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WHY DO WE TEACH PATTERNING AND ALGEBRA?

Mathematics has been called “the science of patterns” (Steen, 1988). Young children enjoy working with patterns, and older students enjoy discovering and manipulating patterns. In fact, it is human nature to find patterns in our everyday experiences. Some educators and mathematicians would go so far as to say that patterning is the foundation of mathematics (Lee, 1996; Mason, 1996). The study of linear growing patterns offers a tangible way for students to think about relationships between quantities. The National Council of Teachers of Mathematics (NCTM) recommends that students participate in patterning activities from a young age, with the expectation that they will be able to “make generalizations about geometric and numeric patterns, provide justifications for their conjectures, and represent patterns and functions in words, tables, and graphs” (NCTM, 2000). Similar expectations can be found in curriculum documents from Ministries of Education across Canada.

Patterning activities are introduced in elementary school so that students can think about relationships between quantities early in their math education, which is intended to help them transition to formal algebra in middle school and high school (Blanton & Kaput, 2004; Carpenter, Franke, & Levi, 2003; Greenes, Chang, & Ben-Chaim, 2007; Kieran, 1990, 1991, 1992; Kieran & Chalouh, 1993; Warren & Cooper, 2006).

How does working with patterns actually support algebraic thinking?

This book explores a research-based developmental approach that takes into account students’ inherent capabilities to think algebraically, which can lead to powerful algebraic understanding. For example:

• We look at some of the reasons why it is important to teach patterning and how working with patterns in a meaningful way can lead to sophisticated algebraic thinking.
• We explore a particular kind of pattern, called a linear growing pattern.
• We review approaches to teaching linear growing patterns that may tend to limit students’ thinking.
• We provide an overview of our lesson sequence, which mathematics educators and researchers have determined to be a comprehensive approach for addressing the topics of patterning and algebraic thinking.
HOW DID WE DEVELOP THE LESSON SEQUENCE?

The main focus of our five-year research study was to develop and assess a new lesson sequence that supports students’ understanding of linear relationships. During the course of our study, we developed an instructional framework to encourage specific kinds of learning. We analyzed students’ thinking, the lesson sequence, and the instructional framework. In each year of our study, we modified the lessons based on how well they were (or were not) helping students learn about linear relationships. The result is a comprehensive series of research-based lessons that are grounded in students’ thinking and have been proven to be effective in a variety of classroom settings (Beatty, 2007, 2010a, 2010b; Beatty & Bruce, 2011; Beatty & Moss, 2006; Moss & Beatty, 2006, 2010).

The goal of the lesson sequence is to allow students to work with linear pattern rules in a variety of different hands-on contexts. Students move from concrete contexts (building patterns with square tiles and position cards) to more abstract representations (graphs and story problems). Finally, students make connections between graphs and more formal algebraic expressions and equations. Thus, the lesson sequence is designed to scaffold students’ understanding of increasingly complex rules and, eventually, to consider and evaluate multiple rules.

Another goal of the lesson sequence is to help teachers introduce concepts related to linear relationships as students develop their mathematics skills. For example, the earliest lessons are appropriate for younger students (Grades 4 to 6) and for students who have not experienced the activities presented or need more practice. Similarly, the final lessons are appropriate for older students (Grades 7 to 10), since the concepts reflect the curriculum outcomes for secondary and high school mathematics. Each lesson includes grade recommendations, and the Curriculum Correlation (available on the Nelson Education website) provides more detailed information about how the lessons support various curricula.

A third goal of the lesson sequence is to foster mathematical discourse, as discussed in How Can You Develop a Successful Math-Talk Community? (page 16). Students are encouraged to discuss different ways of thinking about the concepts and rules, and to identify and describe the strategies they use for problem solving. Although the activities are carefully sequenced, the tasks themselves are open-ended and allow students to investigate a variety of strategies and solutions. These open-ended tasks offer extensive opportunities for rich mathematical discussion.

The Importance of Multiple Representations

When learning a complex concept, such as linear relationships, it is crucial that students have an opportunity to explore multiple representations of this concept. In particular, students need to explore the interactions among representations to learn how changes to one or more representations can affect other representations. Throughout the lesson sequence, students make connections among pattern rules, pictorial and geometric patterns, graphs, context problems (story problems), and equations as a way of developing a deep foundational understanding of linear relationships.
What Makes These Lessons Different

The lesson sequence is based on our analysis of how students think about linear relationships. The result is a series of research-based and field-tested lessons that make sense to students and teachers. Our approach departs from other approaches to teaching linear relationships in five ways:

1. **Concrete and visual representations of linear relationships are emphasized.**
   Students engage in many activities that use concrete representations (linear growing patterns built with tiles) and visual representations (pictorial representations and graphs). These representations help students visualize different aspects of linear relationships, which are not obvious when working with only numeric representations.

