Tour the Text

What are the key features of *Nelson Mathematics 11* that make it inviting and understandable for students?

Preparatory Material

- **Review of Essential Skills and Knowledge**
- **Getting Ready**

Review of Essential Skills and Knowledge

Appears at the beginning of each strand. It reviews content from previous grades, highlighting the required concepts, procedures, and skills needed for the upcoming strand.

Getting Ready

Reviews the important ideas from previous grades and chapters to ensure students have the requisite skills and knowledge for the upcoming chapter. Getting Ready pages appear at the beginning of each chapter.

Review of Essential Skills and Knowledge—Part I

Financial Applications of Sequences and Series

Pattern Recognition

When the independent variable in a relation changes by a steady increment, the dependent variable often changes according to a pattern.

Type of Pattern	Description	Example
Linear	The first differences between dependent variables are constant.	(table below) In a linear relation, each first difference is always the same. Here, each first difference is $10 000.
Quadratic	The first differences between dependent variables change, but the second differences are constant.	(table below) In this case, the first differences are not the same, so the relation is nonlinear. In a quadratic relation, each second difference is always the same. Here, each second difference is –9.8.
Exponential	There is a common multiplier between dependent variables.	(table below)

Linear:

Number of Books (× 1000)	1	2	3	4	5
Cost of Books ($) (× 1000)	60	70	80	90	100
First Differences (× 1000)	10	10	10	10	

Quadratic:

Time (s)	0	1	2	3	4	5
Height of Ball (m)	5	30.1	45.4	50.9	46.6	32.5
First Differences	25.1	15.3	5.5	–4.3	–14.1	
Second Differences	–9.8	–9.8	–9.8	–9.8		

Exponential:

Time (h)	0	1	2	3	4
Number of Bacteria	1000	2000	4000	8000	16 000
Common Multipier	× 2	× 2	× 2	× 2	

Review of Essential Skills and Knowledge – Part I

Getting Ready

In this chapter, you will be working with percents, decimals, rational numbers, exponents and their laws, linear and quadratic equations, zeroes of relationships, and graphs of relationships. These exercises will help you warm up for the work ahead.

1. Extend the pattern to determine the next three numbers in each sequence.
(a) 3, 6, 9, 12, ■, ■, ■
(b) 5, 6, 8, 11, ■, ■, ■
(c) 1, 4, 9, 16, ■, ■, ■
(d) −4, 5, 1, 6, 7, ■, ■, ■
(e) −5, −10, −20, −40, ■, ■, ■
(f) $\frac{2}{3}, \frac{4}{5}, \frac{6}{7}, \frac{8}{9}$, ■, ■, ■

2. Express each percent as a decimal.
(a) 45% (b) 39%
(c) 8% (d) 3%
(e) 98% (f) 4.5%
(g) 5.25% (h) 0.5%
(i) 125% (j) 110%

3. Evaluate.
(a) 5% of 20 (b) 18% of 240
(c) 3.6% of 50 (d) 2.1% of 36
(e) 120% of 80 (f) 135% of 150
(g) 86% of 324 (h) 75% of 60

4. Evaluate.
(a) $\frac{1}{4} + \frac{-3}{4}$ (b) $\frac{1}{2} - \frac{-2}{3}$
(c) $-\frac{3}{4} - \frac{1}{-4}$ (d) $-\frac{3}{5} + \frac{3}{-4}$
(e) $\left(\frac{1}{-2}\right)\left(\frac{-2}{5}\right)$ (f) $-\frac{4}{5} \times \frac{10}{-4}$
(g) $\left(\frac{-5}{12}\right) - 24$ (h) $\left(-2\frac{1}{4}\right)\left(\frac{2}{-9}\right)$
(i) $\frac{-4}{3} \div \frac{2}{-3}$ (j) $-7\frac{1}{8} \div \frac{3}{2}$
(k) $\frac{-2}{3} \div \frac{-3}{8}$ (l) $\frac{-3}{-2} \div \frac{-1}{3}$
(m) $\left(2\frac{3}{4}\right)\left(-1\frac{1}{4}\right)\left(-\frac{3}{8}\right)$
(n) $\left(\frac{-2}{5} + \frac{1}{-2}\right) \div \left(\frac{5}{-8} - \frac{-1}{2}\right)$
(o) $\left(\frac{4}{5} - \frac{3}{-4}\right)\left(-\frac{1}{3} - \frac{1}{2}\right)$

5. Evaluate.
(a) 2^4 (b) $(-3)^2$
(c) -5^3 (d) 6^{-2}
(e) $\left(\frac{1}{2}\right)^3$ (f) $\left(\frac{3}{4}\right)^{-2}$
(g) 13^0 (h) $(1.5)^4$
(i) $(-7)^{-2}$ (j) 4^4
(k) $(2^3)(3^2)$ (l) $(4^{-2})(2^5)$

6. Evaluate.
(a) $4x^2 - 3x + 1$, if $x = 2$
(b) $2n + 3$, if $n = 10$
(c) $2(5)^{n+2}$, if $n = 1$
(d) Prt, if $P = 2000$, $r = 0.08$, and $t = 2$
(e) $P(1 + i)^n$, if $P = 1000$, $i = 0.01$, and $n = 10$
(f) $P(1 + i)^{-n}$, if $P = 5000$, $i = 0.02$, and $n = 5$

7. Simplify.
(a) $(x^4)(x^3)$
(b) $(x^5)(x^{-3})$
(c) $(e^2)^3$
(d) $d^6 \div d^2$
(e) $(5x^2y^3)(-2xy^2)$
(f) $(4x^5)^2$
(g) $(3a^2b^3)^4$
(h) $20m^3n^4 \div 5m^4n^{-2}$
(i) $(-2c^{-3}d^5)^{-3}$
(j) $\left(\frac{3x^2}{4y^3}\right)^2$
(k) $\frac{(2x)(3x^2y)(-5y^3)}{(10x^{-2}y^2)}$
(l) $\frac{(6c^4d^3)^2}{(4c)(3d^4)}$

8. Simplify.
(a) $2x + 3x - 8x$
(b) $3x^2 - 5y^3 - 6x^2 + 7y^3$
(c) $(3x + 5) + (2 - 4x) - (2x - 1)$
(d) $5(2x + 3y)$
(e) $2(3a - 5b) + 4(3a - 3b)$
(f) $3a(5b - 6c + 2) + 4c(2a - b + 1)$
(g) $(x + 5)(x - 2)$
(h) $(2x - 4)^2$
(i) $(3x - 2y)(8x + 4y)$
(j) $(2x + 5)(2x - 5)$

9. Factor fully.
(a) $6x^2 + 9x - 12xy$
(b) $a^2 - 81$
(c) $x^2 + 8x + 12$
(d) $a^2 - 10a + 25$
(e) $10x^2 + 17x + 3$
(f) $6c^2 - 2c - 28$

