Managing Both Courses Using the Same Text

Content Differences

The Functions (University/College) and the Functions and Relations (University) curriculum share three common content strands:

- Financial Applications of Sequences and Series
- Tools for Operating and Communicating with Functions
- Trigonometric Functions

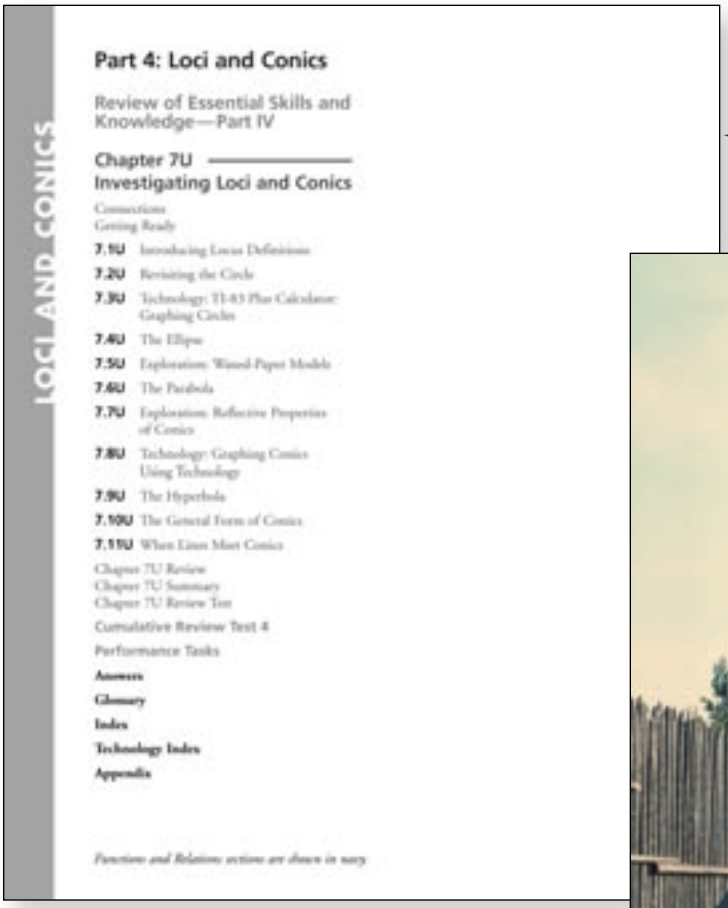
Within these common strands, all overall and specific expectations are the same with the exception of two specific expectations, which are unique to the Functions and Relations (University) course:

- Financial Applications of Sequences and Series: Solving Problems Involving Arithmetic and Geometric Sequences and Series
  – write terms of a sequence, given the formula for the *n*th term or given a recursion formula
  Addressed in *Nelson Mathematics 11* Section 1.3 Sequences and Recursive Formulas
- Tools for Operating and Communicating with Functions: Manipulating Polynomials, Rational Expressions, and Exponential Expressions
  – add, subtract, multiply, and divide complex numbers in rectangular form
  Addressed in *Nelson Mathematics 11* Section 4.5 Adding, Subtracting, and Multiplying Complex Numbers and Section 4.10 Dividing Complex Numbers

The Functions and Relations (University) curriculum also features a fourth content strand:

- Investigations of Loci and Conics
  Addressed in *Nelson Mathematics 11* with a separate chapter: Part 4—Chapter 7: Investigating Loci and Conics

Part 4: Chapter 7 Investigating Loci and Conics is specific to the Functions and Relations (University) course. Students in the Functions (University/College) course should omit this final chapter in the student text.