2. **Students physically manipulate patterns using tiles and position cards.**
   Students are given ownership of the mathematics they are learning as they explore, conjecture, and extend their thinking through building patterns. This physical engagement with complex ideas is integral to developing a deep understanding.

3. **Graphing is introduced through patterning.**
   Most activities that involve graphing linear relationships are not connected to pattern building. In this lesson sequence, all representations (pattern rules, patterns, graphs, story problems, and equations) are integrated so that students can make connections and see the interactions among different representations.

4. **Patterning and graphing are used to teach formal algebraic notation.**
   Instead of relying on memorization, students first develop an understanding of linear relationships and then develop an understanding of how these relationships are represented by algebraic expressions and equations. Equations become meaningful, not memorized.

5. **Negative numbers are explored with graphical representations.**
   By grounding operations with negative numbers in the context of graphing, students explore how the sign of a number (positive or negative) influences the behaviour of a trend line on a graph. Once students make this connection, they can derive meaning for arithmetic operations (addition, subtraction, and multiplication).

Teaching Implications for All Students

The sequential nature of the tasks and the open-endedness of the activities provide all students, including struggling learners, access to aspects of algebraic thinking that are normally challenging. One of the most striking findings of our research with the lesson sequence was the degree to which teachers were amazed and delighted by the capabilities of their lowest-achieving students. This is true for students who struggle within the classroom, and for students who have identified learning needs and are typically withdrawn from the classroom for mathematics instruction. In every lesson, we include some suggested accommodations (Differentiation Opportunities) to ensure that all students can meaningfully engage with the mathematical concepts.
WHAT ARE THE COMPONENTS OF EACH LESSON?

Each of the 22 lessons follows the same format and includes the same components. These components have been field-tested with teachers across Canada, from Grades 4 through 10, and have been revised based on feedback from both teachers and students. Each component of the lesson serves a specific purpose for helping you prepare for and teach the lesson. In addition to the activities, the lessons are supported by culminating tasks at the end of most chapters. These tasks allow students to demonstrate their understanding of what they learned and apply their knowledge in an assessment context.

Each lesson includes sample transcripts and teachers’ experiences to show how it can be implemented in the classroom. Each lesson also includes examples of students’ responses and teachers’ facilitation. These features help give teachers a sense of how the lesson might play out in their own classroom.

<table>
<thead>
<tr>
<th>Component</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>Representations</td>
<td>This component, at the beginning of each lesson, indicates the type of representation that students explore (Guess My Rule, Patterns, Graphs, or Story Problems).</td>
</tr>
<tr>
<td>Recommended Grades</td>
<td>The lessons build sequentially to teach the essential concepts of linear relationships, and each lesson can support various grades. Earlier lessons are suitable for introducing younger students to the concepts or as review for older students.</td>
</tr>
<tr>
<td>Enduring Understanding</td>
<td>Enduring Understanding identifies the key concept that is explored in the lesson. The key concept is the deep learning that students will take away from the lesson.</td>
</tr>
<tr>
<td>Key Math Language</td>
<td>These terms or phrases support the precise use of mathematics language in the classroom. Displaying and referring to the Key Math Language in the classroom, as the lesson develops, helps students better understand the use of these terms in the specific context of the lesson and the larger context of their general understanding of linear relationships and algebra.</td>
</tr>
<tr>
<td>Lesson Overview</td>
<td>The Lesson Overview gives you an overview of the main mathematics concepts that are explored in the lesson. It reminds you how the lesson fits in the sequence and provides a brief rationale that explains the importance of the lesson for students’ learning.</td>
</tr>
<tr>
<td>Teaching the Lesson</td>
<td>Each lesson uses a three-part problem-based approach that includes the following features:</td>
</tr>
<tr>
<td></td>
<td>• an Activation activity, which reminds students what they learned in the previous lesson and introduces the concept that they will explore in the current lesson</td>
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<tr>
<td></td>
<td>• a Development activity, in which students engage in activities to further explore the lesson concept</td>
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<tr>
<td></td>
<td>• a Consolidation activity, which allows students to share their thinking, including their conjectures and questions, and gives you the opportunity to solidify and highlight the learning that has taken place</td>
</tr>
</tbody>
</table>
**Key Questions**

The Key Questions address the overarching ideas that are critical to the lesson. They help you facilitate students’ learning by focusing their attention on specific concepts, and thus are instrumental in encouraging rich math-talk in the classroom. Displaying the Key Questions during the lesson reminds you and students what you need to ask one another.