10. Solve.
(a) $3x + 15 = 5x - 5$
(b) $5(2x - 6) = 3(2x + 1) - 10x - 5$
(c) $2x^2 = 50$
(d) $(x - 5)(x + 3) = 0$
(e) $x^2 - x - 12 = 0$
(f) $3x^2 + 4x = 2$
(g) $(2x + 1)^2 = 1$
(h) $5a^2 - 3a + 2 = 2a^2 + a - 2$

11. Determine the first differences and identify the relationship as linear or nonlinear.

(a)

x	y	First Differences
1	8	
2	11	
3	14	
4	17	
5	20	
6	23	

(b)

t	d	First Differences
6	35	
7	48	
8	63	
9	80	
10	99	
11	120	

(c)

x	y	First Differences
−3	$\frac{1}{8}$	
−2	$\frac{1}{4}$	
−1	$\frac{1}{2}$	
0	1	
1	2	
2	4	

(d)

t	h	First Differences
3	−2	
4	−4	
5	−6	
6	−8	
7	−10	
8	−12	

12. i. Sketch a graph of each relationship.
ii. Use the graph to determine the zeroes of the relationship.
(a) $y = 2x + 6$
(b) $y = x^2 - 4$
(c) $y = x^2 + 3x - 18$
(d) $y = -x^2 - 5x - 6$

Chapter 1 Patterns of Growth: Sequences

Getting Ready

Review Material

- **Chapter Reviews**
- **Chapter Summaries**
- **Chapter Review Tests**
- **Cumulative Review Test**

Chapter Review

Summarizes the key concepts and skills developed within the chapter, and features:

- **Check Your Understanding** questions that help students determine if they understand the big ideas of a chapter
- Additional **Review** and **Practice** questions organized by chapter section
- **Chapter Summary**

Chapter 3 Review

Introducing Functions

Check Your Understanding

1. What is a function? Give an example of a relation that is a function and another example of a relation that is not a function.
2. A function can be described or defined in a number of ways. List these ways and explain how you would use each to determine if a relation is a function.
3. How can you use the graph of a relation to decide if the relation is a function?
4. How do you find the domain and range of a relation?
5. How do you write the domain and the range?
6. How are the rules for solving a linear inequality algebraically different from the rules for solving a linear equation?
7. How do you write the solution to a linear inequality?
8. Describe a non-algebraic method for solving a linear inequality.
9. What is the inverse of a relation?
10. How do you graph the inverse of a relation using the graph of the relation?
11. How do you determine the equation of the inverse from the equation of a relation?
12. Why might $f^{-1}(x)$ be different from the equation of the inverse?
13. Point (a, b) is on the graph of f. Express each of the coordinates of the point using function notation.
14. Why might it be difficult to graph the inverse of a relation on a graphing calculator in function mode (**Func**)? What can you do instead?
15. Compare the domain and range of a relation with the domain and range of the inverse relation.
16. Why may the inverse of a function not be a function? How can you ensure that the inverse of a function is a function?

Chapter Review Test

Designed to help students review the important ideas and concepts of each chapter.

- Questions tie together several expectations from the curriculum.
- Specific questions are identified with Achievement Chart categories.

Chapter 3 Review Test

Introducing Functions

1. **Knowledge and Understanding**
   For each case,
   i. determine $f(5)$
   ii. state the domain and range
   iii. state whether it is a function or not, and justify your answer

   (a)

   x)

   2), (1, 5), (4, 7), (5, −2),

   }

   $3x - 7$

   $\sqrt{x-3}$

   $(x+3)^2 - 5$

   ach inequality.

   each solution set for $x \in \mathbf{R}$.

   $- 2) + 4x \geq -2x$

   $x - 4)$

   $\frac{3(x-5)}{2} \leq x$

3. Determine the equation of the inverse, $f^{-1}(x)$, for each function.
   (a) $f(x) = 5x + 6$
   (b) $f(x) = \sqrt{x+4}$
   (c) $f(x) = x^2 - 5$
4. **Communication:** Will the inverse of a given function also be a function? Clearly explain your reasoning and use an example to support your argument.
5. Explain why the vertical line test is a good test of whether the graph of a relation is a function or not.
6. A relation is defined by $g(x) = -x^2 + 6x - 4$.
   (a) Graph $g(x)$.
   (b) State the domain and range of $g(x)$.
   (c) Determine the equation of $g^{-1}(x)$.
   (d) Graph $g^{-1}(x)$ using the same set of axes used in (a).
   (e) State the domain and range of $g^{-1}(x)$.
   (f) Which relation, $g(x)$ or $g^{-1}(x)$, is a function? Explain.
7. **Application:** A baseball club pays a peanut vendor \$50 per game for selling bags of peanuts for \$2.50 each. The club also pays the vendor a commission of \$0.05 per bag.
   (a) Determine a function that describes the income the vendor makes for each baseball game. Define the variables in your function.

CHAPTER 3 REVIEW TEST

Cumulative Review Test 1

Financial Applications of Sequences and Series

1. A sequence is defined by the general term $t_n = 2n^2 - 4n$.
   (a) Determine the first four terms of the sequence.
   (b) Find t_{15}.
   (c) Graph the first eight terms of the sequence.
   (d) Which term has a value of 880?
2. Martina has won the grand prize in the "Cash Forever" lottery. She will receive \$50 000 the first year and receive an annual increase of \$2000 each year, on top of the \$50 000 annual payment.
   (a) Write the first five terms of the sequence of her annual payments.
   (b) Determine the general term of this sequence.
   (c) Find her annual payment in the tenth year.
   (d) Determine her total winnings over the next 20 years.
3. **U.** Determine the first six terms of the sequence defined by $t_1 = -5$ and $t_n = -3(t_{n-1}) + 8$.
4. **U** Marcus has a bacterial infection and must take 350 mg of medication every six hours. By the time he takes the next dose, 32% of the medication remains in his body.
   (a) Determine a recursive formula that models this situation.
   (b) What will the amount of medication in his body level off to?
   (c) How long will it take for the medication to reach this level?
5. For each sequence, determine
   i. the general term ii. t_{15} iii. S_{25}
   (a) $-18, -54, -162, \ldots$
   (b) $30, 26, 22, \ldots$
6. Determine the number of terms in each sequence.
   (a) $-2, 4, 10, \ldots, 364$
   (b) $3, \frac{3}{2}, \frac{3}{4}, \ldots, \frac{3}{1024}$
7. (a) Determine the amount of a \$12 000 investment at 6%/a compounded quarterly for four years.
   (b) Determine the present value of \$8000 due in 5 years if money is worth 5% compounded semiannually.
8. Evaluate. Round to the nearest hundredth where necessary.
   (a) $25^{\frac{3}{2}}$ (b) $-27^{-\frac{2}{3}}$
   (c) $20^{\frac{4}{5}}$ (d) $200(1.08)^{-10}$
9. Simplify.
   (a) $\dfrac{(25x^6y^4)^{\frac{1}{2}}(9x^4y^2)^{\frac{1}{2}}}{\left(-x^{\frac{1}{2}}y^{\frac{1}{2}}\right)^6}$
   (b) $\dfrac{(49x^{-8}y^2)^{-\frac{1}{2}}(27x^{-3}y^{12})^{\frac{2}{3}}}{\sqrt{16x^8y^{-4}}}$
10. Solve.
    (a) $2^{3x+2} = \frac{1}{32}$ (b) $3^{2x^2+9x} = 243$

CUMULATIVE REVIEW TEST 1

Cumulative Review Test

Cumulative Review Tests are provided at the end of each main strand.