LOCI AND CONICS

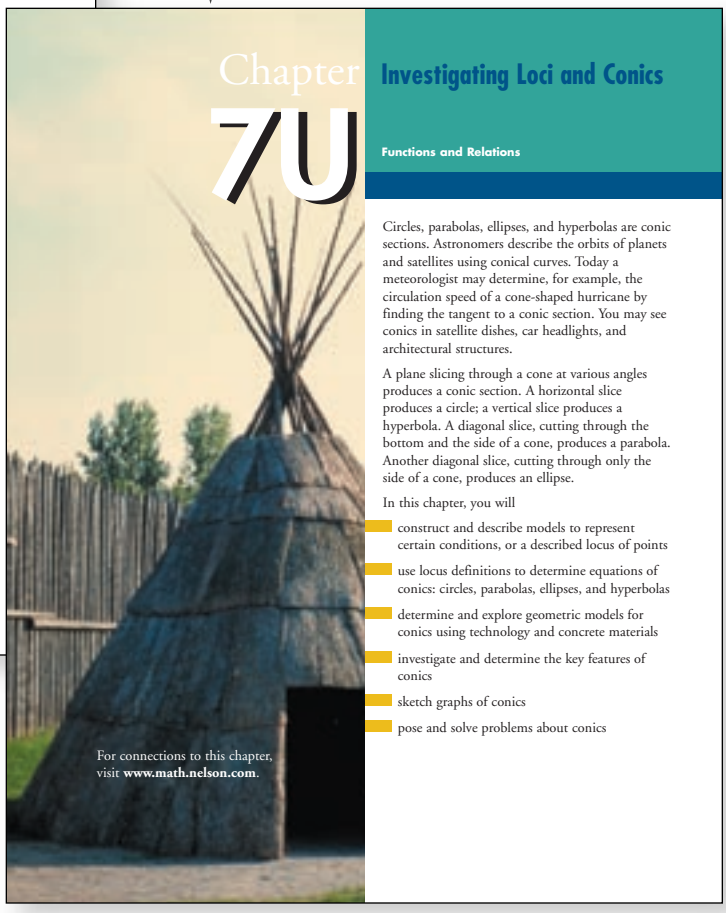
Part 4: Loci and Conics

Review of Essential Skills and Knowledge—Part IV

Chapter 7U
Investigating Loci and Conics

Connections
Getting Ready
7.1U Introducing Locus Definitions
7.2U Revisiting the Circle
7.3U Technology: TI-83 Plus Calculator: Graphing Circles
7.4U The Ellipse
7.5U Exploration: Waxed-Paper Models
7.6U The Parabola
7.7U Exploration: Reflective Properties of Conics
7.8U Technology: Graphing Conics Using Technology
7.9U The Hyperbola
7.10U The General Form of Conics
7.11U When Lines Meet Conics
Chapter 7U Review
Chapter 7U Summary
Chapter 7U Review Test
Cumulative Review Test 4
Performance Tasks
Answers
Glossary
Index
Technology Index
Appendix

Functions and Relations sections are shown in navy.



Functions and Relations (University) Lessons Identified in Student Text

- The three lessons found in Chapter 1 and 4 that are specific to the Functions and Relations (University) course are clearly identified
- The title page for Part 4—Chapter 7 Investigating Loci and Conics, as well as the subsequent lessons are identified for the Functions and Relations (University) course

First page of Section 1.3 Sequences and Recursive Formulas

First page of Section 4.5 Adding, Subtracting, and Multiplying Complex Numbers and Section 4.10 Dividing Complex Numbers

1.3U Functions and Relations

Sequences and Recursive Formulas

Part 1: Depreciation

A new car is rarely resold for its original purchase price. As soon as a new car is driven away from a dealership, the value of the car decreases. This decrease in value is called **depreciation**. The show room price of a new car is $26 000. The salesperson estimates that the value of the car will depreciate by an average of 10% each year.

Think, Do, Discuss

1. Determine the dollar amount of depreciation for the first year.
2. Determine the value of the car at the end of the first year.
3. What per cent of the original purchase price does the value in step 2 represent?
4. Determine the value of the car at the end of the second year.
5. Write the first five terms of the sequence that represents the value of the car at the end of each year for the first five years.
6. Let t_n represent any term of this sequence. How could you represent the term before t_n?
7. For this sequence, write a formula that uses the previous term to calculate any term, except the first term.
8. At the end of which year will the car's value be about one-half of its original purchase price?

Part 2: The Fibonacci Sequence

There are many examples of sequences in nature, and in our everyday lives. A female honeybee hatches from an egg laid by a female honeybee, after the egg has been fertilized by the male. So each female honeybee has two parents. However, a male

4.10U SKILL BUILDER Functions and Relations

Dividing Complex Numbers

In this chapter, you have added, subtracted, and multiplied complex numbers. Dividing complex numbers requires a little mathematical creativity. You cannot directly divide a complex number because the denominator has two independent terms. Overcome this difficulty using the product property of conjugate complex numbers. In other words, recall that $z\bar{z}$ is always a real number.

