Write a problem of your own for which the solution could be the graph of the feasible region shown.



*- Note -*  
 In Constructing Mathematics, Book 1, you discovered that  $y = mx + b$  is the equation of a line, where  $m$  is the slope of the line and  $b$  is its  $y$ -intercept (the point where the line crosses the  $y$ -axis).



## Think about...

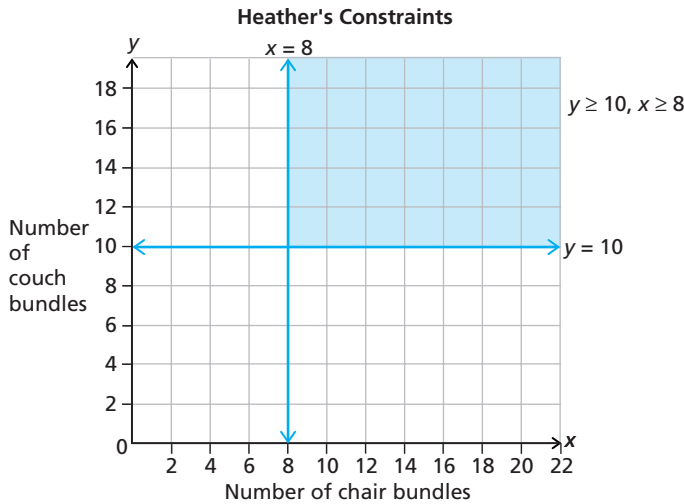


### Question 10 (d)

Compare your answers with others in your class. What do you notice?

9. Round the Bend is a 2000-ha farm. Sue can plant no more than 1500 ha of lettuce and no more than 1000 ha of corn.
- List Sue's constraints.
  - How many hectares of each crop might she plant? Compare your answers with those of other students. Why is it likely that your answers are not the same?
  - Give a combination of lettuce and corn that is not possible. Explain why the combination is not possible.
  - What piece of additional information about the farm might help everyone in your class to get the same answer? Why does this piece of information help?
10. The Sunny Garden Gift Shop is going to mail holiday gift packs of jam and marmalade.
- They have 50 jars of marmalade and 90 jars of jam.
  - The Morning Glory package contains four jars of marmalade and one jar of jam.
  - The Berry Patch package contains four jars of jam and one jar of marmalade.
  - The company earns \$3 profit on the Morning Glory package.
  - The company earns \$4 profit on the Berry Patch package.
- If only Berry Patch packages are made, how many can be produced? How much profit would the company earn?
  - If only Morning Glory packages are made, how many can be produced? How much profit would the company earn?
  - Make a table to record several possible combinations for each type of package that can be made.
  - Find a combination of Morning Glory and Berry Patch packages that can be made to earn more than the amount in parts (a) or (b). Is all of the available jam and marmalade used?

11. When Heather started her job at Spinney Manufacturing, she was told that every two weeks she must cut at least ten couch bundles and at least eight chair bundles. The graph of these constraints is shown below.



She was also told that she could work a maximum of 36 hours in 2 weeks.

- It takes 45 min to cut a chair bundle and 2 h to cut a couch bundle. Express this time constraint as an inequality.
  - Graph  $y \geq 10$ ,  $x \geq 8$  (the constraints on the number of couch bundles and chair bundles that must be cut) and the time constraint on the same axis. Shade the new feasible region.
  - How does this new region compare to the original one? Explain.
12. There is one additional constraint placed on Heather. She is allowed to cut a maximum of 110 m of fabric in a two-week period. It takes 3 m for a chair bundle and 5 m for a couch bundle.
- Express this constraint as an inequality.
  - Add the graph of this inequality to your graph of the feasible region in Question 11.
  - What effect does this have on the feasible region? Explain why this happens.
  - State three possible combinations of couch bundles and chair bundles that Heather could cut in two weeks.

-Note-

It is important for you to complete Questions 11 and 12. They refer to Heather's problem introduced at the beginning of the chapter. Keep your answers from Questions 11 and 12 for use in the next section when you will find Heather's maximum income.

-Note-

For Question 11, all times must have the same units. To convert hours to minutes, multiply by 60.