- Intended for student self-assessment, they combine the important ideas and concepts from each strand.
- Cumulative Review Test after Chapter 7 is for Functions and Relations (University) only.

Chapter Features

- Open and inviting design
- Visuals and technical art to promote readability
- Lessons, examples and questions set in context
- Key Ideas
- Lots of Solved Examples
- Wide Range of Exercises
- Check Your Understanding

Key Ideas focus student learning by summarizing and consolidating key concepts, techniques, terms, and formulas introduced in a section. Terms printed in bold are defined in the glossary.

Focus 1.8

Key Ideas

- In this section, interest is added or compounded to the principal before the interest for the next period is calculated. The formula for **compound interest** is

  $A = P(1 + i)^n$

  where A is the amount, the **future value** of an investment, or a loan,
  P is the original principal invested or borrowed,
  i is the interest rate per conversion period, and
  n is the number of conversion periods.

- You can calculate compound interest in several ways:

annually	once per year	i = annual interest rate	n = number of years
semiannually	2 times per year	i = annual interest rate ÷ 2	n = number of years × 2
quarterly	4 times per year	i = annual interest rate ÷ 4	n = number of years × 4
monthly	12 times per year	i = annual interest rate ÷ 12	n = number of years × 12

- The formula for compound interest is similar to the general term of a geometric sequence. Compare $A = P(1 + i)^n$ to $t_n = ar^{n-1}$.
- The amount that must be invested now, P, that will grow to a specific amount in the future is called the **present value**. The formula for present value is

  $P = \frac{A}{(1 + i)^n}$ or $P = A(1 + i)^{-n}$

  where P is the present value,
  A is the amount that the investment will grow to in the future,
  i is the interest rate for each conversion period, and
  n is the total number of conversion periods.

- A time line is useful for solving a compound interest problem, because you may organize information and clarify whether the problem deals with present value or future value (amount).

Example 1

Martina invests $5000 in a savings account that pays 5.25%/a, compounded annually. She does not make another deposit.

(a) Create the geometric sequence of the year-end balances in the account.

(b) Determine the amount in the account after 20 years.

Chapter 1 Patterns of Growth: Sequences

Solved Examples
Each lesson provides three to five solved examples at progressive levels of difficulty. This provides a frame of reference for all types of questions in the Practise, Apply, Solve section. The solutions include step-by-step annotations.

Example 1

Find, if possible, the next four terms of each sequence. If you cannot find the next four terms, then explain why.

(a) The sequence of the number of seats in each of the first four rows in an auditorium, beginning with the first row, is 17, 22, 27, and 32.

(b) The sequence of points in the last four football games that the Hamilton Tiger Cats played is 13, 21, 35, and 17.

Solution

(a) The numbers increase constantly by 5. The next four terms are then 37, 42, 47, and 52.

(b) The number of points scored in any game depends on many factors. Because the terms do not have a predictable pattern, the next four terms of this sequence cannot be determined.

Example 2

Determine the first four terms of each sequence.

(a) $t_n = 3n^2 - 4$ (b) $t_n = 5^{n-1}$ (c) $t_n = \frac{n-2}{n+2}$

Solution

To find the first four terms of each sequence, substitute 1, 2, 3, and 4 for n.

(a) $t_n = 3n^2 - 4$

$t_1 = 3(1)^2 - 4 = -1$ $t_2 = 3(2)^2 - 4 = 8$ $t_3 = 3(3)^2 - 4 = 23$ $t_4 = 3(4)^2 - 4 = 44$

The first four terms are −1, 8, 23, and 44.

(b) $t_n = 5^{n-1}$

$t_1 = 5^{1-1} = 5^0 = 1$ $t_2 = 5^{2-1} = 5^1 = 5$ $t_3 = 5^{3-1} = 5^2 = 25$ $t_4 = 5^{4-1} = 5^3 = 125$

The first four terms are 1, 5, 25, and 125.

(c) $t_n = \frac{n-2}{n+2}$

$t_1 = \frac{1-2}{1+2} = -\frac{1}{3}$ $t_2 = \frac{2-2}{2+2} = \frac{0}{4} = 0$ $t_3 = \frac{3-2}{3+2} = \frac{1}{5}$ $t_4 = \frac{4-2}{4+2} = \frac{2}{6} = \frac{1}{3}$

The first four terms are $-\frac{1}{3}$, 0, $\frac{1}{5}$, and $\frac{1}{3}$.

1.1 Exploring Patterns and Sequences

Example 3

Determine whether each sequence is finite or infinite.

(a) all positive integers divisible by 5

(b) the leap years from 2000 to the end of the twenty-first century

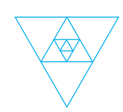
Solution

(a) The sequence is 5, 10, 15, 20, … . You can multiply each term by 5 to get the next term, and you could do this repeatedly, without end. This sequence is then infinite.

(b) The year 2000 is a leap year. Then every fourth year after 2000 is a leap year. The last leap year in the century is 2096. The sequence is 2000, 2004, 2008, 2012, … , 2096. Because the sequence has a last term, it is finite.

Example 4

The area of the largest equilateral triangle in the diagram, triangle 1, is 4096 cm². The second triangle is the result of joining the midpoints of the sides of the first triangle. The third and other triangles were also drawn in this way.



stage 3

(a) To begin a sequence, determine the area of each of the first four triangles.

(b) Find the general term of this sequence.

(c) What is the area of the eighth triangle?

(d) Graph this sequence.

Solution

(a) When you join the midpoints of any equilateral triangle, you divide the triangle into four, congruent, equilateral triangles. Therefore, the area of each of the four triangles is one-quarter the area of the larger triangle. Find the area of *each* smaller triangle by dividing the area of the larger triangle by 4, starting with the area 4096 cm².

