Review of Essential Skills

Slope and Equation of a Line

Slope is a measure of the steepness of a line, expressed as the rise divided by the run between any two points on the line. Since the rise is the difference in corresponding y-values, rise = $y_2 - y_1$. Since the run is the difference in corresponding x-values, run = $x_2 - x_1$. Thus, $m = \frac{y_2 - y_1}{x_2 - x_1}$. The steeper the line is, the larger the magnitude of its slope number.

Example 1

Line 1 passes through $A(-2, 5)$ and $B(-1, 3)$. Line 2 passes through $C(4, -1)$ and $D(-6, 0)$. Which line is steeper?

Solution

line 1: $m_1 = \frac{3-5}{-1-(-2)} = \frac{-2}{1} = -2$

line 2: $m_2 = \frac{0-(-1)}{-6-4} = \frac{-1}{10}$

Since $|-2| > \left|\frac{-1}{10}\right|$, line 1 is steeper than line 2.

A line that rises to the right has a positive slope. A line that falls to the right has a negative slope. A horizontal line has a slope of 0. A vertical line has an undefined slope.

Example 2

Find an equation of the line that passes through $(-1, 3)$ with slope -4.

Solution

Substitute known values into the equation of a line: $m = -4$, $x_1 = -1$, and y_1.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 3 &= -4(x - (-1)) \\ y - 3 &= -4(x + 1) \\ y - 3 &= -4x - 4 \\ 4x + y - 3 + 4 &= 0 \\ 4x + y + 1 &= 0 \end{aligned}$$

The equation is written as the general form of the equation of a line $Ax + By + C = 0$.

Solve for y.

$$\begin{aligned} Ax + By + C &= 0 \\ By &= -Ax - C \\ y &= \frac{-A}{B}x - \frac{C}{B}. \end{aligned}$$

The **slope–y-intercept equation of a line** with slope m and y-intercept b is $y = mx + b$. Thus, $m = \frac{-A}{B}$ and $b = \frac{-C}{B}$.

Summary

- Slope $m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$
- Equations of straight lines:
  - point-slope equation of a line: $y - y_1 = m(x - x_1)$
  - general form of the equation of a line: $Ax + By + C = 0$
  - slope-intercept equation of a line: $y = mx + b$
- Horizontal lines:
  - slope $m = 0$
  - equation $y = b$ where $(0, b)$ is the point at which the line crosses the y-axis.
- Vertical lines
  - slope m is undefined
  - equation $x = a$ where $(a, 0)$ is the point at which the vertical line crosses the x-axis.
- Two lines L_1 and L_2 with slopes m_1 and m_2 are
  - parallel if and only if $m_1 = m_2$
  - perpendicular if and only if $m_1m_2 = -1$; that is, their slopes are negative reciprocals, $m_2 = \frac{-1}{m_1}$

Example 3

Line L_1 passes through (1, 2) and is parallel to L_2, which passes through (–4,1) and (2, –5). Find the equation of L_1.

Solution

The slope of L_2 is

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-5-1}{2-(-4)} \\ &= \frac{-6}{6} \\ &= -1 \end{aligned}$$

L_2 is parallel to L_1, so the slope of L_1 is –1.
Using $m = -1$ and point (1, 2).

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 2 &= -1(x - 1) \\ y - 2 &= -x + 1 \\ y &= -x + 3 \end{aligned}$$

Example 4

Line L_1 passes through (–5, 2) and is perpendicular to L_2, given by $3x - 4y + 2 = 0$. Find the equation of L_1.

Solution

Find the slope of L_2.

$$
\begin{aligned}
3x - 4y + 2 &= 0 \\
-4y &= -3x - 2 \\
y &= \frac{3}{4}x + \frac{1}{2}
\end{aligned}
$$

So, $m_2 = \frac{3}{4}$.

$L_1 \perp$ to L_2, so $m_1 = \frac{-1}{m_2} = \frac{\pm 1}{\frac{3}{4}} = -\frac{4}{3} = \frac{-4}{3}$. Therefore.

$$
\begin{aligned}
y - y_1 &= m(x - x_1) \\
y - 2 &= \frac{-4}{3}(x + 5) \\
3y - 6 &= -4x - 20 \\
4x + 3y - 6 + 20 &= 0 \\
4x + 3y + 14 &= 0
\end{aligned}
$$

Practice

1. Find the slope, if it exists, of the line that passes through each pair of points.

(a) (1, 1), (4, 9) **(b)** (–2, 3), (5, 7) **(c)** (1, –6), (–1, –4) (d) (–5, –3), (2, –10)

(e) (–3, 2), (6, 4) **(f)** (0, 8), (–3, 8) **(g)** (–4, –6), (–4, 0) (h) (5, 0), (–4, –1)

2. Sketch the graph of **(a)** $x = 3$ and **(b)** $y = -4$.

3. Find the slope and y-intercept, if they exist, of the line for each equation.

(a) $y = 3x + 1$ **(b)** $y = -2x + 5$ **(c)** $x + 2y = 0$

(d) $x - 4y - 3 = 0$ **(e)** $x = -3y + 7$ **(f)** $4x + 5y = 10$

4. Find the equation of the line

(a) through (–2, 1) with slope 6

(b) through (1, 6) with slope $\frac{2}{3}$

(c) through (–4, 0) and (3, 1)

(d) through (–1, –5) and (–2, 7)

(e) with slope 3, y-intercept –1

(f) with slope –9, x-intercept 4

(g) through (–8, 7), parallel to x-axis

(h) through (0, 6), parallel to y-axis

(i) through (0, 0), parallel to line $x = 5$

(j) with y-intercept 6, parallel to line $2x + 3y + 4 = 0$

(k) through (–1, –2) perpendicular to line $y = 4$

(l) through $\left(\frac{1}{4}, \frac{-2}{3}\right)$ perpendicular to line $4x - 2y + 1 = 0$

Simplifying Algebraic Expressions: Adding, Subtracting, and Multiplying Polynomials

Polynomials are algebraic expressions resulting from adding or subtracting terms such as $2x$, $-5y$, $4xy$, and $-8t^2$. A **monomial** has one term, a **binomial** has two terms, and a **trinomial** has three terms. To add or subtract polynomials, combine **like** terms**.** Use the **distributive property** to multiply a polynomial by a monomial.

Example 1

Simplify $(-4x^2 + 3x - 1) + (x^2 - 7x + 5)$.

Solution

$$\begin{aligned}(-4x^2 + 3x - 1) + (x^2 - 7x + 5) &= -4x^2 + 3x - 1 + x^2 - 7x + 5 \\ &= -3x^2 - 4x + 4\end{aligned}$$

The process of multiplying polynomials is called **expanding**. The **degree** of a polynomial is the highest exponent or sum of exponents.

Special Products

$(a + b)(a - b) = a^2 - b^2$
$(a + b)^2 = a^2 + 2ab + b^2$
$(a - b)^2 = a^2 - 2ab + b^2$

Exponent Laws

$a^m a^n = a^{m+n}$
$(am)^n = a^{mn}$
$(ab)^m = a^m b^m$

Example 2

Simplify $(3 - 2t)(t^2 + 4t - 1)$.

Solution

$$\begin{aligned}(3 - 2t)(t^2 + 4t - 1) &= 3t^2 + 12t - 3 - 2t^3 - 8t^2 + 2t \\ &= -2t^3 - 5t^2 + 14t - 3\end{aligned}$$

Practice

1. Simplify.

(a) $(n^2 - 2n + 3) + (4n^2 + 2n - 1)$
(b) $(2x^2 - 4x + 5) + (-x^2 - x + 3)$
(c) $(-3w^2 + w - 7) + (2w^2 - 3w + 6)$
(d) $(x^3 - 4x^2 + 2x - 1) - (3x^3 + 3x^2 - 5x + 8)$

2. Simplify. State the degree of each simplified polynomial.

(a) $2(3t+1) - 5(t - 3)$
(b) $-2x(4 - 3x) + x(2x + 5)$
(c) $2x(x^2+3) - x(5 - 3x)$
(d) $(3 - 10b + 4b^2) - 2(3b^2 - b)$
(e) $m(3m^2-9m+7) + 2m[m + 4m(1 - m)]$
(f) $-x[5x - x(1 - 2x)] - 2x[x - 4x(x + 1)]$

3. Simplify.

(a) $(k + 4)(k - 5)$
(b) $4(2x + 5y)(x - 3y)$
(c) $(c + 3)(c - 3)$
(d) $(x + 2)(x - 2) - 3(x + 4)(x - 1)$
(e) $(x + 3y + 2z)(3x + 2y + z)$
(f) $(h - 7)^2$

Solving Linear and Quadratic Equations

Linear equations are of the form $ax + b = 0$, $a \neq 0$. When solving linear equations, remember to move all variables to one side of the equation and constants to the other.

