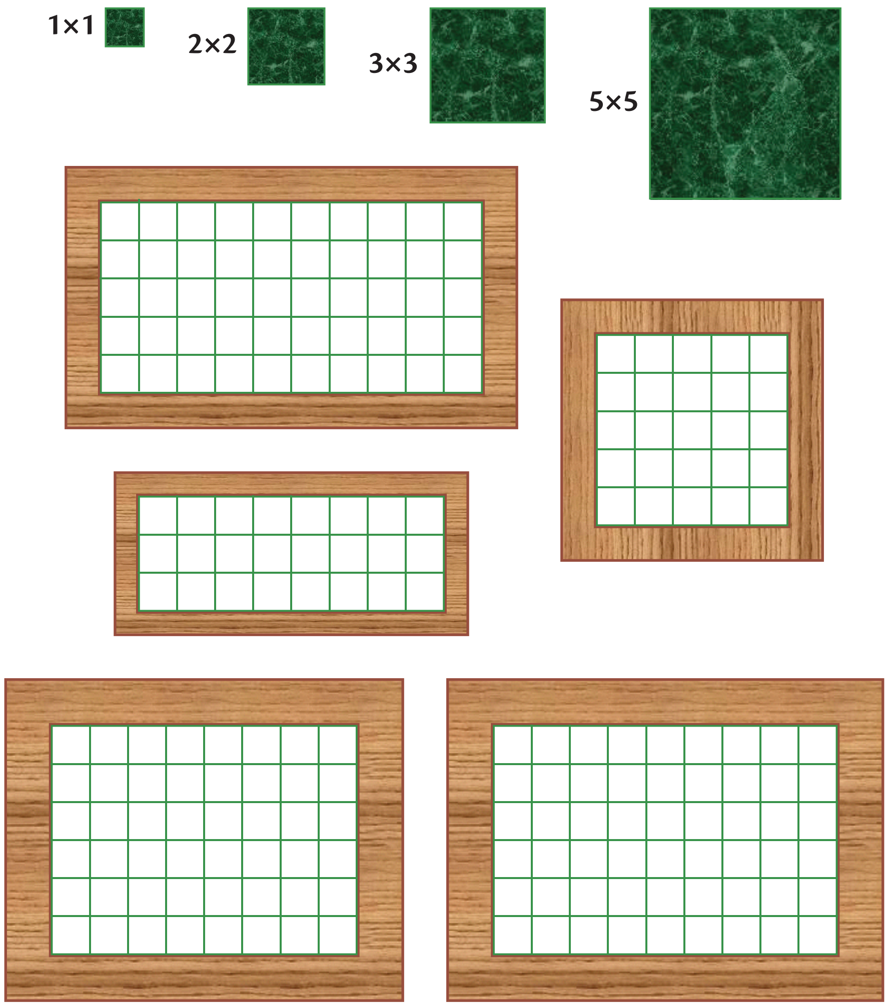
CHAPTER 4

Grades 6–8

COMMON FACTORS



Which tiles can be used (without cutting) to perfectly fit each of these rectangle frames?

TO SIMPLIFY FRACTIONS and to solve certain types of problems, students need to know about common factors. For example, it is useful to know that 3 is a common factor of 6 and 9 when writing the fraction $\frac{6}{9}$ as the simpler equivalent $\frac{2}{3}$. It is also useful to know that 3 is a common factor of 6 and 9 when trying to determine what size tile could be used, without cutting any tiles, to cover an area that is 6 units wide by 9 units long.

Students learn to determine common factors either by guessing and testing or by factoring each number and looking for factors the numbers have in common. Students should be aware that 1 is always a common factor of any two numbers and that any common factor is less than or equal to the lesser of the two numbers being factored. The topic of common factors is addressed in **Common Core State Standards 6.NS**.

The picture provided here is designed to focus students on the notion that square tiles of different sizes can often exactly fit rectangular spaces, but the tile edge lengths are limited to those that are factors of both the length and the width, if the tiles are not to be cut.

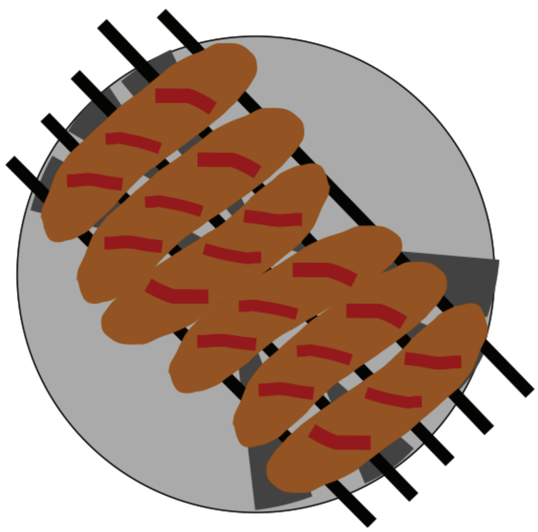
QUESTIONS to supplement the question with the picture and to include in a conversation about the picture include