Triangle	1	2	3	4
Area (cm²)	4096	$\frac{4096}{4} = 1024$	$\frac{1024}{4} = 256$	$\frac{256}{4} = 64$

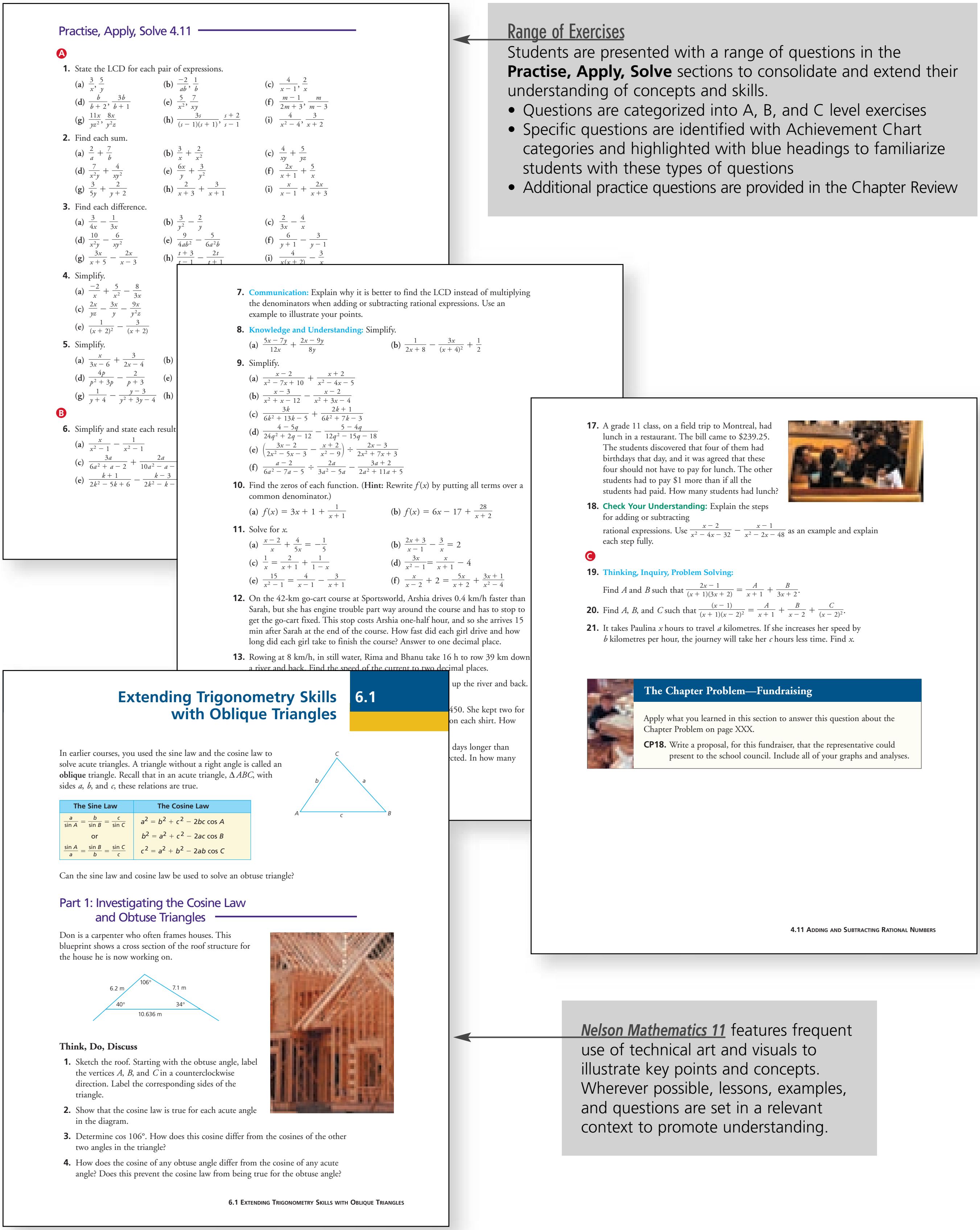
(b) The sequence is 4096, 1024, 256, 64, … . You could also generate each term by dividing the first term, 4096, by a multiple of 4, or a power of 4. Each exponent is 1 less than the position, n, of the term.

$4096, \frac{4096}{4}, \frac{4096}{16}, \frac{4096}{64}, \ldots$

$\frac{4096}{4^0}, \frac{4096}{4^1}, \frac{4096}{4^2}, \frac{4096}{4^3}, \ldots$

The general term is $t_n = \frac{4096}{4^{n-1}}$.

Chapter 1 Patterns of Growth: Sequences



Practise, Apply, Solve 4.11

A

1. State the LCD for each pair of expressions.
(a) $\frac{3}{x}, \frac{5}{y}$ (b) $\frac{-2}{ab}, \frac{1}{b}$ (c) $\frac{4}{x-1}, \frac{2}{x}$
(d) $\frac{b}{b+2}, \frac{3b}{b+1}$ (e) $\frac{5}{x^2}, \frac{7}{xy}$ (f) $\frac{m-1}{2m+3}, \frac{m}{m-3}$
(g) $\frac{11x}{yz^2}, \frac{8x}{y^2z}$ (h) $\frac{3s}{(s-1)(s+1)}, \frac{s+2}{s-1}$ (i) $\frac{4}{x^2-4}, \frac{3}{x+2}$

2. Find each sum.
(a) $\frac{2}{a}+\frac{7}{b}$ (b) $\frac{3}{x}+\frac{2}{x^2}$ (c) $\frac{4}{xy}+\frac{5}{yz}$
(d) $\frac{7}{x^2y}+\frac{4}{xy^2}$ (e) $\frac{6x}{y}+\frac{3}{y^2}$ (f) $\frac{2x}{x+1}+\frac{5}{x}$
(g) $\frac{3}{5y}+\frac{2}{y+2}$ (h) $\frac{2}{x+3}+\frac{3}{x+1}$ (i) $\frac{x}{x-1}+\frac{2x}{x+3}$

3. Find each difference.
(a) $\frac{3}{4x}-\frac{1}{3x}$ (b) $\frac{3}{y^2}-\frac{2}{y}$ (c) $\frac{2}{3x}-\frac{4}{x}$
(d) $\frac{10}{x^2y}-\frac{6}{xy^2}$ (e) $\frac{9}{4ab^2}-\frac{5}{6a^2b}$ (f) $\frac{6}{y+1}-\frac{3}{y-1}$
(g) $\frac{3x}{x+5}-\frac{2x}{x-3}$ (h) ... (i) ...

4. Simplify.
(a) $\frac{-2}{x}+\frac{5}{x^2}-\frac{8}{3x}$
(c) $\frac{2x}{yz}-\frac{3x}{y}-\frac{9x}{y^2z}$
(e) $\frac{1}{(x+2)^2}-\frac{3}{(x+2)}$

5. Simplify.
(a) $\frac{x}{3x-6}+\frac{3}{2x-4}$ (b)
(d) $\frac{4p}{p^2+3p}-\frac{2}{p+3}$ (e)
(g) $\frac{1}{y+4}-\frac{y-3}{y^2+3y-4}$ (h)

B

6. Simplify and state each result
(a) $\frac{x}{x^2-1}-\frac{1}{x^2-1}$
(c) $\frac{3a}{6a^2+a-2}+\frac{2a}{10a^2-a-...}$
(e) $\frac{k+1}{2k^2-5k+6}-\frac{k-3}{2k^2-k...}$

7. Communication: Explain why it is better to find the LCD instead of multiplying the denominators when adding or subtracting rational expressions. Use an example to illustrate your points.