Example 1

Solve $2(4x - 1) = 10(x + 2)$ for x.

Solution

$$\begin{aligned} 2(4x - 1) &= 10(x + 2) \\ 8x - 2 &= 10x + 20 \\ 8x - 10x &= 20 + 2 \\ -2x &= 22 \\ x &= -11 \end{aligned}$$

Quadratic equations are of the form $ax^2 + bx + c = 0$, $a \neq 0$. To solve a quadratic equation either factor it or use the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The term $b^2 - 4ac$ is the **discriminant**. If the discriminant is positive, there are two different real roots. If it is equal to 0, there are two equal real roots. If it is negative, there are two complex conjugate roots.

Example 2

(a) Solve $18x^2 - 32 = 0$ for x.

(b) Solve $5x^2 + 3x - 3 = 0$ for x.

Solution

(a)

$$\begin{aligned} 18x^2 - 32 &= 0 \\ 2(9x^2 - 16) &= 0 \\ 2(3x - 4)(3x + 4) &= 0 \\ 3x - 4 &= 0 \text{ or } 3x + 4 = 0 \\ 3x &= 4 \text{ or } 3x = -4 \\ x &= \frac{4}{3} \text{ or } -\frac{4}{3} \end{aligned}$$

(b) Use the quadratic formula.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-3 \pm \sqrt{3^2 - 4(5)(-3)}}{2(5)} \\ &= \frac{-3 \pm \sqrt{9 + 60}}{10} \\ &= \frac{-3 \pm \sqrt{69}}{10} \end{aligned}$$

Practice

1. Solve.

(a) $-7(x + 3) = 4(x + 1)$ **(b)** $3(a + 2) - 8(a - 3) = 0$ **(c)** $5(3t + 14) - 2(t - 15) = 9(2t + 5)$

2. Solve and check.

(a) $x^2 - 14x + 33 = 0$ **(b)** $q^2 - 2q - 15 = 0$ **(c)** $2m^2 + 11m + 12 = 0$

3. Find the discriminant of each equation and determine the nature of the roots.

(a) $2x^2 - 7x + 15 = 0$ **(b)** $x^2 + 2x - 5 = 0$ **(c)** $16x^2 + 8x + 1 = 0$

4. Find the roots.

(a) $x^2 + 2x - 2 = 0$ **(b)** $-3x^2 - 4x + 5 = 0$ **(c)** $4x^2 + 20x + 21 = 0$

Graphing Linear and Quadratic Functions

Graphing Linear Functions

There are four ways to graph a linear function such as $y = 2x - 1$.

(a) Use a table.

x	y
–2	–5
–1	–3
0	–1
1	1
2	3

(b) Find two points on the line.

Let $x = 1$.

$y = 2(1) - 1$

$y = 2 - 1$

$y = 1$

One point is (1, 1).

Let $x = -3$.

$y = 2(-3) - 1$

$y = -6 - 1$

$y = -7$

One point is (–3, –7).

(c) Find the *x*- and *y*-intercepts.

For *x*-intercept, $y = 0$.

$0 = 2x - 1$

$2x = 1$

$x = \frac{1}{2}$

One point is $\left(\frac{1}{2}, 0\right)$.

For *y*-intercept, $x = 0$.

$y = 2(0) - 1$

$y = -1$

One point is (0, –1).

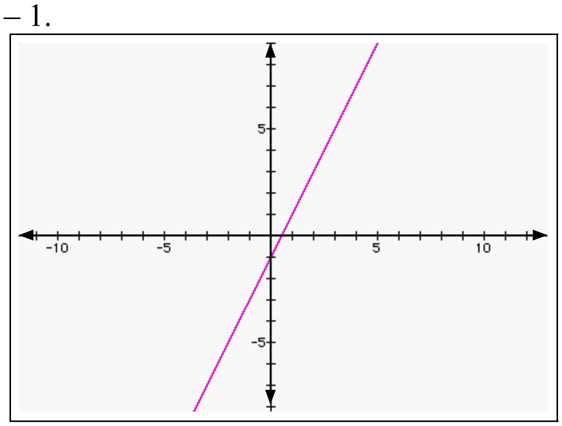
(d) Use the slope–*y*-intercept form $y = mx + b$.

Compare $y = 2x - 1$ with $y = mx + b$.

$m = 2$ (rise = 2, run = 1)
$b = -1$
Plot (0, –1) and use a slope of 2 to graph the line.

Each method gives the same graph. You can graph a linear relation when it is in the general form $Ax + By + C = 0$, but it is usually helpful to rearrange it into $y = mx + b$ form.

Sketch the graph of $y = 2x - 1$.



Graphing Quadratic Functions

The graph of a quadratic function is a **parabola.** There are three main forms for the equation of a quadratic function.

- In **standard** form, $f(x) = ax^2 + bx + c$, the y-intercept coordinate, $(0, c)$ is clearly visible.
- In **factored** form, $f(x) = a(x - p)(x - q)$, the x-intercept coordinates, $(p, 0)$ and $(q, 0)$ are clearly visible.
- In **vertex** form, $f(x) = a(x - h)^2 + k$, the coordinates of the vertex (h, k) are clearly visible. The maximum or minimum value of the function is k.

Complete the square to write the equation of a quadratic function in vertex form.

$$\begin{aligned} f(x) &= ax^2 + bx + c \\ &= a\left(x^2 + \frac{b}{c}x\right) + c \\ &= a\left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right] + c \\ &= a\left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right] - a\left(\frac{b}{2a}\right)^2 + c \\ &= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c \end{aligned}$$

Therefore, the coordinates of the vertex are $\left(\frac{-b}{2a}, \frac{-b^2}{4a} + c\right)$. A quadratic function in vertex form, $f(x) = a(x - h)^2 + k$, has two zeros if a and k have opposite signs; one zero if $k = 0$; and no zeros if a and k have the same sign.

Example 1

Graph $y = -3x^2 - 2x + 7$.

Solution

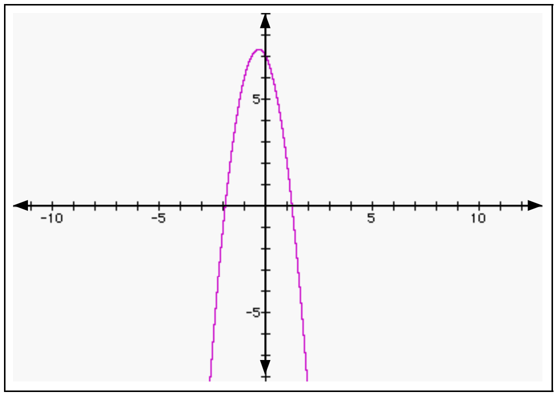
Factor partially.

$$y = -3x^2 - 2x + 7$$
$$y = x(-3x - 2) + 7$$

$a = -3$, so the parabola opens downward.

Let $x = 0$ or $-3x - 2 = 0$ to find two points on the curve. When $x = 0$, then $y = 7$. When $3x - 2 = 0$, then $x = \frac{-2}{3}$ and $y = 7$. The axis of symmetry is halfway between $(0, 7)$ and $\left(\frac{-2}{3}, 7\right)$. So, $x = \dfrac{0 + \left(\frac{-2}{3}\right)}{2} = \frac{-1}{3}$. To find the vertex, substitute $x = \frac{-1}{3}$ into $y = -3x^2 - 2x + 7$. Then $y = -3\left(\frac{-1}{3}\right)^2 - 2\left(\frac{-1}{3}\right) + 7 = 7\frac{1}{3}$. The curve opens downward so the vertex occurs at $\left(\frac{-1}{3}, 7\frac{1}{3}\right)$, which is a maximum point.