**Differentiation Opportunities**

Our research has shown that these lessons are effective with students at all levels. Differentiation Opportunities address some common issues that arose during our field tests and provide suggestions to support students’ learning. They are based on ideas that teachers successfully used in their classroom to individualize the lessons for their students.

**Assessment Strategies**

Assessment Strategies include suggestions for diagnostic and formative assessment (for student learning) and summative assessment (of student learning). The type of assessment that is emphasized depends on the content of the lesson and where it fits in the lesson sequence.

**Possible Lesson Extensions**

In each lesson, we share a variety of possible extensions that maintain the sequence of the lesson but add instructional flair. Our hope is that these extensions will inspire you to adapt the delivery of the lesson to meet the needs of your classroom and students. The extensions incorporate some of the most engaging activities that other teachers have used in their classrooms to extend student thinking.

**Examples of Students’ Thinking**

Each lesson includes examples of the wide variety of thinking that takes place during the open-ended activity. These examples represent the type of thinking that occurs with each lesson, which will help you anticipate possible responses from students and encourage them to articulate and discuss their understanding.

**Teachers’ Experiences**

These examples provide additional insights into issues that might arise and how you can address them. They also illustrate what other teachers have discovered about their students’ learning, as well as the math concepts, as they implemented the lessons.

Many of the approximately 50 teachers we worked with during our research and field tests commented on the effectiveness of the lessons. One Grade 6 teacher noted:

The biggest difference for me was seeing kids who are normally petrified of math and not terribly successful, and who believe that they can’t do it, leading the discussion. One of my self-proclaimed weak math students got the concept and was questioning typically stronger students in class about their patterns and explaining why their patterns were not growing patterns. He’s the one who double-checks around him to make sure his answer is right before his hand goes up. It’s a big class with lots of learning needs and they’re all there—they all get it!
CHAPTER 1

Guess My Rule Game and Linear Growing Patterns

EXPLORING RELATIONSHIPS IN PATTERNS

The first four lessons of the sequence build a foundation for students’ understanding of the mathematical relationships in patterns. Through hands-on practice and classroom discussions, students learn how to solve problems about patterning and then invent meaningful procedures based on what they have learned. This approach helps students see why the procedures work, which gives them a deep understanding of both the concepts and procedures.

Lessons 1 to 4 can be used with students in Grades 4 through 10. In Grades 4 to 6, these lessons can be used to form the basis of the patterning and algebra unit. In Grades 7 to 10, these lessons can be used to review concepts and assess students’ understanding. The four lessons in Chapter 1 have also been used successfully as remediation for older students who have not yet grasped the fundamentals of multiplication or linear relationships.

IMPROVING MULTIPLICATIVE THINKING

Multiplicative thinking is an overarching concept that is found in many areas of mathematics, including multiplication, ratios, algebra, and geometry. Multiplicative thinking goes beyond knowing multiplication facts, to a deeper understanding and the ability to unitize, which is explored in Lesson 2. Our studies have shown that playing the Guess My Rule game (Lessons 1 and 3) and building linear growing patterns (Lessons 2 and 4) helps to foster multiplicative thinking.

In Lessons 1 and 3, students play the Guess My Rule game, in which they determine the mathematical rule that generates output numbers for randomly suggested input numbers. This game strengthens their number sense and their facility with multiplication facts. During Lessons 2 and 4, students build patterns that allow them to see the relationship between the position number and the number of repetitions of the pattern core, which is the core element or unit of the pattern. The number of repetitions of the pattern core is determined by the position number (Figure 1.1).
MULTIPLE REPRESENTATIONS

Exploring multiple representations of linear relationships helps students develop a deep conceptual understanding. In the first four lessons, students explore how linear rules connect input values and output values, as well as position numbers and numbers of tiles. These algebraic connections are explained in detail in each lesson, in the section called What Mathematics Are Students Learning? Allowing students to work with both numeric and visual representations is a powerful way to help them develop their understanding of algebraic relationships.