8. Knowledge and Understanding: Simplify.
(a) $\frac{5x-7y}{12x}+\frac{2x-9y}{8y}$ (b) $\frac{1}{2x+8}-\frac{3x}{(x+4)^2}+\frac{1}{2}$

9. Simplify.
(a) $\frac{x-2}{x^2-7x+10}+\frac{x+2}{x^2-4x-5}$
(b) $\frac{x-3}{x^2+x-12}-\frac{x-2}{x^2+3x-4}$
(c) $\frac{3k}{6k^2+13k-5}+\frac{2k+1}{6k^2+7k-3}$
(d) $\frac{4-5q}{24q^2+2q-12}-\frac{5-4q}{12q^2-15q-18}$
(e) $\left(\frac{3x-2}{2x^2-5x-3}-\frac{x+2}{x^2-9}\right)\div\frac{2x-3}{2x^2+7x+3}$
(f) $\frac{a-2}{6a^2-7a-5}\div\frac{2a}{3a^2-5a}-\frac{3a+2}{2a^2+11a+5}$

10. Find the zeros of each function. (**Hint:** Rewrite $f(x)$ by putting all terms over a common denominator.)
(a) $f(x)=3x+1+\frac{1}{x+1}$ (b) $f(x)=6x-17+\frac{28}{x+2}$

11. Solve for x.
(a) $\frac{x-2}{x}+\frac{4}{5x}=-\frac{1}{5}$ (b) $\frac{2x+3}{x-1}-\frac{3}{x}=2$
(c) $\frac{1}{x}=\frac{2}{x+1}+\frac{1}{1-x}$ (d) $\frac{3x}{x^2-1}=\frac{x}{x+1}-4$
(e) $\frac{15}{x^2-1}=\frac{4}{x-1}-\frac{3}{x+1}$ (f) $\frac{x}{x-2}+2=\frac{5x}{x+2}+\frac{3x+1}{x^2-4}$

12. On the 42-km go-cart course at Sportsworld, Arshia drives 0.4 km/h faster than Sarah, but she has engine trouble part way around the course and has to stop to get the go-cart fixed. This stop costs Arshia one-half hour, and so she arrives 15 min after Sarah at the end of the course. How fast did each girl drive and how long did each girl take to finish the course? Answer to one decimal place.

13. Rowing at 8 km/h, in still water, Rima and Bhanu take 16 h to row 39 km down a river and back. Find the speed of the current to two decimal places.

... up the river and back.

...450. She kept two for ... on each shirt. How ...

... days longer than ...cted. In how many ...

17. A grade 11 class, on a field trip to Montreal, had lunch in a restaurant. The bill came to \$239.25. The students discovered that four of them had birthdays that day, and it was agreed that these four should not have to pay for lunch. The other students had to pay \$1 more than if all the students had paid. How many students had lunch?

18. Check Your Understanding: Explain the steps for adding or subtracting rational expressions. Use $\frac{x-2}{x^2-4x-32}-\frac{x-1}{x^2-2x-48}$ as an example and explain each step fully.

C

19. Thinking, Inquiry, Problem Solving:
Find A and B such that $\frac{2x-1}{(x+1)(3x+2)}=\frac{A}{x+1}+\frac{B}{3x+2}$.

20. Find A, B, and C such that $\frac{(x-1)}{(x+1)(x-2)^2}=\frac{A}{x+1}+\frac{B}{x-2}+\frac{C}{(x-2)^2}$.

21. It takes Paulina x hours to travel a kilometres. If she increases her speed by b kilometres per hour, the journey will take her c hours less time. Find x.

The Chapter Problem—Fundraising

Apply what you learned in this section to answer this question about the Chapter Problem on page XXX.

CP18. Write a proposal, for this fundraiser, that the representative could present to the school council. Include all of your graphs and analyses.

4.11 Adding and Subtracting Rational Numbers

Extending Trigonometry Skills with Oblique Triangles 6.1

In earlier courses, you used the sine law and the cosine law to solve acute triangles. A triangle without a right angle is called an **oblique** triangle. Recall that in an acute triangle, ΔABC, with sides a, b, and c, these relations are true.

The Sine Law	The Cosine Law
$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$	$a^2=b^2+c^2-2bc\cos A$
or	$b^2=a^2+c^2-2ac\cos B$
$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$	$c^2=a^2+b^2-2ab\cos C$

Can the sine law and cosine law be used to solve an obtuse triangle?

Part 1: Investigating the Cosine Law and Obtuse Triangles

Don is a carpenter who often frames houses. This blueprint shows a cross section of the roof structure for the house he is now working on.

6.2 m, 7.1 m, 106°, 40°, 34°, 10.636 m

Think, Do, Discuss

1. Sketch the roof. Starting with the obtuse angle, label the vertices A, B, and C in a counterclockwise direction. Label the corresponding sides of the triangle.
2. Show that the cosine law is true for each acute angle in the diagram.
3. Determine cos 106°. How does this cosine differ from the cosines of the other two angles in the triangle?
4. How does the cosine of any obtuse angle differ from the cosine of any acute angle? Does this prevent the cosine law from being true for the obtuse angle?

6.1 Extending Trigonometry Skills with Oblique Triangles

Range of Exercises

Students are presented with a range of questions in the **Practise, Apply, Solve** sections to consolidate and extend their understanding of concepts and skills.

- Questions are categorized into A, B, and C level exercises
- Specific questions are identified with Achievement Chart categories and highlighted with blue headings to familiarize students with these types of questions
- Additional practice questions are provided in the Chapter Review

Nelson Mathematics 11 features frequent use of technical art and visuals to illustrate key points and concepts. Wherever possible, lessons, examples, and questions are set in a relevant context to promote understanding.

Check Your Understanding questions, highlighted by green headings, appear in each Practise, Apply, Solve section (refer to example on page 9 of brochure). These questions will help students determine if they understand the main ideas of each section. A full page of Check Your Understanding questions appears at the end of each chapter in the Chapter Review section (see page 17 of brochure for example).

Varied Instructional Approaches

Recognizing the need to accommodate different teaching and learning styles, *Nelson Mathematics 11* offers a variety of lesson types. The varied instructional approaches provide students the opportunities to explore concepts individually and cooperatively; independently and with teacher direction; through hands-on activities; and through the study of examples followed by practice.

The five main kinds of lessons found in *Nelson Mathematics 11* are:

- Concept Lesson with Think, Do, Discuss
- Concept Lesson
- Skill Builder Lesson
- Technology Lesson
- Exploration Lesson

Concept Lesson

Concepts and ideas are presented to students through the use of Key Ideas, solved examples, and questions from the Practise, Apply, Solve sections.

Functions: Concept and Notation 3.2

In this section, you will develop the concept of a function. You will also learn how to write a function and how to work with functions.

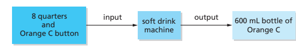
Part 1: The Function

A vending machine behaves like a function. In other words, you get a predictable and reliable output from a vending machine depending on what you put into it.

A soft-drink vending machine allows the customer to put in nickels, dimes, quarters, loonies, or toonies, and then make a choice.

When you input the correct combination of coins and push a button, you expect to get the correct soft drink. The "input" for this function is a number of coins and a button on the machine. The "output" is a soft drink.

A different input might produce the same output. For example:



However, you would not expect the same inputs to produce different outputs. For example, suppose that you input two loonies and pushed the Orange C button. Would you expect to get a bottle of Fizz?