Sketch the graph of $y = -3x^2 - 2x + 7$.



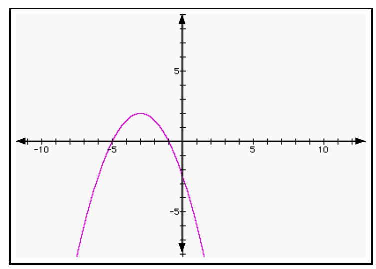
Example 2

Graph $y = \frac{-1}{2}(x+3)^2 + 2$.

Solution

The coordinates of the vertex are (–3, 2). Determine one point on the curve and use symmetry to find a second point. When $x = 0$, $y = \frac{-1}{2}(0+3)^2 + 2 = -2\frac{1}{2}$. So $\left(0, \pm 2\frac{1}{2}\right)$ is a point on the curve. Since $x + 3 = 0$ is the axis of symmetry, $\left(-6, \pm 2\frac{1}{2}\right)$ is also on the parabola. The vertex (–3, 2) is a maximum point.

Sketch the graph of $y = \frac{-1}{2}(x+3)^2 + 2$.



Practice

1. Graph each function.

   (a) $y = 3x$ **(b)** $y = 4x + 3$ **(c)** $y = -x - 2$ **(d)** $y = x^2$

   (e) $x + y = 2$ **(f)** $3x + 4y - 5 = 0$ **(g)** $y = -x^2 - x + 30$ **(h)** $y = -3x^2 + 9x - 2$

2. Sketch the graph of each parabola. Show the coordinates of the vertex; the equation of the axis of symmetry; the coordinates of two points on the graph; and x- and y-intercepts, if any.

   (a) $y = (x - 2)^2 + 3$ **(b)** $y = 2(x + 1)^2 + 4$ **(c)** $y = -3(x + 5)^2 - 1$

   (d) $k = -2(l - 4)^2 + 2$ **(e)** $v = \frac{1}{2}(u - 6)^2 - 10$ **(f)** $m = -(t + 3)^2 + 1.5$

3. Write the equation of the parabola with vertex (–1, 5) that opens up and is congruent to $y = 2x^2$.

4. A theatre owner charges \$5 per ticket and sells 250 tickets. After conducting a survey, the owner notices that for every one dollar she raises the ticket price, she will lose 10 customers. If she raises the price by x dollars, explain how she determines the revenue function $R(x) = (5 + x)(250 - 10x)$. Use a sketch of the graph of the revenue function to determine how much she should charge to maximize revenue.

Factoring Common Trinomials, Perfect Squares, and Differences of Squares

The opposite of expanding is **factoring,** which is the process of changing a sum or difference of terms into a product. For example, $2x(x - 1) = 2x^2 - 2x$ is expanding, whereas $2x^2 - 2x = 2x(x - 1)$ is factoring. Six different methods of factoring are shown. **ALWAYS** look for a common factor first!

Common Factoring

Remove the greatest common factor from the polynomial.

Example: $4x^2 - 12x = 4x(x - 3)$ since the g.c.f is $4x$.

Factoring Trinomials of the Form $x^2 + bx + c$

Look for two numbers whose product is c and whose sum is b.

Example: $t^2 + 2t - 3 = (t + 3)(t - 1)$

Factoring Trinomials of the Form $ax^2 + bx + c$

Look for two numbers whose product is ac and whose sum is b; also called **factoring by decomposition**.

Example:
$$\begin{aligned} 2a^4 + 5a^2b + 3b^2 &= (2a^4 + 2a^2b) + (3a^2b + 3b^2) \\ &= 2a^2(a^2 + b) + 3b(a^2 + b) \\ &= (a^2 + b)(2a^2 + 3b) \end{aligned}$$

Factoring by Grouping

Look for groups of similar terms; rearrange if necessary.

Example:
$$\begin{aligned} 6cd - 5 - 10d + 3c &= (6cd + 3c) - (5 + 10d) \\ &= 3c(2d + 1) - 5(1 + 2d) \\ &= 3c(2d + 1) - 5(2d + 1) \\ &= (2d + 1)(3c - 5) \end{aligned}$$

Factoring Perfect Square Trinomials

Look for a trinomial whose quadratic term (first) and constant (last) term are perfect squares.

Example: $9a^2 - 24a + 16 = (3a - 4)^2$

Factoring a Difference of Squares

Look for two perfect squares with opposite signs and use the general rule:
$a^2 - b^2 = (a - b)(a + b)$.

Example: $9x^2 - 4y^2 = (3x - 2y)(3x + 2y)$

Practice

1. Factor.

(a) $3x - 9xy$ **(b)** $-4x^2y - 12xy^2$ **(c)** $18a^3b + 27a^3b^2 - 3ab$

2. Factor.

(a) $x^2 - x - 6$ **(b)** $y^2 + 2y - 24$ **(c)** $b^2 + 7b - 44$ **(d)** $r^2 - 5r - 36$

3. Factor.

(a) $3s^2 - 3s - 18$

(b) $2x^2 + 7x + 5$

(c) $6t^2 - 7t - 5$

(d) $8 - 14x + 6x^2$

(e) $-10x^4 - 22x^2 - 4$

4. Factor.

(a) $4a + 3b + 6 + 2ab$ **(b)** $12x^2y - 20ax - 9axy + 15a^2$

(c) $2y^3 + 5y - 4y^2 - 10$ **(d)** $-10r - 9q^2 + 30q + 6qr - 25$

5. Factor.

(a) $25x^2 - 20x + 4$ **(b)** $9r^2 + 6r + 1$

(c) $x^2 - 2xy + y^2$ **(d)** $144t^2 + 72t + 9$

6. Factor

(a) $4a^2 - 1$ **(b)** $16p^2 - 25q^2$

(e) $z^4 - 256$ **(d)** $36x^2 - (2y + 7)^2$

7. Factor fully.

(a) $x^2 - 5x + 6$ **(b)** $3x(x - 2y) - y(x - 2y)$

(c) $-60 + 30a^2 - 5a$ **(d)** $m^2p^2 - 100a^2$

(e) $2t^3 + 5t - 4t^2 - 1$ **(f)** $18q^3 - 33q^2 + 9q$

(g) $\frac{x^2}{81} - \frac{y^2}{36}$ **(h)** $6m^2n^2 - 12mn^2 + 20n^2 - 10mn$

(i) $a^3b^2 + 2a^3b + b^3$ **(j)** $(x^2 - 2x)^2 - 9$

(k) $25(3x + 1)^2 - 16(2x + 1)^2$ **(l)** $6(x - 2)^2 + 17(x - 2) + 5$

(m) $x^2 - y^2 + z^2 - 2xy$ **(n)** $\frac{x^2}{3} + \frac{17x}{12} + \frac{5}{4}$

Exponent Rules

Positive Exponent Rule	**Zero Exponent Rule**
If $n > 0$, then $x^n = x \cdot x \cdot x \cdot x \cdot x \cdot \quad \cdot x \cdot x$. **Example:** $2^4 = 2 \times 2 \times 2 \times 4 = 16$	If $x \neq 0$, then $x^0 = 1$. 0^0 is undefined. **Examples:** $2^0 = 1$, $\pi^0 = 1$
Negative Exponent Rule If $n > 0$ and $x \neq 0$, then $x^{-n} = \frac{1}{x^n}$. **Example:** $4^{-2} = \frac{1}{4^2}$ $= \frac{1}{16}$	**Rational Exponent Rule** If m and n are integers, where $x \neq 0$ and $x^{1/n}$ exist, then: $x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m$ or $x^{\frac{m}{n}} = \left(x^m\right)^{\frac{1}{n}} = \sqrt[n]{x^m}$ **Examples:** $4^{\frac{3}{2}} = \left(\sqrt{4}\right)^3 = 2^3 = 8$ $(-8)^{\frac{2}{3}} = (64)^{\frac{1}{3}} = \sqrt[3]{64} = 4$