GENERALIZING

In our experience, most patterning activities support recursive thinking, or thinking about the previous position or number in a pattern and then adding to it (see page 4). Although this approach allows students to determine the next few positions in a pattern, it becomes very cumbersome when trying to predict the number of tiles for the 100th or 356th position. Students who use this approach may have difficulty determining a rule that will enable them to predict the number of tiles for any position in the pattern (generalization).

Our approach to the Guess My Rule game does not support recursive thinking. Students volunteer random input numbers, which results in a non-ordered table of values. To figure out the rule, students must look across the columns to find the relationship between the input numbers (independent variable) and the output numbers (dependent variable). This sets the stage for the activity in Lesson 2, in which students consider the relationship between the position numbers (independent variable) and the numbers of tiles (dependent variable) in a linear growing pattern.

Through the patterning activities, students look for the pattern rule that will give them the number of tiles for any position number. This is known as explicit reasoning, because students are identifying the explicit rule (or mathematical structure) that underlies the linear growing pattern. Rather than knowing the pattern “grows by 3 each time,” students begin to understand that the number of tiles represents the pattern rule as it applies to each position number. By applying explicit reasoning, students can predict the number of tiles for any position number of the pattern. They can also accurately predict what any position of the pattern will look like.

Going Slowly

Throughout the lesson sequence, students explore complex concepts that lead to incredibly sophisticated mathematical understanding. It is important to give numerous opportunities for students to practise thinking, conjecturing, and justifying. This ensures that students build a solid foundation not only for algebraic reasoning, but also for multiplicative thinking, mathematical discussion, data management, and other complex mathematical concepts that they will use as they advance through high school.
In this lesson, the two different components of a pattern rule (the multiplier and the constant) are represented using tiles of two different colours. For example, Figure 1.10 represents the composite rule “number of tiles = position number × 5 + 3.” In this example, the multiplier (5) is represented by the red tiles, which increase by 5 at each successive position number. In other words, the number of red tiles at each position is equal to the position number multiplied by 5. The constant (3) is represented by the yellow tiles that stay the same at each position. Notice that the red tiles at the first position form the pattern core. If we isolate the multiplication component of the rule, there is only one pattern core (1 × 5) in the first position. There are two repetitions of the pattern core (2 × 5) in the second position. The number of repetitions of the pattern core corresponds to the position number.

What Mathematics Are Students Learning?

In Lesson 4, students learn that numeric rules for composite (two-step) growing patterns can be represented visually. Students also learn that the arrangement of the two colours of tiles shows predictable growth, so the pattern core is continued (and amplified) in each consecutive position. Using coloured tiles to create a concrete representation of the linear growing pattern helps students make mathematical connections to the Guess My Rule game that they played in Lesson 3.

When students describe their pattern rule using the composite rule statement, they are laying the foundation to connect this language to the language of standard algebraic notation (which is introduced in Lesson 20). The underlying structure of the composite rule statement and standard algebraic notation is almost identical:

“The number of tiles is equal to the position number times _____ plus _____.”

\[ y = mx + b \]

TEACHING THE LESSON

During the Development section of the lesson, students create three different patterns for their one rule. There are several reasons for doing this:

- It shows students that they can represent the same underlying rule in a variety of ways (multiple tile arrangements for one rule). It is part of the process of developing the ability to take a generalized pattern rule and represent the rule with a linear growing pattern.
- By the third pattern, students usually start thinking about how to build a pattern beyond the usual arrays, while maintaining predictable growth. They realize that their pattern must be consistent in every position to be a linear growing pattern.
Creating different patterns encourages flexibility and emphasizes creativity, which are important to students’ progress in mathematical thinking. By asking students to create the patterns themselves, they develop a sense of what part of the pattern is growing (multiplicative thinking) and what part is staying the same. Students develop a great deal of ownership for their mathematical thinking through their creation of unique (and often beautiful) patterns.

**Activation**

1. Explain that students are going to figure out the pattern rule for some patterns. Make sure that all the students can clearly see the pattern building by using magnetic tiles to build the pattern on the board, using a document camera, or using virtual tiles on an interactive whiteboard.

2. Use tiles and position cards (BLM 3) to build the first three positions in the rocket pattern (Figure 1.10). Scaffold students’ understanding of what is growing (red tiles) and what is staying the same (yellow tiles) by building the pattern as follows:
   - For the first position, place 5 red tiles and then 3 yellow tiles.
   - For the second position, place 10 red tiles and then 3 yellow tiles.
   - For the third position, place 15 red tiles and then 3 yellow tiles.