Example 1

Express each of the following in the form $a + bi$.

(a) $\frac{2+3i}{5-2i}$ (b) $\frac{2}{1-i}$ (c) $\frac{3+i}{i}$

Solution

In each case, multiply by $\frac{\bar{z}}{\bar{z}}$, which is the same as multiplying by 1.

(a) In this case, $5 + 2i$ is the complex conjugate of $5 - 2i$.
Multiply the numerator and denominator by $5 + 2i$.

$$\frac{2+3i}{5-2i} = \frac{2+3i}{5-2i} \times \frac{5+2i}{5+2i}$$
$$= \frac{(2+3i)(5+2i)}{(5-2i)(5+2i)} \quad \text{Expand.}$$
$$= \frac{10+4i+15i+6i^2}{25+10i-10i-4i^2} \quad \text{Substitute } -1 \text{ for } i^2.$$
$$= \frac{10+19i+6(-1)}{25-4(-1)} \quad \text{Simplify.}$$
$$= \frac{4+19i}{29}$$
$$= \frac{4}{29} + \frac{19}{29}i$$

(b) $\frac{2}{1-i} = \frac{2}{1-i} \times \frac{1+i}{1+i}$
$= \frac{2(1+i)}{(1-i)(1+i)}$
$= \frac{2+2i}{1-i^2}$
$= \frac{2+2i}{1-(-1)}$
$= \frac{2+2i}{2}$
$= 1+i$

(c) $\frac{3+i}{i} = \frac{3+i}{i} \times \frac{-i}{-i}$
$= \frac{(3+i)(-i)}{i(-i)}$
$= \frac{-3i-i^2}{-i^2}$
$= \frac{-3i-(-1)}{-(-1)}$
$= \frac{1-3i}{1}$
$= 1-3i$

4.5U SKILL BUILDER Functions and Relations

Adding, Subtracting, and Multiplying Complex Numbers

Because complex numbers have two parts, the real part and the imaginary part, they are ideal for describing real-life quantities that have two components. For example, in electronics, the resistance of a circuit is expressed as impedance. Impedance has two parts, resistance, R, and a reactance, X. Impedance is represented by the complex number, $R + Xi$.

In this section, you will learn how to add, subtract, and multiply complex numbers. You may be interested to know that a calculator uses complex arithmetic algorithms to perform many simple operations.

Adding and Subtracting Complex Numbers

When adding or subtracting two complex numbers, use the laws of algebra and treat the symbol i as if it were a variable.

Example 1

(a) Add $(5 - 2i)$ and $(-3 + 7i)$.

(b) Find the sum of $(-4 - 3i)$ and its complex conjugate.

Solution

(a) $(5 - 2i) + (-3 + 7i)$
$= 5 - 2i - 3 + 7i$ (real, imaginary)
$= 5 - 3 - 2i + 7i$
$= 2 + 5i$

(b) $(-4 - 3i) + (-4 + 3i)$
$= -4 - 3i - 4 + 3i$
$= -4 - 4 - 3i + 3i$
$= -8 + 0i$
The sum is a real number.

Example 2

(a) Simplify $(7 - 4i) - (2 + 3i)$.

(b) Subtract $(3 - 5i)$ from its complex conjugate.

Solution

(a) $(7 - 4i) - (2 + 3i)$
$= 7 - 4i - 2 - 3i$
$= 7 - 2 - 4i - 3i$
$= 5 - 7i$

(b) $(3 + 5i) - (3 - 5i)$
$= 3 + 5i - 3 + 5i$
$= 3 - 3 + 5i + 5i$
$= 0 + 10i$
$= 10i$
The difference is an imaginary number.

Managing Both Courses—Other Support Features

Course Considerations Provided in Teacher's Resource

The front matter of each chapter in the *Nelson Mathematics 11* Teacher's Resource provides a summary table of key considerations for teaching Functions (University/College) and Functions and Relations (University).