Exponent Properties

- $x^m \cdot x^n = x^{m+n}$ **Example:** $x^2 \cdot x^3 = x^5$
- $x^m \div x^n = x^{m-n}$ **Example:** $x^8 \div x^6 = x^{8-6} = x^2$
- $(x^m)^n = x^{mn}$ **Example:** $(x^4)^2 = x^8$
- $(xy)^m = x^m y^m$ **Example:** $(xy)^7 = x^7 y^7$
- $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$, $y \neq 0$ **Example:** $\left(\frac{x}{y}\right)^4 = \frac{x^4}{y^4}$

Practice

1. Simplify. Express answers without negative exponents.

(a) $x^2 x^7$ **(b)** $(y^3)^5$ **(c)** $(x + 2)^4(x + 2)$ **(d)** $\frac{a^9}{a^3}$ **(e)** $\frac{y^4}{y^7}$

(f) $\frac{(x^2 y)^2}{(xy^3)^4}$ **(g)** 8^0 **(h)** $(2^0 + 2)^0$ **(i)** 4^{-1} **(j)** -4^{-5}

(k) $\left(\frac{1}{10}\right)^{-1}$ **(l)** $\left(\frac{4}{3}\right)^{-2}$ **(m)** $(a^3 b c^0)^{-2}$ **(n)** $\left(\frac{x^2 y^{-5}}{x^{-2} y^{-3}}\right)^{-4}$ **(o)** $\frac{(y+1)^3 (y+2)^4}{(y+1)^5 (y+2)}$

Solving Exponential Equations with and without Graphing Technology

In an **exponential equation,** one or more exponent has a variable. For example, $3^{4x+2} = 27^{x-2}$ is an exponential equation.

- To solve an exponential equation approximately, graph the equation either by hand, a graphing calculator, or software with graphing capabilities, and then interpolate or extrapolate.
- If both sides of an exponential equation have the same base, it can be solved exactly with algebra, using the fact that if the bases are the same, the exponents are equal. That is, if $m^x = n^y$ and $m = n$, then $x = y$.
- Verify the solution to an exponential equation by substitution.

Example 1

Solve for x algebraically.

(a) $3^{4x} = 729$ **(b)** $5^{4x+1} = \frac{1}{125}$ **(c)** $8^x = 16\sqrt[3]{2}$ **(d)** $2^{x^2} = (16^{x-1})(2^x)$

Solution

(a)

$$\begin{aligned} 3^{4x} &= 729 \\ 3^{4x} &= 3^6 \\ 4x &= 6 \\ x &= \frac{6}{4} \\ x &= \frac{3}{2} \end{aligned}$$

(b)

$$\begin{aligned} 5^{4x+1} &= \frac{1}{125} \\ 5^{4x+1} &= 5^{-3} \\ 4x+1 &= -3 \\ 4x &= -3-1 \\ 4x &= -4 \\ x &= -1 \end{aligned}$$

(c)

$$\begin{aligned} 8^x &= 16\sqrt[3]{2} \\ (2^3)^x &= (2^4)(2^{\frac{1}{3}}) \\ 2^{3x} &= 2^{\frac{13}{3}} \\ 3x &= \frac{13}{3} \\ 9x &= 13 \\ x &= \frac{13}{9} \end{aligned}$$

(d)

$$\begin{aligned} 2^{x^2} &= (16^{x-1})(2^x) \\ 2^{x^2} &= (2^4)^{x-1}(2^x) \\ 2^{x^2} &= 2^{4x-4}2^x \\ 2^{x^2} &= 2^{5x-4} \\ x^2 &= 5x-4 \\ x^2 - 5x + 4 &= 0 \\ (x-4)(x-1) &= 0 \\ x &= 4 \text{ or } x = 1 \end{aligned}$$

To verify each solution, substitute it into the original equation to see if the left-hand side of the equation equals the right-hand side of the equation.

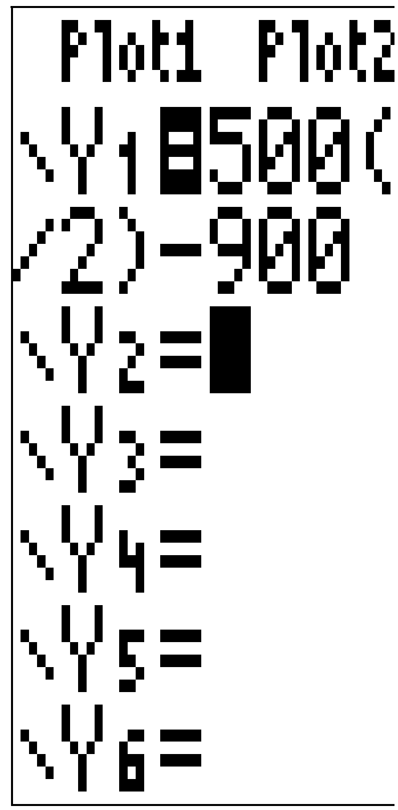
Example 2

June deposits \$500 in an account paying 8% per annum compounded semi-annually. How long will it take for the deposit to increase to \$900? Use a graphing calculator to determine the zeros.

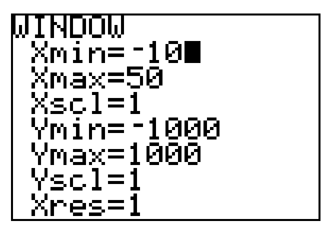
Solution

Find the zeros of the graph of $y = 500(1.08)^{\frac{n}{2}}$ to determine the solution. Using the TI-83 Plus, ensure the selected graphing mode (**MODE**) is **function**. Follow these steps:

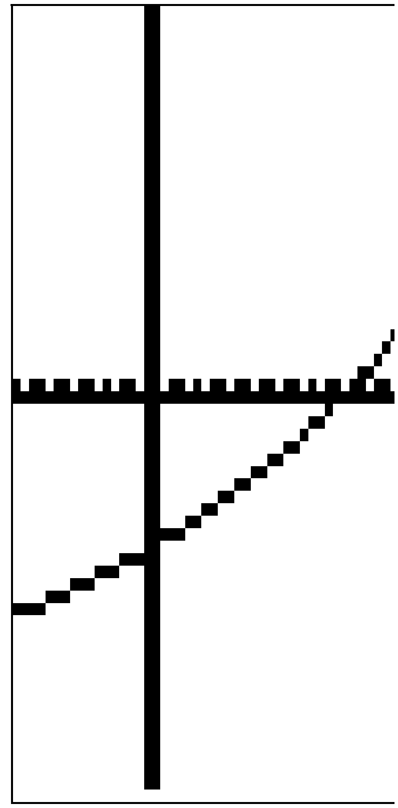
Step 1: Press Y= and enter the equation as shown.



Step 2: Press WINDOW and adjust the window settings as shown.



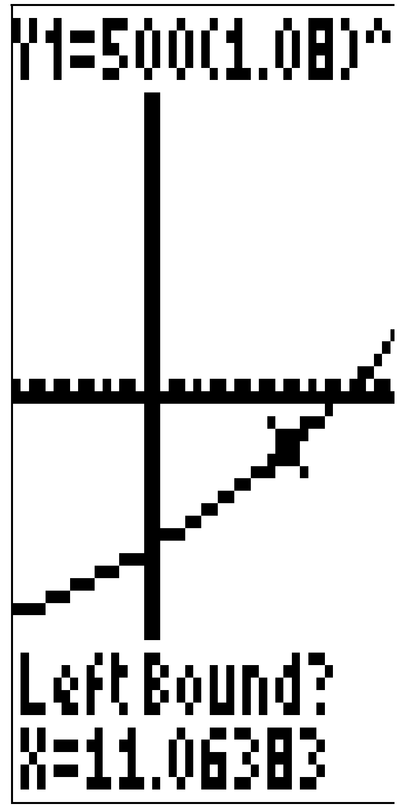
Step 3: Press GRAPH to draw the graph.