3. Once you have built the pattern, ask the following Key Question:
   - How many tiles would you need for position 4 in the pattern (or for a rocket with 4 astronauts)? For position 10? For position 15?
   - How many tiles would you need for the 100th position in the pattern (or for a rocket with 100 astronauts)? What would it look like?

4. Encourage students to look at the pattern in different positions to help them visualize and describe it accurately. Using this arrangement of tiles helps students articulate “groups of tiles” at each position. For example, at position 20 there would be 20 groups of 5 red tiles and 3 yellow tiles. Students may also articulate that the pattern is growing “by 5 red tiles each time” and that the 3 yellow tiles “stay the same.” Help students focus on groupings of tiles to develop their multiplicative thinking.
5. Ask volunteers for the pattern rule. As students respond, model stating and recording the pattern rule as follows: “number of tiles = position number × 5 + 3.”

6. Once students correctly identify the pattern rule, ask the following Key Questions:
   • What changes in each position of the rocket pattern? How does it change? What doesn’t change in the pattern?
   • Which part of the rule is represented by the red tiles? Which part is represented by the yellow tiles? How do you know?

7. Use the last Key Question and the discussion to help solidify students’ understanding that one colour of tile represents the multiplier (the part that grows) and the other colour represents the constant (the part that stays the same). Use the rule to show how the position number is related to the number of tiles. Indicate that 1 × 5 = 5 by pointing to the first position card and then to the five red tiles. Then emphasize that the three yellow tiles represent the constant. Finally, indicate that all the tiles together (the red and the yellow) make eight tiles in total. Do the same for the second and third positions.

Development

1. Distribute position cards and two different colours of tiles to each pair of students. Display the rule “number of tiles = position number × 5 + 2.” Ask students to build the first three positions in the pattern using one colour of tile for the multiplier and the other colour for the constant.

2. Once students have built the pattern, have them do a brief “gallery walk” to see how other pairs built their patterns for the same rule. As students do their gallery walk, look at their patterns and listen to their discussions to determine if they understand the relationship between the pattern rule and the linear growing pattern. Check that students have identified the multiplier and built a pattern that increases by five tiles each time and includes two tiles in a second colour to represent the constant. If the pattern does not seem to be following the pattern rule, encourage students to build the multiplier first (for position 1) using five tiles of one colour and then add two tiles in another colour. Ask students how many tiles there are, in total, at the first position. Do the same for the second and third positions. Then ask students to identify the colour of tile that shows groups of 5, which are growing in each position, and the colour of tile that shows two tiles, which stay the same in each position.

3. Have students return to their partners. Ask them to think of a pattern rule and record it using the composite rule statement: “number of tiles = position number × ____ + ____.” Make sure that the multiplier and the constant are no more than 10, and that students have enough tiles to make patterns with multipliers and constants up to 10.
4. Have students build the first three positions in a pattern that follows their rule. Remind them to use one colour of tile to represent the multiplier of the pattern and another colour of tile to represent the constant. They should also use their position cards to show the positions in their patterns.

5. Have students build the first three positions in another tile pattern that follows the same rule but looks completely different. Once they have built the first three positions, ask them to build a third pattern that follows the same rule but looks completely different. When students are finished, they should have three different linear growing patterns that represent the same rule.

6. Once students have created three different patterns that all follow the same rule, have them do a “gallery walk.” Have one partner take the tour, while the other partner stays to discuss the pair’s pattern with other students. After 5 min, ask the partners to switch roles.

7. While students are doing their gallery walk, gather information about their learning by asking the following Key Questions:
   - What is your rule?
   - How do you know that your pattern follows your rule?
   - How did you figure out how to build a pattern for your rule?
   - How many tiles would there be in the 10th position? How many tiles would there be in the 100th position?
   - If you had [XX] tiles, could you build position [XX] in the pattern? Why or why not?

Consolidation

1. After all the students have had a chance to explore other students’ patterns and answer questions about their own patterns, bring the class together to discuss their experiences with building patterns. Connect this pattern-building activity to the Guess My Rule game by asking the following Key Questions:
   - What did we learn today?
   - How are building patterns and the Guess My Rule game similar? How are they different?

DIFFERENTIATION OPPORTUNITIES

Encourage all students to participate in Lesson 4 because some key features of this lesson, such as using colour scaffolding to distinguish between the two parts of the rule, may help struggling students. Orientation can also be used to distinguish between the two parts of the rule. For example, turning the tiles that represent the constant so they are at a different angle (like a “diamond”) distinguishes the constant and the multiplier through both colour and orientation (Figure 1.10).
The following suggestions may help you vary or differentiate the instruction to ensure that all students successfully participate in the lesson.