The combination of coins and the soft-drink button used are the **independent variables**. The soft drink received is the **dependent variable**. The machine operates as a **function** because the machine produces a guaranteed output for a specific input. In other words, a unique value of the dependent variable is produced for a specific set of values of the independent variables. The soft drink received is a function of the combination of coins and the soft-drink button pushed.

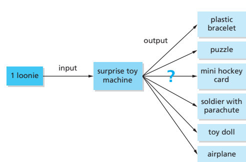
3.2 Functions: Concepts and Notation

A **function** produces a unique value of the dependent variable for each value of the independent variable.

The set of all possible values of the independent variable is called the **domain** of the function. The set of all possible values of the dependent variable is called the **range** of the function.

There are some vending machines that are not functions. One example is a machine that rewards a child with a surprise toy when the child inserts a loonie.

The prize is not a function of the coin inserted in the machine since there are a variety of toys that the child could receive for $1. One value of the independent variable produces more than one value of the dependent variable.



Example 1

For the soft-drink machine,

(a) what is the domain of the function? (b) what is the range of the function?

Solution

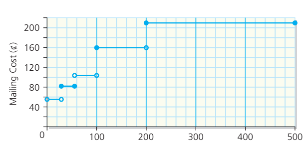
(a) The domain is the set of all possible combinations of coins, which total $2 or $1.25, and a specific button.

(b) The range is the set of all possible types of drinks that the machine contains.

Example 2

The table and the graph show the cost of mailing a first-class letter in Canada in 2001.

Mass of Letter	Mailing Cost
less than 30 g	47¢
30 g to 50 g	73¢
over 50 g but less than 100 g	92¢
over 100 g but less than 200 g	$1.50
200 g to 500 g	$2.00



Chapter 3 Introducing Functions

The first two pages of Section 3.2 Functions: Concepts and Notation. Other lesson pages follow with additional solved examples, Key Ideas and Practise, Apply, Solve questions.

Concept Lesson with Think, Do, Discuss

Concepts and ideas are explored through inquiry and dialogue, with the teacher acting as coach and guiding students to an understanding of concepts and skills. This type of lesson is supported with a summary of Key Ideas, multiple solved examples, and a bank of levelled questions in the Practise, Apply, Solve section.

- Depending on time constraints or personal choice, teachers may choose to teach these lessons using the Key Ideas, solved examples, and practise questions rather than using the Think, Do, Discuss questions

The first three pages of Section 5.3 Trigonometric Functions illustrate the **Think, Do, Discuss** feature. Other pages follow in the lesson with solved examples, Key Ideas summary and Practise, Apply, Solve questions

Trigonometric Functions 5.3

The London Eye is a Ferris wheel on the south bank of the Thames River in London, England. It is 135 m high, as high as a 40-storey building, and can carry a thousand riders at once.

Sarah and her younger brother Billy visited England and rode the London Eye. During the ride, Billy kept asking, "How high are we now?" Trigonometry can be used to model the height of Sarah and Billy at different times.

Part 1: Collecting Data for the Model

Follow these steps to gather data about Sarah and Billy's height as their capsule rotates.

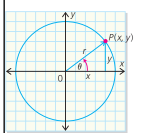
Think, Do, Discuss

1. Draw a circle to model the London Eye. Centre the circle at the origin of the x- and y-axes. At the bottom of the circle, draw a horizontal line to represent the ground.
2. Draw points on the circle to indicate different positions on the ride. Choose points that are easy to estimate, such as points on the x-axis, the y-axis, and the rotation angle halfway between the x- and y-axes in each quadrant.
   (a) How high above the ground will Sarah and Billy be when they are at the top of the ride? halfway to the top? Show these points on the diagram. What are the coordinates of each point? How can you use each coordinate to determine the height above the ground?
   (b) What acute angle corresponds to halfway between each quadrant? How many points on the ride will this angle help determine?
   (c) Draw right triangles to the circle that include each related acute angle.
   (d) How long is the hypotenuse of each right triangle? Explain your answer.
   (e) What trigonometric ratio combines the acute angle and the hypotenuse to give a second side of the triangle that represents vertical distance?
   (f) How could you use this second side length to determine Sarah and Billy's location on the London Eye with respect to the x-axis? How could you use this second side length to find their height above the ground?
   (g) How many different positions on the Ferris wheel have you represented on your diagram? What principal angles in standard position do these positions represent? By using the angles in standard position, what assumption have you made about where Billy and Sarah get on the ride?
   (h) Determine the height above the ground, to the nearest decimal, for each angle in standard position. Create a table in your notebook to record the data.

Angle of Rotation (degrees)									
Height Above Ground (m)									

Part 2: Extending the Primary Trigonometric Ratios Beyond Triangles

Suppose Sarah and Billy were on any Ferris wheel of radius r and their location on the wheel is $P(x, y)$, as shown. θ represents the angle of rotation.



In quadrant I, the related acute angle and the angle in standard position are the same. In the right triangle for angle θ, the opposite side has length y and the adjacent side has length x. r is the hypotenuse of the right triangle, where $r = \sqrt{x^2 + y^2}$. The length r is always positive and in quadrant I, x and y are also positive.

Then,

$$\sin\theta = \frac{\text{OPP}}{\text{HYP}} = \frac{y}{r} \qquad \cos\theta = \frac{\text{ADJ}}{\text{HYP}} = \frac{x}{r} \qquad \tan\theta = \frac{\text{OPP}}{\text{ADJ}} = \frac{y}{x}$$

In general, any point, $P(x, y)$, on a circle of radius r can be expressed as a function of θ. In other words, as θ changes, corresponding changes occur in x and y, both in magnitude and sign.

Graphing the Sine and Cosine Functions

For any point $P(x, y)$ on the terminal arm of an angle θ in standard position, the trigonometric functions are defined as follows:

$$\sin\theta = \frac{y}{r},\ \cos\theta = \frac{x}{r},\ \text{and } \tan\theta = \frac{y}{x},\ x \neq 0, \text{ where } r = \sqrt{x^2 + y^2}$$

Help Sarah determine her position anywhere on a Ferris wheel with respect to the centre of the wheel.

APTER 5 MODELLING PERIODIC FUNCTIONS

Think, Do, Discuss

1. (a) Which of the primary trigonometric functions best links the distance of a rider on the Ferris wheel from the horizontal diameter of the wheel, the size of the Ferris wheel, and the angle of rotation? Solve the equation for this distance in terms of the other two variables.
   (b) Complete the table to show this distance for the London Eye as a function of angular rotation. Use increments of 30° for one complete revolution.