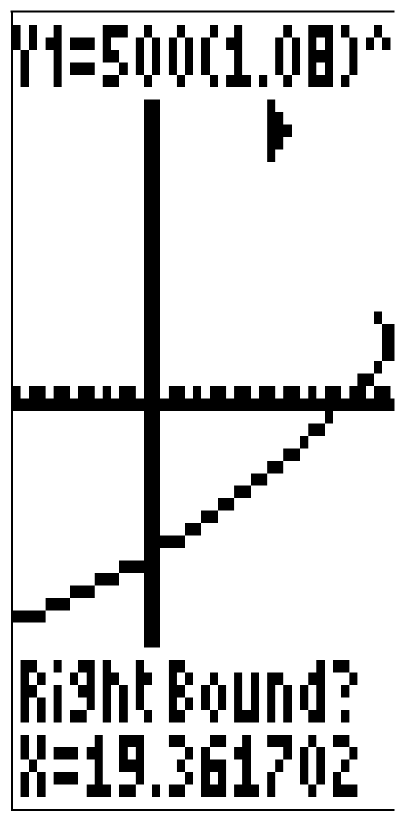
Step 4: Determine the zero using the zero function.

(a) Press 2nd, TRACE, 2, ENTER.

(b) To respond to “Left Bound?”, cursor to any point on the curve below the x-axis and press ENTER.

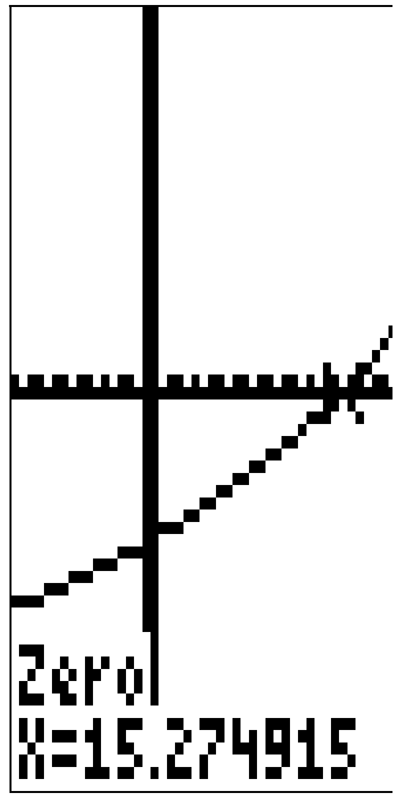


(c) To respond to “Right Bound?”, cursor to any point on the curve above the x-axis and press ENTER.



(d) Respond to "Guess?" by pressing Enter.

The calculator shows that the \$500 will take about 15.3 years to increase to \$900 in an account paying 8% per annum compounded semiannually.



Practice

1. Express each value as a power of 3.

(a) 27

(b) $\frac{1}{9}$

(c) 243

(d) $\left(\frac{1}{81}\right)^{2x}$

2. Solve for *x* algebraically.

(a) $5^x = 5^3$

(b) $2^7 = 2^x$

(c) $10^{-3} = 10^x$

(d) $3^{x+6} = 3^{-2}$

(e) $4^{2x} = 4^{16}$

(f) $6^{x-1} = 6^{3x+4}$

(g) $3^x = 9^{x-1}$

(h) $5^x = \frac{1}{125}$

(i) $6^x = \sqrt[3]{6}$

(j) $4(2^x) = 32$

(k) $2^{-x} = 32$

(l) $125^{x} = 25\sqrt{5}$

3. Determine approximate solutions using graphing technology. Round to two decimal places.

(a) $3^{x} = 30$

(b) $5.8^{x} = 70$

(c) $1.08^{x} = 3$

(d) $5^{2x-1} = 35$

(e) $9.1^{3x} = 23$

(f) $30^{-x} = 12$

4. Solve $2^{2x} - 33(2^{x}) + 32 = 0$.

5. Solve $(5^{x^2})(625) = (\frac{1}{125})^{2x}$ for x. Round to two decimal places.

Finite Differences

When values of *x* are spaced evenly in a table, the **first differences** are the differences between consecutive values of *y*. The **second differences** are differences between consecutive first differences, and so on.

For a linear function, such as $y = 5x - 3$, the first differences are constant.

x	$y = 5x - 3$	First difference
2	5	12 – 7 = 5
3	12	17 – 12 = 5
4	17	22 – 17 = 5
5	22	27 – 22 = 5
2	27	

For a quadratic function, such as $y = 2x^2$, the second differences are constant.

x	$y = 2x^2$	First difference	Second difference
0	0	2 – 0 = 2	6 – 2 = 4
1	2	8 – 2 = 6	10 – 6 = 4
2	8	18 – 8 = 10	14 – 10 = 4
3	18	32 – 18 = 14	
4	32		

For an exponential function, such as $y = 3^x$, the first differences form a **geometric sequence**.

x	$y = 3^x$	First difference
0	1	3 – 1 = 2
1	3	9 – 3 = 6
2	9	27 – 9 = 16
3	27	81 – 27 = 54
4	81	

The **rate of change** for a linear function is the slope of the line. It describes how quickly *y* changes with respect to *x*. For example, if a car travels 80 km in one hour, its rate of change (or speed) is 80 km/h. An example of a rate for a quadratic function is an apple falling to the ground with an acceleration of 10 m/s^2. The number of viruses doubling every hour describes the rate of change for an exponential function.

Practice

1. Create the difference table for each relation. Is the relation linear, quadratic, or exponential?

(a)

x	y
1	5
2	10
3	15
4	20
5	25

(b)

x	y
1	4
2	8
3	12
4	16
5	20

(c)

x	y
2	0
4	1
6	4
8	9
10	16

(d)

x	y
0	500
1	250
2	125
3	62.5
4	31.25

(e)

x	y
0	1
2	4
4	16
6	64
8	256

(f)

x	y
5	0
4	4
3	–6
2	5
1	–3

(g)

x	y
3	100
4	50
5	25
6	12.5
7	6.25

(h)

x	y
5	25
6	36
7	49
8	64
9	81

2. A bacteria culture grows in a laboratory dish. The table shows the number of bacteria in the dish at various times.

Time (h)	0	10	20	30	40
Number of Bacteria	8	64	512	4096	32768

(a) Is the relation linear, quadratic, or exponential? How can you tell?

(b) Graph the data and state the rate of change.

(c) How many bacteria are in the dish after 80 min?

(d) About when does the population reach one million?

3. The velocity of a sound wave depends on the temperature of the medium in which it travels. The table shows the speed of sound in air.

Temperature of Air(°C)	40	30	20	10	0	–10
Speed of Sound in Air (m/s)	356	350	344	338	332	326

(a) Is the relation linear, quadratic, or exponential? How can you tell?

(b) Graph the data and state the rate of change.

(c) What is the speed of sound in air when the air temperature is –25°C?

(d) At what temperature is the speed of sound 329 m/s?

4. Describe a realistic situation that shows

(a) a linear function **(b)** a quadratic function **(c)** an exponential function

5. Create a table with $-3 \le x \le 3$ for each function. Graph each function and describe any patterns.

(a) $y = x^2$ **(b)** $y = x^3$ **(c)** $y = \frac{1}{x}$ **(d)** $y = \sqrt{x}$

6. Graph $y = 4x$, $y = 4x^2$, and $y = 4^x$ on the same set of axes. Compare their rates of change.

Scatter Plots and Lines and Curves of Best Fit

In a **scatter plot**, two or more variables are plotted on a coordinate grid. A straight line that approximates a trend for the data in a scatter plot is called a **line of best fit**. For curves, it is called a **curve of best fit**.

Example 1: Line of Best Fit

An archeologist recovers eight male skeletons from an ancient battle site and records this data.

Length of tibia bone *l* (cm)	36	30	41	45	52	33	28	46
Height of male *H* (cm)	164.4	151	176.4	186.0	202.8	157.3	143.7	189.4

(a) Plot the points on a scatter plot and draw a line of best fit.

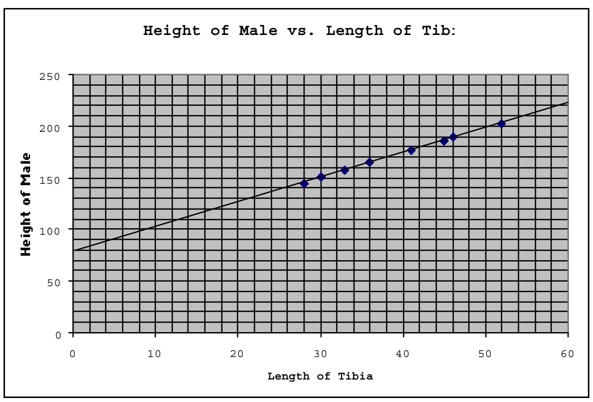
(b) What type of relation is shown?