<table>
<thead>
<tr>
<th>Situation</th>
<th>Instructional Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students make a pattern but only consider the number of tiles and do not distinguish the multiplication and constant parts of the rule with different colours or orientations.</td>
<td>It is extremely important that students use colour (and possibly orientation) scaffolding when building composite patterns at this stage in the lesson sequence. Encourage students to use two colours of tiles, and scan the room periodically to ensure that all students are doing this. If students do not understand why they are using different colours of tiles, try explaining as follows: “When you use two colours of tiles, you can quickly see which part of your rule involves multiplication and which part involves addition.”</td>
</tr>
</tbody>
</table>
| Students keep saying, “I don’t get it” or “I can’t figure out the rule.” | Teach students specific strategies for determining the rule:  
  • Look at positions 1 and 2. Which colour of tiles never changes in number? This is the constant. Which colour of tiles is increasing in number at each position? This is the multiplier. How many tiles of this colour do you see at position 1? What does this tell you?  
  • Encourage students to build patterns using simple multipliers and simple constants (such as 1, 2, and 3).  
  • Have students use BLM 7: Composite Rule Tiles Chart to record the values for a linear growing pattern. For example, for the pattern rule “number of tiles = position number × 2 + 3,” ask students to look at position 1 and determine the number of red tiles (2) and green tiles (3), and the total number of tiles (5). Then ask students to look at position 2 and determine the number of red tiles (4) and green tiles (3), and the total number of tiles (7). When students have filled in the chart for the first three positions, have them look at the chart and the pattern together. Ask them what numbers increase and what numbers stay the same. Encourage them to make connections among the numbers in the pattern rule, the number of tiles in each position of the pattern, and the values in the chart. |
| Students quickly find this activity too easy and are ready for a challenge. | Encourage students to build only the second and third, or third and fourth, positions in a linear growing pattern. Have them ask their peers to guess what the missing positions would look like. Alternatively, students could cover up positions 1 and 2, and have their peers guess the rule by looking at only positions 3 and 4. |

**ASSESSMENT STRATEGIES**

As the last lesson in Chapter 1, Lesson 4 presents the opportunity to determine whether students are making connections between the pattern rule, the visual representation of the pattern, and the numeric representation. The following summative tasks (assessment of learning) can be used to gauge students’ progress.

1. During the Development section of the lesson, have students record both the patterns they build and the rule for each pattern. You may wish to have students complete and hand in BLM 8: Composite Rule Assessment or record their patterns and rules in their Math Log.
2. Give students a drawing that represents the first three positions in a composite (two-step) pattern and have them determine the rule. You may wish to use BLM 9: Sample Pattern Assessment or create your own pattern. Have students write about the pattern (on the line master or in their Math Log) using the Key Math Language from the lesson. Check to see if students have correctly identified the pattern rule, that their explanation shows logical reasoning, and that they are using the Key Math Language appropriately.

3. You may wish to have students complete the Lesson 4 Exit Task (BLM 10) to assess their overall understanding. Ask students to write the rule, answer the questions, and then hand in their work.

4. Consider using the Culminating Task for Lessons 1 through 4 (BLM XX) and the accompanying scoring guide (BLM XX). Attaching the rubric to students’ work, with highlighted descriptors and comments, can help students recognize what they are currently doing and how they can move forward.

POSSIBLE LESSON EXTENSIONS

Some of our field-test teachers took photographs of students’ patterns and posted them on a bulletin board. Then they had students write down their guesses for the pattern rules on removable notes and attach their guesses to photographs. The class then discussed the guesses, which led to rich math-talk about

- the strategies that students used to guess the pattern rules
- what makes a linear growing pattern
- how the multiplier is related to the growth of a pattern
- how the constant is related to the tiles that stay the same

EXAMPLES OF STUDENTS’ THINKING

By the end of this lesson, most students can make the connection between playing Guess My Rule and building linear growing patterns. One student explained the connection as follows:

“In pattern building, the input part is the position number and the output part is the pattern you build. So, in the middle is the operation or the rule you have to use. In the Guess My Rule game, you have to use the input, apply the rule, and then you get the output. The same is true with patterns: you have to apply the rule to the position number to get your answer and make the pattern.”