- In general, students in the Functions and Relations (University) course should be able to work with abstract concepts and work at a faster, more independent pace. Students in the Functions (University/College) course may benefit from more time to explore concepts, solve problems and practise skills.

Front matter from the Teacher's Resource for Chapter 1 Patterns of Growth: Sequences featuring key considerations for teaching both courses

Using This Chapter with Functions/Relations and Functions Classes

Functions and Relations (University)	
Students in this course must be prepared for the abstract mathematical concepts found in the Geometry and Discrete Mathematics course. These students will be required to broaden their skills in solving complex problems. To help prepare these students for this course, you could consider the following:	
1.1	Students in both courses recognize and describe patterns. Expect U students to develop algebraic models more frequently as a method of describing a pattern.
1.3	This section introduces U students to recursive formulas. Omit this section, and all subsequent references to it, in U/C courses.
1.6, 1.7	U students will benefit from opportunities to develop a wide range of tools but will be expected to become proficient in use of algebraic models.
1.8, 1.9	Students in both courses are provided with an understanding of the mathematics underlying many common financial situations. Reliance on, and connection to, theory may allow U students to move at a faster pace
1.10	Use this section judiciously with either group of students. Drill and practise to consolidate and develop mastery of skills may require wide variations in time and amount.
1.11	Encourage U students to recognize the common elements in various situations and to consider the validity of steps in a solution. U students will turn to algebraic explanations of their thinking.

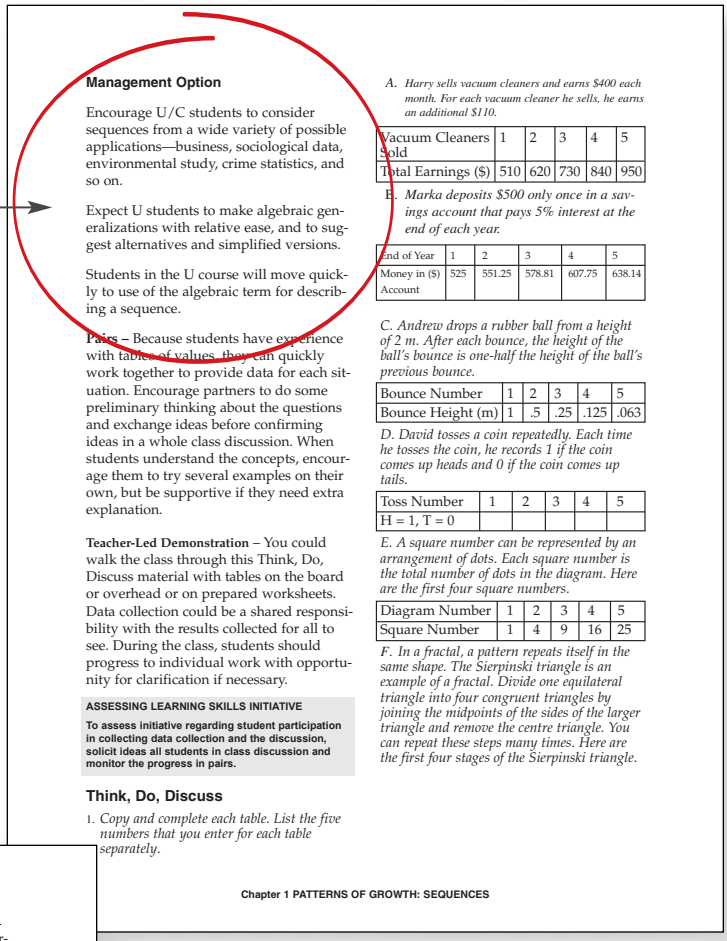
Functions (University/College)	
Students in the University/College course must be prepared to build on theor experience with functions if they move onto the Advanced Functions and Introductory Calculus course in grade 12. It should be noted, that these students will not encounter trigonometric functions in the calculus course. However, these students will be required to use polynomial, rational, exponential, and logarithmic functions to model real-world problems in a range of applications. To help prepare these students for this course you could consider the following:	
1.1	Give multiple examples, and encourage students to use diagrams, technology, words, and algebra to describe the patterns.
1.3	This section is intended for students in the U course; omit it from U/C courses.
1.6, 1.7	In these sections, U/C students have many opportunities to recognize and manipulate patterns from various sources. Encourage them to use algebraic, technological, and other methods for describing and investigating.
1.8, 1.9	Students in both courses are provided with an understanding of the mathematics underlying many common financial situations. Use a spreadsheet with U/C students to enhance their understanding of the concepts.
1.11	In this section, U/C students may rely more on technology or multiple examples.