(c) Choose two points on the line of best fit to find the slope of the line. What does this rate of change represent?

(d) The archaeologist finds the tibia of another male. Determine an equation the archaeologist could use to predict the male's height. That is, find the equation for a line of best fit.

Solution

(a)



(b) A linear relation is shown.

(c) Two points on the line are (32, 155) and (50, 198). The slope of the line of best fit is

$$m = \frac{198 - 155}{50 - 32}$$
$$= \frac{43}{18}$$
$$= 2.4$$

This means that 1 cm of tibia bone length corresponds to about 2.4 cm in the person's height.

(d) Using (32, 155) and $m = 2.4$, the equation of a line of best fit is:

$$H - H_1 = m(l - l_1)$$
$$H - 155 = 2.4(l - 32)$$
$$H - 155 = 2.4l - 76.8$$
$$H = 2.4l + 78.2$$

Example 2: Curve of Best Fit

A baseball player hits a homerun. The table gives the height of the ball above the ground, from the time it is hit until it touches the ground.

Time t (s)	0	1	2	3	4	5	6
Height, H (m)	2	27	42	48	43	29	5

(a) Plot the points on a scatter plot and draw a curve of best fit.

(b) What type of relation is shown?

(c) Use the approximate vertex (3, 48) and another point on the curve to find the equation of the curve of best fit.

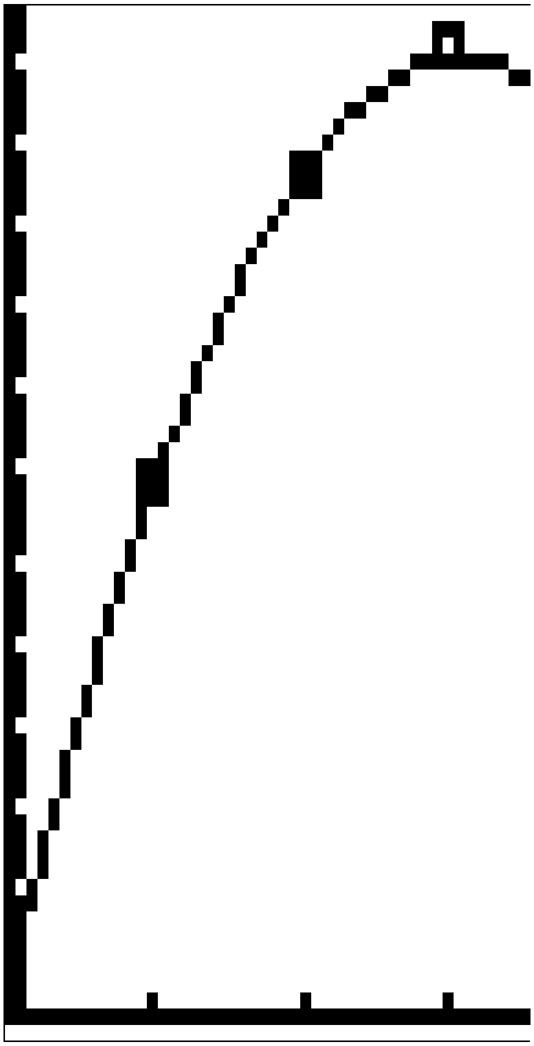
Solution

(a)

(b) A quadratic relation is shown.

(c) The vertex is (3, 48) and another point on the curve is (0, 2).

$$\begin{aligned} H &= a(t-h)^2 + k && \text{Substitute known values.} \\ 2 &= a(0-3)^2 + 48 \\ 2 &= 9a + 48 \\ 9a &= 2 - 48 \\ 9a &= -46 \\ a &= -5.1 \end{aligned}$$



The equation of the curve of best fit is $H = -5.1(t - 3)^2 + 48$, or $H = -5.1t^2 + 30.6t + 2.1$.

Practice

1. In each case, create a scatter plot and determine the equation of the line of best fit.

(a)

x	y
0	5
1	12
2	17
3	25
4	33
5	36

(b)

Time (s)	Temperature (°C)
30	29.6
60	29.9
90	29.5
120	29.2
150	29.4
180	29.1

2. In each case, create a scatter plot and determine the equation of the curve of best fit.

(a)

x	y
–2	8
–1	–1
0	–4
1	–1
2	8

(b)

Time (s)	Height (m)
0	20
0.5	18.8
1	15.1
1.5	8.9
2	0.4

Simplifying Rational Expressions

Rational expressions are algebraic expressions that can be written as the quotient of two polynomials, such as $\frac{x-1}{5x^2-3}, \frac{x^2+3x+2}{2x^2-x-1}, 7x^2-\frac{5}{x}$. Rational expressions can be reduced to lower terms by dividing both the numerator and denominator by any common non-zero factor. That is, $\frac{a(x)d(x)}{b(x)d(x)}=\frac{a(x)}{b(x)}$ if $a(x)$, $b(x)$, $c(x)$, and $d(x)$ are rational expressions and $b(x) \neq 0$ and $d(x) \neq 0$. A rational expression is **undefined** if the denominator equals 0.

Example 1

Simplify. Factor if possible and then reduce factors common to both the numerator and denominator. State restrictions.

(a) $\frac{21m^3n^2}{6mn^4}$ **(b)** $\frac{x^2-1}{x^2+4x+3}$ **(c)** $-\frac{2x-y}{y-2x}$ **(d)** $\frac{10s^4t+15s^3t^2}{4s^2+12st+9t^2}$

Solution

(a)
$$\frac{21m^3n^2}{6mn^4}=\frac{(3mn^2)(7m^2)}{(3mn^2)(2n^2)}=\frac{7m^2}{2n^2}$$
$m \neq 0, n \neq 0$

(b)
$$\frac{x^2-1}{x^2+4x+3}=\frac{(x+1)(x-1)}{(x+1)(x+3)}=\frac{x-1}{x+3}$$
$x \neq -1, -3$

(c)
$$-\frac{2x-y}{y-2x}=\frac{-2x+y}{y-2x}=\frac{y-2x}{y-2x}=1$$
$x \neq 2y$

(d)
$$\frac{10s^4t+15s^3t^2}{4s^2+12st+9t^2}=\frac{5s^3t(2s+3t)}{(2s+3t)(2s+3t)}=\frac{5s^3t}{2s+3t}$$
$s \neq -\frac{3}{2}t$

Practice

1. State the restrictions on each rational expression, if any.

(a) $\frac{5}{x}$ **(b)** $\frac{x}{5}$ **(c)** $\frac{17}{x-2}$ **(d)** $\frac{3x}{x^2-36}$

2. Simplify and state restrictions. Write your answers without negative exponents.

(a) $\frac{2ab}{4b}$ **(b)** $\frac{12m^7n^4}{15m^2n^5}$ **(c)** $\frac{35p^2q^2}{28pq^4}$ **(d)** $\frac{16xyz^3t}{24x^2y^2zt^2}$

(e) $\frac{abc-bc^2}{arc-rc^2}$ **(f)** $\frac{x^2+5x}{x^2+4x-5}$ **(g)** $\frac{3h^2+6h}{h^2+4h+4}$ **(h)** $\frac{x^2-1}{2xy+2y}$

(i) $\frac{b^3-a^2b}{b^2-2ab+a^2}$ **(j)** $\frac{x^2+3x+2}{x^2+5x+6}$ **(k)** $\frac{t^3+t^2}{t-t^3}$ **(l)** $\frac{r^2-4}{r^2+r-6}$

Adding and Subtracting Rational Expressions

To add or subtract rational expressions, find a **lowest common denominator (l.c.d)** and combine the numerators. In other words, if $a(x)$, $b(x)$, $c(x)$, and $d(x)$ are rational expressions with $b(x) \neq 0$ and $d(x) \neq 0$, $\frac{a(x)}{b(x)}+\frac{c(x)}{b(x)}=\frac{a(x)+c(x)}{b(x)}$ and $\frac{a(x)}{b(x)}-\frac{c(x)}{b(x)}=\frac{a(x)-c(x)}{b(x)}$.

Example 1

Add or subtract as required and then simplify. State restrictions.