It is important that students make the connection between the input and output numbers in Guess My Rule and the position number and number of tiles at each position in a pattern. You can scaffold this connection by creating posters of both ways to represent a linear growing rule, colour-coding them as follows:

- output number = input number \( \times 3 + 2 \)
- number of tiles = position number \( \times 3 + 2 \)
Students Articulate Their Thinking

The following transcript demonstrates the type of thinking that developed in one Grade 6 classroom during Lesson 4. Notice how the teacher spent time supporting students in articulating their thinking processes as they tried to determine the pattern rule for the rocket pattern (Figure 1.10). As well, the teacher emphasized both the numeric and visual aspects of the pattern and the pattern rule.

Teacher: When you were looking at the first position in this pattern, what were you thinking? What was going through your mind?
Realisa: I was figuring out what the second position would look like.
Giveega: I was counting the number of tiles.
Teacher: How many of you were counting the tiles? [Most students raise their hands.]
Suthan: I was trying to figure out which way the pattern was growing.
Teacher: Which way it would grow, yes. You’ve had enough experience to know that this will probably be a pattern that grows. So, then I built the second position. What was going through your mind when you saw that?
Niroshan: I was counting how many more tiles for the second position, and I was looking for a rule that would fit both positions.
Teacher: Why is this important?
Niroshan: Because if it doesn’t work for both positions, then it’s not the rule. So, I was comparing the two positions.
Teacher: Then I built the third position in the pattern. What was going through your mind then?
Nishanthan: I was trying to figure out if my rule was correct.
Teacher: How were you checking?
Nishanthan: I guessed that the rule was “times 5 plus 3,” so first I compared position 1 and position 2 and they both equalled “times 5 plus 3.” Then I checked out position 3, and it equalled “times 5 plus 3.” When I did “times 5 plus 3” to position 3, I got 18.
Teacher: What did you do to position 3?
Nishanthan: I multiplied 3 times 5, which is 15. 15 red tiles plus 3 is 18, which is adding the 3 yellow tiles.
Teacher: What do we know about this rule, just by looking at the different positions?
Myra: It’s growing. Like, the columns are growing.
Kelly: There’s a pattern. It grows by 5 each time.
Rejuana: All the red tiles, at each position, are growing by 5. And the yellow tiles stay the same.
Teacher: Just by looking at the pattern, can you find the rule?
Mayuran: For position 1, there are 5 red tiles. For position 2, there are 10 red tiles. And for position 3, there are 15 red tiles. So, if you divide 15 by 3, you get 5. If you divide 10 by 2, you get 5. And if you divide 5 by 1, you get 5. So it’s “times 5” for the red.
Teacher: What’s multiplied by 5?
Mayuran: Times the position number. And then there’s always 3 tiles, and we can’t find any other way to times it—“times 3” or something like that—so we just need to add 3 at the end. So, the rule is “times 5 plus 3.”
Teacher: So, if we were going to build position 4, what would it look like?
Ally: Four rows of 5 red tiles. And one yellow diamond shape on top and two on the bottom, so it looks like the fourth position in the pattern.
Teacher: What part of the pattern is the times part? What are you looking at when you want to find the times part of the rule?
Kholid: The red part. The red part is the only one that’s growing, right? There are 3 that stay the same, so you add the 3 every time. I’m looking at the part that stays the same in the pattern.
Teacher: Is the position number important when you’re trying to apply the rule? Do you need it?
Tamara: Yes! If you don’t have the position number, you won’t know what to multiply by. You won’t be able to times it by what it’s supposed to be. If you have the rule “times 5 plus 3” . . . well, what times 5? It’s the position number times 5.
Teacher: So, if we were going to state the rule using our math language, what is the rule?
Nicola: “The number of tiles is equal to the position number times 5 plus 3.”

TEACHERS’ EXPERIENCES

The following experiences show different ways of teaching the lesson and may give you some ideas for implementing the lesson in your own classroom.

Ensuring That Students Are Engaged

In her classroom, Adeline noticed several students who were more interested in building structures with the tiles than engaging in pattern building. To get these students engaged in pattern building, Adeline challenged them to build “structures” that followed a composite linear rule. The students fully participated in this challenge (Figures 1.11 and 1.12).

Matching Tile Patterns to the Pattern Rule

Bev found that some of her students loved the visual aspect of the lesson. They tended to build the pattern first and then try to fit a rule to it. She found that they had difficulty finding a rule for the patterns they built because their patterns did not always depict linear growth. Having these students articulate the connection to the rule allowed them to discover the mismatch between their pattern rule and the pattern they created. They could then change the rule, change their pattern, or change both to match.