Angle of Rotation	0°	30°	60°	90°	120°	…	…
Distance on Ferris Wheel from the Horizontal Diameter of the Wheel							

   (c) Graph the distance from the horizontal diameter versus the angle of rotation.
   (d) What is the greatest distance above the horizontal diameter? below the horizontal diameter? How can these values be used to determine the amplitude of the curve?
   (e) What is the period of the curve?
   (f) How would the graph look for two revolutions of the Ferris wheel? What type of model would describe the data? What equation models the distance from the horizontal diameter for any angle of rotation for the Ferris wheel?
   (g) What is Sarah and Billy's distance from the horizontal diameter when the ride has rotated 130°? 280°? 420°? 550°? What is their height above the ground for each of these positions of the Ferris wheel if it almost touches the ground?
   (h) Which coordinate is used to represent distance from the horizontal diameter?
   (i) Write an equation to model their height above the ground for any angle of rotation of the Ferris wheel.
2. (a) Which primary trigonometric function best links the distance of a rider from the vertical diameter of the Ferris wheel, the size of the wheel, and the angle of rotation? Solve the equation for this distance in terms of the other two variables.
   (b) Complete a table as in step 1(b) to show distance on the London Eye from the vertical diameter of the wheel for the given angles of rotation of the ride.
   (c) Graph the distance from the vertical diameter versus the angle of rotation.
   (d) What is the greatest horizontal distance to the right of the vertical diameter? to the left of the vertical diameter? How do these distances compare to those in step 1 (d)? Explain. What is the amplitude of the curve?
   (e) What is the period of the curve?
   (f) Extend the graph to represent two revolutions of the Ferris wheel.
   (g) What is Sarah and Billy's distance from the vertical diameter when the ride has rotated 130°? 280°? 420°? 550°?

5.3 TRIGONOMETRIC FUNCTIONS

SKILL BUILDER 4.1

Extending Algebra Skills: Completing the Square

The same relationship can be expressed in several different algebraic forms. You have seen that quadratic relations can be written several ways:

$y = -2x^2 + 16x - 24$ (standard form) $y = -2(x - 6)(x - 2)$ (factored form) $y = -2(x - 4)^2 + 8$ (vertex form)

Each form provides different information about the relationship and its graph, you must be able to move algebraically from one form to another.

You have already completed the square to express a quadratic function in vertex form; however, when you work with real data, the coefficients in the model are often not integers. You will need to use rational numbers to complete the square in these cases.

Example 1

Put $f(x) = -3x^2 + 13x - 11$ in **vertex form**, $f(x) = a(x - h)^2 + k$, by completing the square. State the coordinates of the vertex.

Solution

Complete the square.

$f(x) = -3x^2 + 13x - 11$ — Factor the coefficient of x^2 from the first two terms.

$f(x) = -3(x^2 - \frac{13}{3}x) - 11$ — Add and subtract the square of half the coefficient of x inside the brackets.

$f(x) = -3\left[x^2 - \frac{13}{3}x + \left(-\frac{13}{6}\right)^2 - \left(-\frac{13}{6}\right)^2\right] - 11$ — The first three terms inside the square brackets form a perfect square. Multiply the fourth term by the coefficient of x^2. Then move this term outside the brackets.

$f(x) = -3\left[x^2 - \frac{13}{3}x + \left(-\frac{13}{6}\right)^2\right] - (-3)\left(-\frac{13}{6}\right)^2 - 11$ — Factor the perfect square.

$f(x) = -3\left(x - \frac{13}{6}\right)^2 + \frac{169}{12} - 11$ — Simplify.

$f(x) = -3\left(x - \frac{13}{6}\right)^2 + \frac{37}{12}$

The vertex form of $f(x) = -3x^2 + 13x - 11$ is $f(x) = -3\left(x - \frac{13}{6}\right)^2 + \frac{37}{12}$.

The vertex is $\left(\frac{13}{6}, \frac{37}{12}\right)$.

Example 2

Find the coordinates of the vertex for the graph of $f(x) = -0.16x^2 + 9.76x - 9.408$ by completing the square.

Solution

$f(x) = -0.16x^2 + 9.76x - 9.408$ — Factor the coefficient of x^2 from the first two terms.

$f(x) = -0.16(x^2 - 61x) - 9.408$ — Add and subtract the square of half the coefficient of x inside the bracke

$f(x) = -0.16[x^2 - 61x + (-30.5)^2 - (-30.5)^2] - 9.408$ — Group the three terms that form the perfect square, then multiplied by -0.16.

$f(x) = -0.16[x^2 - 61x + (-30.5)^2] - (-0.16)(-30.5)^2 - 9.408$ — Factor.

$f(x) = -0.16(x - 30.5)^2 + 148.84 - 9.408$ — Simplify.

$f(x) = -0.16(x - 30.5)^2 + 139.432$

The coordinates of the vertex are (30.5, 139.432).

Focus 4.1

Key Ideas

- The same relationship can take several different algebraic forms.
- Transform the equation of a quadratic function in standard form, $f(x) = ax^2 + bx + c$ to vertex form, $f(x) = a(x - h)^2 + k$, by **completing the square**.

$f(x) = ax^2 + bx + c$ — Factor the coefficient of x^2 from the first two terms.

$f(x) = a\left(x^2 + \frac{b}{a}x\right) + c$ — Add and subtract the square of half the coefficient of x inside the bracke

$f(x) = a\left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right] + c$ — Group the three terms that form the perfect square. Multiply the fourth term by a and move it outside the brackets.

$f(x) = a\left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right] - a\left(\frac{b}{2a}\right)^2 + c$ — Factor the perfect square and simplif

$f(x) = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$

4.1 EXTENDING ALGEBRA SKILLS: COMPL

Practise, Apply, Solve 4.1

A

1. Factor the coefficient of x^2 from each expression.
   (a) $2x^2 - 6x$ (b) $3x^2 + 2x$ (c) $-4x^2 + 2x$
   (d) $\frac{1}{2}x^2 - 5x$ (e) $-1.2x^2 - 6.6x$ (f) $\frac{2}{3}x^2 + \frac{5}{6}x$
   (g) $-\frac{4}{3}x^2 - 2x$ (h) $2.7x^2 - 0.9x$ (i) $-6x^2 + \frac{3}{5}x$
2. Find the value of d that makes each expression a perfect square.
   (a) $x^2 + 3x + d$ (b) $x^2 - 7x + d$ (c) $x^2 + \frac{2}{3}x + d$
   (d) $x^2 - \frac{3}{5}x + d$ (e) $x^2 - 2.5x + d$ (f) $x^2 + 0.45x + d$
   (g) $x^2 - 22.4x + d$ (h) $x^2 - \frac{5}{9}x + d$ (i) $x^2 + \frac{x}{100} + d$
3. Express each function in vertex form by completing the square.
   (a) $f(x) = x^2 - 4x + 3$ (b) $f(x) = x^2 + 6x - 8$
   (c) $f(x) = 3x^2 - 6x + 1$ (d) $f(x) = x^2 - x$
   (e) $f(x) = x^2 + 3x + 5$ (f) $f(x) = -x^2 + 5x - 2$
   (g) $f(x) = x^2 - 2.6x - 2.7$ (h) $f(x) = x^2 + 5.4x - 1.8$
   (i) $f(x) = x^2 - 3.4x - 2.8$
4. Express $g(x) = 2x^2 + 5x + 4$ in vertex form. Explain your steps.