Additional Support Integrated in Teaching Notes

Further support is provided in the teaching notes for each section, including:

- Additional example for both the Functions (University/College) and the Functions and Relations (University) courses
- Specific questions from the Practise, Apply, Solve sections are identified within the Teacher's Resource as being more appropriate for the Functions and Relations (University) or the Functions (University/College) course.
- Additional teaching suggestions distinguishing between Functions (University/College) and the Functions and Relations (University) courses

Management Options from the Teacher's Resource for Section 1.1 offer suggestions for U and U/C



Management Option

Encourage U/C students to consider sequences from a wide variety of possible applications—business, sociological data, environmental study, crime statistics, and so on.

Expect U students to make algebraic generalizations with relative ease, and to suggest alternatives and simplified versions.

Students in the U course will move quickly to use of the algebraic term for describing a sequence.

Pairs – Because students have experience with tables of values, they can quickly work together to provide data for each situation. Encourage partners to do some preliminary thinking about the questions and exchange ideas before confirming ideas in a whole class discussion. When students understand the concepts, encourage them to try several examples on their own, but be supportive if they need extra explanation.

Teacher-Led Demonstration – You could walk the class through this Think, Do, Discuss material with tables on the board or overhead or on prepared worksheets. Data collection could be a shared responsibility with the results collected for all to see. During the class, students should progress to individual work with opportunity for clarification if necessary.

ASSESSING LEARNING SKILLS INITIATIVE

To assess initiative regarding student participation in collecting data collection and the discussion, solicit ideas all students in class discussion and monitor the progress in pairs.

Think, Do, Discuss

1. *Copy and complete each table. List the five numbers that you enter for each table separately.*

A. Harry sells vacuum cleaners and earns $400 each month. For each vacuum cleaner he sells, he earns an additional $110.

Vacuum Cleaners Sold	1	2	3	4	5
Total Earnings ($)	510	620	730	840	950

B. Marka deposits $500 only once in a savings account that pays 5% interest at the end of each year.

End of Year	1	2	3	4	5
Money in ($) Account	525	551.25	578.81	607.75	638.14

C. Andrew drops a rubber ball from a height of 2 m. After each bounce, the height of the ball's bounce is one-half the height of the ball's previous bounce.

Bounce Number	1	2	3	4	5
Bounce Height (m)	1	.5	.25	.125	.063

D. David tosses a coin repeatedly. Each time he tosses the coin, he records 1 if the coin comes up heads and 0 if the coin comes up tails.