(a) $\frac{1}{x}+\frac{3}{x}$ **(b)** $\frac{x^2+x+1}{x^2+1}-\frac{x+1}{x^2+1}$

Solution

(a)
$$\frac{1}{x}+\frac{3}{x}=\frac{1+3}{x}$$
$$=\frac{4}{x}$$

$x \neq 0$

(b)
$$\frac{x^2+x+1}{x^2+1}-\frac{x+1}{x^2+1}=\frac{x^2+x+1-(x+1)}{x^2+1}$$
$$=\frac{x^2+x+1-x-1}{x^2+1}$$
$$=\frac{x^2}{x^2+1}$$

no restrictions, since $x^2+1 \neq 0$ for $x \in$ **R**.

Practice

1. State the LCD.

(a) $\frac{3}{x},\frac{5}{y}$ **(b)** $\frac{-2}{mn},\frac{1}{n}$ **(c)** $\frac{12}{xy},\frac{2}{x^2}$ **(d)** $\frac{21x}{x-1},\frac{4}{x}$

(e) $\frac{a}{a+1},\frac{4}{a-2}$ **(f)** $\frac{q}{2q+3},\frac{-8q}{q+2}$ **(g)** $\frac{3t}{(t-1)(t+1)},\frac{-t}{t+1}$

2. Add.

(a) $\frac{-2a}{y}+\frac{3}{y}+\frac{a}{y}$ **(b)** $\frac{3x}{a-b}+\frac{5x}{b-a}$ **(c)** $\frac{1}{x+8}+\frac{1}{x-2}$ **(d)** $\frac{3mn}{4y^2}+\frac{2mn}{4y}$

3. Subtract.

(a) $\frac{5x}{2}-\frac{3x}{2}$ **(b)** $\frac{-7}{x+2}-\frac{5}{x+2}$ **(c)** $\frac{a+b}{3}-\frac{a-b}{2}$ **(d)** $\frac{2a}{x(x-1)}-\frac{3a}{x^2-x}$

4. Simplify. State restrictions and reduce the answer to lowest terms.

(a) $\frac{x^2-4y^2}{x^2+2xy}+\frac{x^2-2xy+4y^2}{2xy}$ **(b)** $\frac{x^2-1}{(x+1)^2}-\frac{x^2-5x+6}{x^2-4x+4}+\frac{x+3}{x^2+4x+3}$

Multiplying and Dividing Rational Expressions

If $a(x)$, $b(x)$, $c(x)$, and $d(x)$ are rational expressions; and $b(x) \neq 0$, $c(x) \neq 0$, and $d(x) \neq 0$; then $\frac{a(x)}{b(x)} \frac{c(x)}{d(x)} = \frac{a(x)c(x)}{b(x)d(x)}$ and $\frac{a(x)}{b(x)} \div \frac{c(x)}{d(x)} = \frac{a(x)d(x)}{b(x)c(x)}$.

To multiply rational expressions:

1. Factor the numerators and denominators.
2. State all the restrictions on the variables.
3. Divide out any factors common to numerators and denominators.
4. Multiply the numerators, then denominators.
5. Write the result as a single rational expression.

To divide rational expressions:

1. Factor the numerators and denominators.
2. State all the restrictions on the variables.
3. Take the reciprocal of the second rational expression and change division to multiplication.
4. State any new restrictions.

Example 1

Multiply and simplify.

$$\frac{x^2+3x+2}{2x^2-x-1} \times \frac{x-1}{x+1} = \frac{(x+1)(x+2)(x-1)}{(2x+1)(x-1)(x+1)}$$
$$= \frac{x+2}{2x+1}$$

Example 2

Divide and simplify.

$$\frac{a^3-2a^2+a}{a^2+2a} \div \frac{a-1}{a^2-4} = \frac{a^3-2a+a}{a^2+2a} \times \frac{a^2-4}{a-1}$$
$$= \frac{a(a^2-2a+1)(a-2)(a+2)}{a(a+2)(a-1)}$$
$$= \frac{a(a-1)^2(a-2)(a+2)}{a(a+2)(a-1)}$$
$$= (a-1)(a-2)$$

Practice

1. Simplify and state any restrictions.

(a) $\frac{3ac}{4b} \frac{4x}{2ay}$ **(b)** $\frac{3x^2y^2z^3}{4a^2b^2c^2} \frac{8a^3b^2c^2}{9x^2yz^3}$ **(c)** $\frac{4a^3x}{6dy^2} \div \frac{2a^2x^2}{8a^2y}$

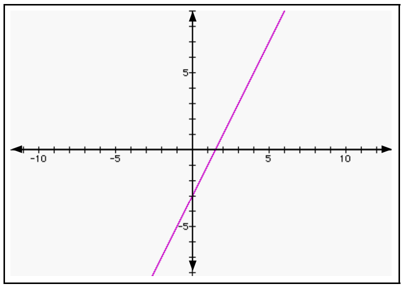
(d) $\frac{10x}{x+2} \div \frac{5}{2(x+2)}$ **(e)** $\frac{12y}{3y-9} \frac{4y-12}{6y^2}$ **(f)** $\frac{6p-12}{5p+5} \frac{2p+2}{3p-6}$

(g) $4x^2-9y^2 \frac{2}{3xy+2x^2}$ **(h)** $\frac{21p-3q^2}{16p^2+4p^3} \frac{12+7p+p^2}{14-9p+p^2}$

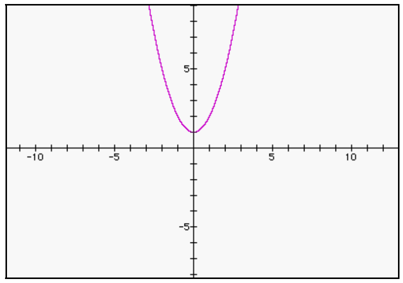
Graphing Functions and Their Reciprocals

Here are the graphs of some common functions.

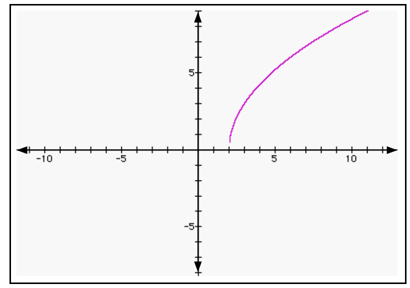
linear function



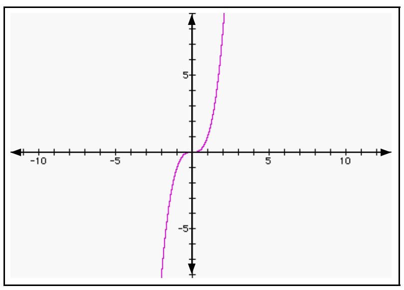
quadratic function



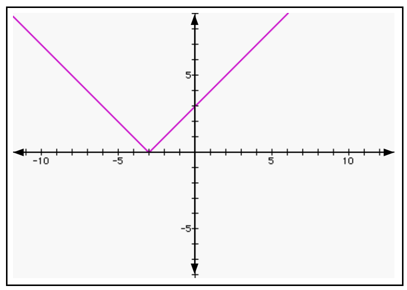
square-root function



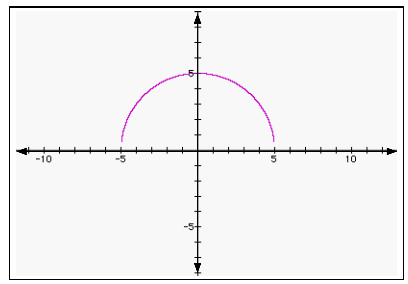
cubic function



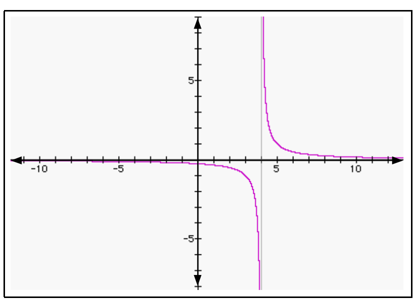
absolute value function



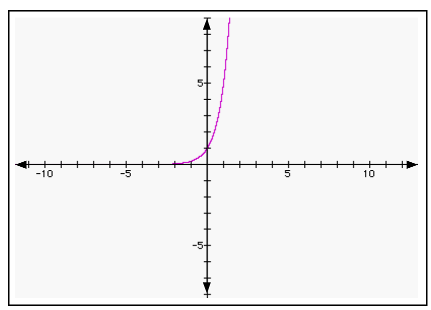
semi-circle function



reciprocal function



exponential function



Graphing Reciprocals of Functions

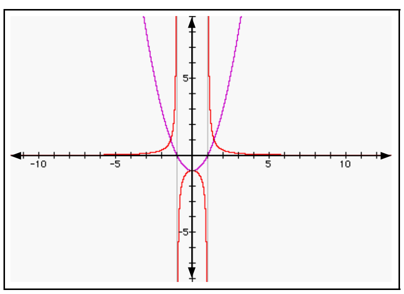
Every function $f(x)$ has a reciprocal, $\frac{1}{f(x)}$. The graph of $f(x)$ can be used to graph $\frac{1}{f(x)}$.