B

5. Express each equation in vertex form by completing the square.
   (a) $f(x) = x^2 - 3x + 2$ (b) $f(x) = 2x^2 + 5x - 4$
   (c) $f(x) = 3x^2 - 4x + 1$ (d) $f(x) = 5x^2 - x - 5$
   (e) $f(x) = -2x^2 + 3x + 5$ (f) $f(x) = -4x^2 - 7x - 2$
   (g) $f(x) = 1.4x^2 - 4.9x - 2.7$ (h) $f(x) = -3.6x^2 + 5.4x - 1.67$
   (i) $f(x) = \frac{1}{2}x^2 - 4x + 1$ (j) $f(x) = \frac{2}{3}x^2 - 5x - 2$
   (k) $f(x) = -\frac{3}{2}x^2 - \frac{7}{8}x + \frac{1}{2}$ (l) $f(x) = -\frac{4}{5}x^2 + \frac{2}{3}x - \frac{3}{10}$
6. i. Find the coordinates of the vertex by completing the square.
   ii. State the domain and the range of $f(x)$.
   iii. Sketch the graph for each quadratic function.
   (a) $f(x) = x^2 + 10x + 24$ (b) $f(x) = 4x^2 + 8x + 7$
   (c) $f(x) = -2x^2 + 5x - \frac{1}{8}$ (d) $f(x) = \frac{1}{2}x^2 - 3x + 4$
   (e) $f(x) = -2x^2 + 7x - 12$ (f) $f(x) = -2x^2 + 7x$

CHAPTER 4 QUADRATIC AND RATIONAL EXPRESSIONS

Skill Builder Lesson

As the name implies, Skill Builder lessons focus on the development of mathematical skills. Skills are presented through the use of solved examples, and then consolidated through the use of Key Ideas and Practise, Apply, Solve sections.

The first three pages of a Skill Builder Lesson from Section 4.1 Extending Algebra Skills: Completing the Square.

Technology Lesson

Technology Lessons develop skills with the use of technology, including graphing calculators (TI-83 Plus), digital probes, spreadsheets, and graphing software. Detailed instructions and screen captures are provided to lead students through these activities.

- Technology Lessons are denoted with a green side bar and a calculator icon in the section box.

Technology Lesson Section 1.2 TI-83 Plus Calculator: Generating the Terms of a Sequence.

TI-83 Plus Calculator: Generating the Terms of a Sequence

1.2

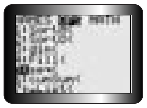
TECHNOLOGY

On the TI-83 Plus calculator, you can create a list from the general term that displays the specific terms of the sequence.

Generate or list the first five terms of the sequence defined by $t_n = n^2$.

1. Select sequence from the List OPS menu.

Press [2nd] [STAT] [▸]. Scroll down to sequence and press [ENTER].



step 1

2. Enter the information for sequence.

You will need to enter the following:

- the expression of the general term
- the variable n — let [X,T,Θ,n] represent n
- the first position number
- the last position number
- the increment — the increment is 1, because the difference between each pair of consecutive natural numbers is always 1

Press [X,T,Θ,n] [x²] [,] [X,T,Θ,n] [,] [1] [,] [5] [,] [1] [)].

step 2

3. Generate the first five terms of the sequence.

Press [ENTER].

step 3

Practice 1.2

List the first ten terms of each sequence.

(a) $t_n = 2n + 4$ (b) $t_n = n^3$ (c) $t_n = \frac{n+2}{n}$ (d) $t_n = \sqrt{n}$

(e) $t_n = -5n - 2$ (f) $t_n = (0.5)^n$ (g) $t_n = 2(3)^n$ (h) $t_n = n^n$

Exploration Lesson

Often used to introduce a new idea or explore a concept, Exploration Lessons provide students with an opportunity to investigate ideas and relationships through a hands-on experience.

- Exploration Lessons are denoted with a purple side bar and are optional

The first two pages of Exploration Lesson Section 7.7 Reflective Properties of Conics is shown.

Reflective Properties of Conics

7.7U

Functions and Relations

EXPLORATION

This chapter has referred to satellites, conical reflectors in flashlights and headlights, and other reflection devices. But why are these shapes conics? You will need to …e the reflective properties of conics to understand their significance.

… easily create a model of a parabolic reflector. Sketch the graph of $x^2 = 8y$ on …heet of graph paper and mark the focus. (You could choose any other …c equation, but make sure the graph will fit on the paper.) Bend a thin strip …oard and place it accurately and securely along the parabola. Carefully roll a …toward the curve making sure to roll the marble in a direction that is parallel …-axis. Roll the marble several times, starting from different points, but always …marble in a direction that is parallel to the y-axis.

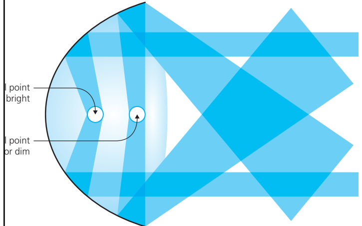
… Do, Discuss

…es the marble's path cross the focus? Explain.

…aw a diagram of this model. Using the diagram, suggest and demonstrate why …marble's path crosses the focus.

…e reflective properties of the parabola appear in the design of flashlights, car …dlights, and satellite dishes. Can you find other instances of parabolic …ectors?

…ou flick your headlights from bright to dim, think about conics. The bright …created by the light source at the focal point; its rays are parallel to the …'s axis of symmetry. When you dim the headlights, the light source changes …and the rays point down and up. The rays pointing up are shielded—only …pointing down are reflected out.

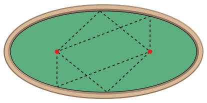


EXPLORATION

4. Try to create a similar model of an ellipse. This time, start with one marble at one focal point and "shoot" it toward any point on the ellipse. Shoot the marble from this focal point at several different points along the ellipse. What do you notice?

5. Draw a diagram of this model. Discuss why you think that, if you were to roll another marble from the other focal point, then the paths of the marbles should cross.

Imagine an elliptical pool table with only one pocket located at one of the foci. If you place a ball at the other focus and roll it toward the cushion of the pool table, the ball will always be reflected to the pocket.



If a tangent is drawn to the curve at the point where the ball strikes the cushion, the angle at which the ball strikes the cushion is equal to the angle at which it rebounds from the cushion. In this way, the ball will always roll toward the pocket.

6. If you wanted to build an elliptical pool table with a length of 2 m and a width of 1.8 m, then where would you place the "pocket"?

Did You Know?

A "whispering gallery" is another instance of the reflective properties of an ellipse. A whispering gallery is a large room where the walls of the gallery, or the dome above it, are in the shape of an ellipse. If a person stands at one focal point of the ellipse and whispers, then someone standing at the other focal point can hear the whisper. There are many whispering galleries in the world. The ceiling in St. Paul's Cathedral in London, England is a great dome, 34 m in diameter. The dome's shape is an ellipse. If someone, facing the wall of the cathedral's whispering gallery, whispers, then the whisper is clearly audible on the opposite side of the cathedral.

One room in the National Statuary Hall in Washington, D.C., was unintentionally constructed as a whispering gallery in the nineteenth century. However, the room was unsuitable for meetings because of the echoes and was reconstructed.

The room where the city council meets in Coquitlam, British Columbia, has an elliptical dome ceiling. This room was constructed so that it would not be a whispering gallery.