- ***Which tiles can you use for the 6 × 8 picture? Why?*** [We want students to notice that a tile with a linear dimension of 3 or 5 will not work because neither dimension is a factor of both 6 and 8.]
- ***Which tiles can you use for the 6 × 9 picture? Why?*** [We want students to notice that a tile with a linear dimension of 1 or 3 will work because either dimension is a factor of both numbers, but a 2 × 2 tile will not work because 2 is a factor of 6 but not of 9.]
- ***In which frames will the 5 × 5 tile fit? Explain.*** [We want students to realize that the only possibility is one where both dimensions are multiples of 5, namely 5 × 5 or 5 × 10 in the picture provided.]
- ***Which tile always works? Why?*** [We want students to recognize that 1 is a factor of every whole number.]
- ***In which other sizes of frames, not in the picture, would the 3 × 3 tile fit?*** [We want students to recognize that there is an infinite number of possibilities that can be found by using many different multiples of 3 as lengths and widths.]

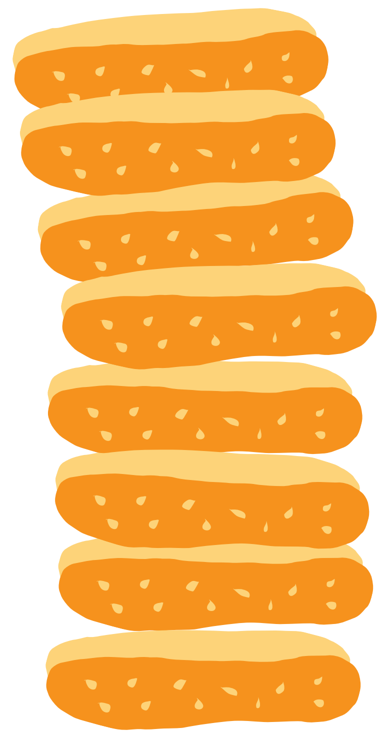
EXTENSION Present the following problem: Classes of two different sizes were each divided into the same number of groups. No students were left out in either class. The group sizes were the same in each class, but different from one class to the next. First ask students how big the classes might have been and how many groups were in each. Then ask how this problem is like the tile problem. The objective is for students to see that this is another common factor problem since both classes had a common group size.

COMMON MULTIPLES

Hot dogs come in packages of 6



Hot dog buns come in packages of 8



How many packages of buns and packages of hot dogs would you need to buy to have a bun for each dog and none left over?

STUDENTS USE COMMON MULTIPLES both to determine common denominators for computations with fractions (e.g., adding, subtracting, or possibly dividing fractions) and to solve certain types of problems.

One way to calculate common multiples is to factor each number into primes and use as few of each required prime as possible. In the hot dog/bun example, students might recognize that they need a common multiple of 6 and 8. They might just know that 24 works, or they might factor hot dogs and buns as follows:

$$6 = 3 \times 2$$
$$8 = 2 \times 2 \times 2$$

Thus, the least common multiple is made up of three 2s (since 8 has three 2s, even though 6 has only one) and one 3 (since 6 has one 3, even though 8 has none). We want students to recognize that each common multiple is at least as great as the greater of the two numbers, and that there is an infinite number of common multiples. The topic of common multiples is first addressed in **Common Core State Standards 6.NS**.

The picture provided here is designed to help students recognize when a common multiple is useful for solving a problem.

QUESTIONS to supplement the question with the picture and to include in a conversation about the picture include

- ***Why can't you buy exactly 12 hot dog buns?*** [We want students to realize that you can only buy multiples of 8 buns in this situation.]
- ***How do you know which numbers of hot dog buns you could buy?***
- ***Why can't you buy exactly 16 hot dogs?*** [We want students to realize that you can only buy multiples of 6 hot dogs in this situation.]
- ***How do you know which numbers of hot dogs you could buy?***
- ***Why are there a lot of possible answers to the question with the picture?*** [We want students to realize that as soon as you calculate one common multiple, you can multiply it by any whole number at all to get another common multiple.]
- ***What do you notice about the numbers of packages of buns and hot dogs?*** [We want students to notice that the number of packages of buns is always a multiple of 3 (to ensure that the total number of buns is a multiple of 6) and that the number of packages of hot dogs is always a multiple of 4 (to ensure that the total number of hot dogs is a multiple of 8).]

EXTENSION Present the following problem: Ellen works at a shelter every 10th day and Lisa works there every 6th day. If they both work there today, how many days will it be before they work together again? Ask how this problem is related to the problem of the hot dogs and buns. Students should realize that it would have to be the 20th, 30th, 40th, . . . day (multiples of 10) for Ellen to be at the shelter and would have to be the 12th, 18th, 24th, . . . day (multiples of 6) for Lisa to be at the shelter. Clearly, a common multiple is required.