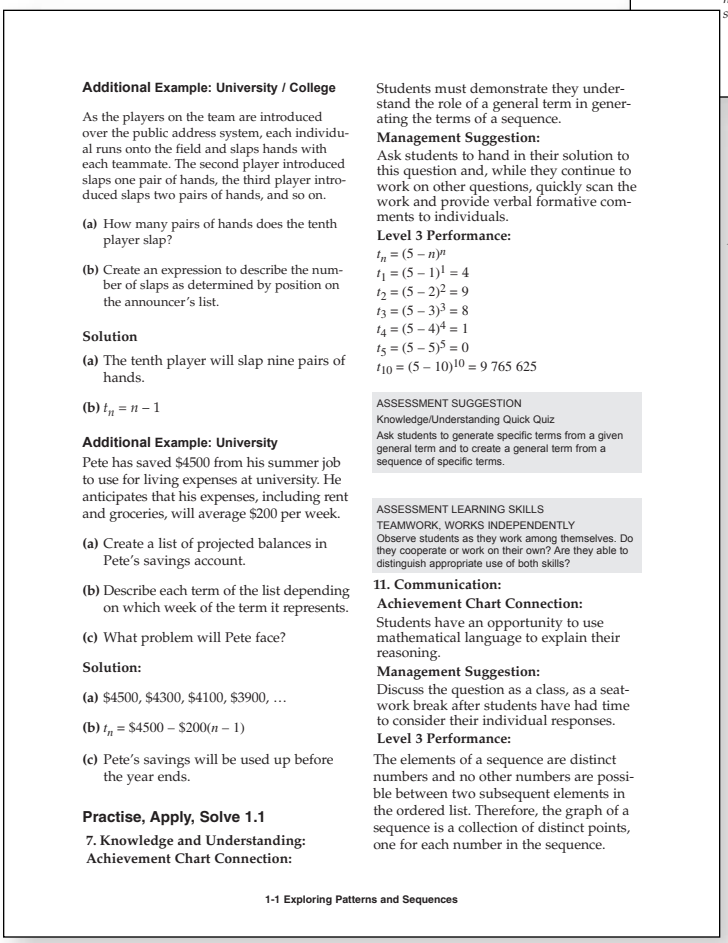
Toss Number	1	2	3	4	5
H = 1, T = 0					

E. A square number can be represented by an arrangement of dots. Each square number is the total number of dots in the diagram. Here are the first four square numbers.

Diagram Number	1	2	3	4	5
Square Number	1	4	9	16	25

F. In a fractal, a pattern repeats itself in the same shape. The Sierpinski triangle is an example of a fractal. Divide one equilateral triangle into four congruent triangles by joining the midpoints of the sides of the larger triangle and remove the centre triangle. You can repeat these steps many times. Here are the first four stages of the Sierpinski triangle.

Chapter 1 PATTERNS OF GROWTH: SEQUENCES



Additional Example: University / College

As the players on the team are introduced over the public address system, each individual runs onto the field and slaps hands with each teammate. The second player introduced slaps one pair of hands, the third player introduced slaps two pairs of hands, and so on.

(a) How many pairs of hands does the tenth player slap?

(b) Create an expression to describe the number of slaps as determined by position on the announcer's list.

Solution

(a) The tenth player will slap nine pairs of hands.

(b) $t_n = n - 1$

Additional Example: University

Pete has saved $4500 from his summer job to use for living expenses at university. He anticipates that his expenses, including rent and groceries, will average $200 per week.

(a) Create a list of projected balances in Pete's savings account.

(b) Describe each term of the list depending on which week of the term it represents.

(c) What problem will Pete face?

Solution:

(a) $4500, $4300, $4100, $3900, …

(b) $t_n = \$4500 - \$200(n - 1)$

(c) Pete's savings will be used up before the year ends.

Practise, Apply, Solve 1.1

7. Knowledge and Understanding: Achievement Chart Connection:

Students must demonstrate they understand the role of a general term in generating the terms of a sequence.

Management Suggestion:

Ask students to hand in their solution to this question and, while they continue to work on other questions, quickly scan the work and provide verbal formative comments to individuals.

Level 3 Performance:

$t_n = (5 - n)^n$
$t_1 = (5 - 1)^1 = 4$
$t_2 = (5 - 2)^2 = 9$
$t_3 = (5 - 3)^3 = 8$
$t_4 = (5 - 4)^4 = 1$
$t_5 = (5 - 5)^5 = 0$
$t_{10} = (5 - 10)^{10} = 9\ 765\ 625$

ASSESSMENT SUGGESTION

Knowledge/Understanding Quick Quiz

Ask students to generate specific terms from a given general term and to create a general term from a sequence of specific terms.

ASSESSMENT LEARNING SKILLS

TEAMWORK, WORKS INDEPENDENTLY

Observe students as they work among themselves. Do they cooperate or work on their own? Are they able to distinguish appropriate use of both skills?

11. Communication:

Achievement Chart Connection:

Students have an opportunity to use mathematical language to explain their reasoning.

Management Suggestion:

Discuss the question as a class, as a seatwork break after students have had time to consider their individual responses.

Level 3 Performance:

The elements of a sequence are distinct numbers and no other numbers are possible between two subsequent elements in the ordered list. Therefore, the graph of a sequence is a collection of distinct points, one for each number in the sequence.

1-1 Exploring Patterns and Sequences

An additional example for both the University/College and University course is shown for Section 1.1