FIGURE 1.11
Students built stacked tile structures that represented linear growing patterns.

FIGURE 1.12
One student built bridge-type structures that represented linear growing patterns.
## POSITION CARDS

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COMPOSITE RULES TILES CHART

Colour the tiles at the top of the chart to match the multiplier and constant in the pattern. Use the chart to record the position number, number of multiplier tiles, number of constant tiles, and the total number of tiles in each position.

Position Number | × | + | Number of Tiles
---|---|---|---

Rule:
COMPOSITE RULE ASSESSMENT

My rule: _____________________________________________________________

My linear growing pattern: ____________________________________________

I know that my linear growing pattern is correct because
SAMPLE PATTERN ASSESSMENT

Rule:

I know that the rule is correct because

What part of the pattern represents the multiplier in the pattern rule? What part of pattern represents the constant in the pattern rule?

What would position 20 in the pattern look like?

If you had 35 tiles, could you build position 10 in the pattern? Why or why not?

If you had 100 tiles, which position in the pattern could you build?
LESSON 4 EXIT TASK

Rule:

There would be ________ tiles at the 100th position in the pattern.

Describe what the 56th position in the pattern would look like.

Could you build a position in the pattern with exactly 154 tiles? Why or why not?
Dr. Ruth Beatty
Faculty of Education
Lakehead University, Orillia Campus

Dr. Beatty teaches the mathematics methods course for preservice P/J teacher candidates in the Faculty of Education at Lakehead University (Orillia). Her seven-year research study of how children learn complex mathematical concepts, particularly in the domain of “early algebra”, has been supported by a number of federal grants including a three-year Canadian Graduate Scholarship (CGS) awarded by SSHRC.

The third year of her CGS study formed the basis for her doctoral thesis, which was selected as OISE/UT’s Outstanding Thesis of the Year. Her work has resulted in a comprehensive curriculum for students in Grades 4 to 10, as well as a model of teacher professional development that links research and practice. It has also resulted in a collaborative partnership with the Ministry of Education to produce online learning objects for linear relations: www.oame.on.ca/clips. In addition, Dr. Beatty was invited by the International Commission on Mathematics Education (ICMI) to contribute to an international study of technology and mathematics education based on her work utilizing online discourse spaces to support students’ collaborative mathematical problem solving. The scope of Dr. Beatty’s research studies has provided her the opportunity to work with a vast range of students, teachers, consultants, researchers, and math educators in various settings throughout North America, Asia, and Europe.

Dr. Catherine D. Bruce
Associate Professor
School of Education and Professional Learning
Trent University

Dr. Bruce began working at Trent University at the inception of the Bachelor of Education program, where she developed the guiding frameworks for the program philosophy and curricula.

She has been at Trent now for eight years, receiving merit for teaching and research in 2008. Dr. Bruce teaches and coordinates the Mathematics methods courses in the School of Education and professional learning at Trent. She also taught in mathematics classrooms for 14 years, including working as a consultant for the Kawartha Pine Ridge District School Board. Dr. Bruce is passionate about understanding the complexities of how teachers and students learn mathematics. Both her Masters degree and her PhD, from the University of Toronto, inquired into mathematics teaching and related teacher efficacy. In 2008, Dr. Bruce was awarded a SSHRC grant for a three-year study on teacher candidate efficacy in mathematics and building peer coaching relationships between preservice and inservice teachers. Her current research program examines the effects of several different teacher professional development models, teacher and student efficacy, and student achievement in mathematics. Dr. Bruce’s web-based framework for effective interactive whiteboard use and Digital Research Papers are examples of how she attempts to connect research and practice. These can be found at her website: www.tmerc.ca
From Patterns to Algebra is an innovative professional learning resource that identifies how students construct algebraic reasoning beginning with patterning in the early grades.

Written by Dr. Ruth Beatty and Dr. Catherine D. Bruce, two of Canada’s leading math educators, From Patterns to Algebra builds on students’ experience with patterning to construct algebraic reasoning that will support their exploration of linear relationships in later grades.

Key Features:
- Maps out a highly effective lesson sequence for teaching to the big ideas related to linear relationships in patterning and algebra
- Includes math background, ready-to-use lesson plans, and assessment tools to help students develop and build on key concepts over multiple years
- Helps teachers discover ways to link math learning from the elementary to the secondary grades
- Based on Canadian curriculum, Canadian research, and seven years of field-testing with over 1000 students

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