- The reciprocal of 0 is undefined. An asymptote is drawn where the domain of the function is undefined.
- The reciprocal of a positive number is positive, and that of a negative number is negative.
- As x approaches 0, the reciprocal $\frac{1}{x}$ becomes larger. If x is close to 1, then the reciprocal $\frac{1}{x}$ is also close to 1. As x gets larger, the reciprocal $\frac{1}{x}$ gets smaller toward 0.

Example 1

Graph $f(x) = x^2 - 1$ and its reciprocal function $\frac{1}{f(x)} = \frac{1}{x^2 - 1}$ on the same set of axes. State the domain and range of both functions.

x	$f(x) = x^2 - 1$	$\frac{1}{f(x)} = \frac{1}{x^2-1}$
–3	8	$\frac{1}{8}$
–2	3	$\frac{1}{3}$
–1	0	undefined
–1/2	– 3/4	$\frac{-4}{3}$
0	–1	-1
1/2	–3/4	$\frac{-4}{3}$
1	0	undefined
2	3	$\frac{1}{3}$
3	8	$\frac{1}{8}$



For $f(x)$, domain: $\{x \mid x \in \mathbf{R}\}$, range: $\{y \mid y \geq -1, y \in \mathbf{R}\}$

For $\frac{1}{f(x)}$, domain: $\{x \mid x \neq \pm 1, x \in \mathbf{R}\}$,
range: $\{y \mid y \neq 0, y \in \mathbf{R}\}$

Practice

1. Graph each function. State the domain and range.

(a) $y = x$ **(b)** $y = x^2 + 4$ **(c)** $y = \sqrt{x + 6}$

(d) $y = x^3$ **(e)** $y = |2x - 1|$ **(f)** $y = 2^x$

(g) $y = 7^x - 2$ **(h)** $y = \frac{1}{x + 7}$ **(i)** $y = \frac{1}{x^2}$

(j) $y = \frac{1}{x^2 + 1}$ **(k)** $y = \sqrt{4 - x^2}$ **(l)** $y = \frac{1}{2}x - 1$

(m) $y = |3(x + 2)|$ **(n)** $y = -4x^3 + 3$ **(o)** $y = \sqrt{49 - x^2}$

2. Given $f(x)$, write the equation of the reciprocal function $\frac{1}{f(x)}$. Simplify.

(a) $f(x) = 4x - 7$ **(b)** $f(x) = 2x^2 - 5x + 6$ **(c)** $f(x) = \sqrt{x+4}$

(d) $f(x) = |2(x + 1)|$ **(e)** $f(x) = \sqrt{64 - (x+3)^2}$ **(f)** $f(x) = \frac{1}{2}x^3 + 1$

3. Graph each function and its reciprocal on the same set of axes. State their domain and range.

(a) $y = x^2 + 5$ **(b)** $y = (x - 3)^2 - 4$ **(c)** $y = \sqrt{2x+5} - 3$

(d) $y = \left|\frac{1}{3}x - 1\right| + 2$ **(e)** $y = \sqrt{x} + 8$ **(f)** $y = 3 - x$

Inverse Functions

- The **inverse** of a relation maps each output of the original relation back onto the corresponding input value; it is the reverse of the original relation.
- The inverse of a function $y = f(x)$ is denoted y^{-1}. The inverse may be function or it may not. If it is a function, the notation "$f^{-1}(x)$" is used.
- If $(a, b) \in f$, then $(b, a) \in f^{-1}$.
- Given a table for a relation, interchange x and y to obtain a table for the inverse relation.
- The domain of f is the range of f^{-1}. The range of f is the domain of f^{-1}.
- The graph of $f^{-1}(x)$ is the reflection of the graph of $f(x)$ in the line $y = x$.
- The graphs of $f(x)$ and $f^{-1}(x)$ intersect at points on the line $y = x$.
- To determine the equation of the inverse in function notation, interchange x and y, and solve for y.

Example 1

For $f(x) = 2(x - 1)^2 + 4$

i. determine the inverse relation algebraically

ii. write a table for f and for the inverse for $-2 \leq x \leq 2$.

iii. draw an arrow diagram for f and for the inverse, then determine whether the inverse is a function

iv. graph f and the inverse on the same set of axes

v. state the domain and range of f and the inverse

Solution

i.

$$y = f(x) = 2(x-1)^2 + 4$$
$$x = 2(y-1)^2 + 4$$
$$x - 4 = 2(y-1)^2$$
$$(y-1)^2 = \frac{x-4}{2}$$
$$y - 1 = \sqrt{\frac{x-4}{2}}$$
$$y = \sqrt{\frac{x-4}{2}} + 1$$

ii.

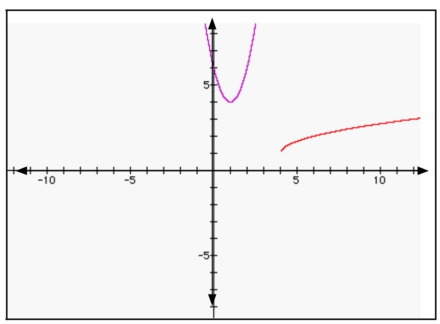
x	$f(x)$	x	$f^{-1}(x)$
–2	22	22	–2
–1	12	12	–1
0	6	6	0
1	4	4	1
2	6	6	2

iii.

$x \to f(x)$	$x \to f^{-1}(x)$
$-2 \to 22$	$22 \to -2$
$-1 \to 12$	$12 \to -1$
$0 \to 6$	$6 \to 0$
$1 \to 4$	$4 \to 1$
$2 \to 6$	$6 \to 2$

The arrow diagram reverses for the inverse. Both are functions.

iv.



v. for $f(x)$, domain: $\{x \mid x \in \mathbf{R}\}$, range: $\{y \mid y \geq 4, y \in \mathbf{R}\}$;
for $f^{-1}(x)$, domain: $\{x \mid x \geq 4, x \in \mathbf{R}\}$, range: $\{y \mid y \in \mathbf{R}\}$

Practice

1. Describe in words the inverse of each case.

   (a) multiply by 6, then subtract 4

   (b) divide by 2, then add 7

   (b) add 1, then multiply by 5

   (d) subtract 9, then divide by 3

2. Find each inverse algebraically.

   (a) $f(x) = -2x + 3$

   (b) $g(x) = 5x - 8$

   (c) $h(x) = 2(x + 1)$

   (d) $k(x) = \frac{-1}{4}x - 10$

   (e) $l(x) = 4x^2$

   (f) $m(x) = 5 - 2x^2$

   (g) $n(x) = \frac{1}{3}(x - 1)^2$

   (h) $p(x) = -(x + 3)^2 + 1$

3. Write a table for each function and its inverse. Then graph both on the same set of axes, and check on a graphing calculator. State the domain and range.

   (a) $f(x) = 3x + 4$

   (b) $g(x) = \frac{1}{3 - x}$

   (c) $h(x) = \sqrt{x - 2}$

   (d) $k(x) = -2(x + 1)^2 + 3$

   (e) $l(x) = \sqrt{9 - (x - 2)^2}$

   (f) $m(x) = -4x^3$

   (g) $n(x) = \frac{3}{2}x^2 - 10$

   (h) $p(x) = -\sqrt{x} + 5$

